

New Design Formula of the Cylindrical Hollow Dielectric Resonator Antenna Using Covariance Matrix Adaptation Evolutionary Strategy

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Abstract:

Cylindrical hollow dielectric resonator (DR) antenna (CHDRA) has been widespread studied not only because it has wideband characteristics but also can be used as a package cover. To reduce the difficulty of antenna design, a new design formula of CHDRA using covariance matrix adaptation evolutionary strategy (CMA-ES) is investigated in this paper. Both dielectric waveguide model (DWN) and transverse transmission line (TTL) technique are used to derive characteristic equations of the cylindrical hollow structure. By solving these equations, we determine the resonate frequency of the CHDRA and get the design formula of the CHDRA using by the method of CMA-ES. The maximum error of the formulas is 2.48% over the range of $9 \leq r \leq 27$, $0.4 \leq h_2/r_2 \leq 2$, $0.3 \leq h_1/h_2 \leq 0.8$, and $0.3 \leq r_1/r_2 \leq 0.8$, where ϵ_r , r_1 , r_2 , h_1 , and h_2 are the dielectric constant, inner and outer radius, inner and outer height of the CHDRA, respectively.

Keywords: Dielectric resonator antenna, Covariance matrix adaptation evolutionary strategy, Hollow, Dielectric waveguide model, Transverse transmission line.

I. INTRODUCTION

Since Dielectric resonator (DR) antenna (DRA) was proposed in 1983 by Long et al. [1], it has been widely studied by many researchers because of its advantages such as small size, low loss, low cost, light weight and ease of excitation. The basic shapes of DRA such as hemispherical, cylindrical or rectangular were very popular in the early days because they are easy to do theoretical analysis [2, 3]. With the development of wideband wireless communication system, people pay more and more attention to wideband DRAs. In order to design wideband DRA, various new structures have been proposed. A rotationally symmetric bowtie DRA with two triangular dielectrics was proposed by Z.-P. Zhao, J. Ren, Y. Liu, and Y-Z Yin from Xidian University for wideband circularly polarized operation [4]. Benefit from this bowtie structure, it has an axial-ratio bandwidth of 27.1%. S. Gao, N-W Liu, G. Fu from the same university presented a circular disk with an annular ring loaded cylindrical DRA with a bandwidth of 62% due to introducing tri-modes simultaneously [5]. Research group from City University of Hong Kong designed new wideband substrate-integrated DRAs by filling nanoparticles into PCB substrate [6,7]. A wideband circularly polarized DRA consisting of a stair-shaped dielectric resonator and an open-ended slot

ground was proposed by L. Lu, Y.-C. Jiao, H. Zhang, R-Q Wang, and T. Li, which has an axial-ratio bandwidth of 46% [8].

Recently hollow DRA becomes very popular in the antenna design, as it not only can enhance the bandwidth compared with the solid one [9,10], but also can be used as a package cover [10-14]. In [15], K. Lu, K.-W. Leung and Y.-M. Pan give a theoretical analysis of rectangular hollow DRA. So, calculating the resonant frequency of rectangular hollow DRA becomes easy. However, no method on the resonant frequency of cylindrical hollow DRA (CHDRA) was developed yet.

In this paper, the dielectric waveguide model (DWN) and transverse transmission line (TTL) technique are used to analysis characteristic equations of the CHDRA. By solving these equations, the resonant frequency of the CHDRA can be determined. Using the results, an engineering formula of calculating the resonant frequency is given by the method of covariance matrix adaptation evolutionary strategy (CMA-ES). This formula will reduce the difficulty of CHDRA design and drive the promotion of DRA to the industrial field.

II. THEORY AND METHODS

Fig. 1. (a) shows a CHDRA resting on top of a ground plane, with its inner and outer radial, hollow cavity height, dielectric superstrate height, and dielectric constant given by r_1, r_2, h_1, h_2 , and ϵ_r , respectively. As shown in Fig. 1. (b), we divide the DRA into two parts: solid region Part A and ring region Part B. By extending the outer radius of the CHDRA to infinitely in the radial direction, a three-layer horizontal slab guide model is obtained shown in Fig. 2(a), where ϵ_{ez} is the effective dielectric constant of Part B. Fig. 2(b) shows its equivalent two-wire transmission line of Fig. 2(a). Referring to the figure, we note that the characteristic impedance Z_{0i} of the i th transmission line section must correspond to the TM_{01} mode for a HEM_{111} mode DRA. It is given by [15]:

$$Z_{0i} = k_{zi} / \omega \epsilon_0 \epsilon_{ri} \quad (1)$$

where, $\epsilon_{r1} = \epsilon_{ez}$, $\epsilon_{r2} = \epsilon_r$, $\epsilon_{r3} = 1$, and k_{zi} is wavenumber the i th transmission line section, in which ϵ_{ez} and k_{z3} can be calculated by

$$\epsilon_{ez} = \frac{k_{z1}^2 - k_{z2}^2}{k_0^2} + \epsilon_r \quad (2)$$

$$k_{z3} = -j \sqrt{k_0^2 (\epsilon_r - 1) - k_{z2}^2} \quad (3)$$

where $k_0 = 2\pi f/c$ is the wavenumber in air, with c and f being the speed of light in vacuum and the resonant frequency of the HEM_{111} mode CHDRA. By using the transverse resonance condition, we can get the following characteristic equation:

$$Z_U + Z_L = 0 \tag{4}$$

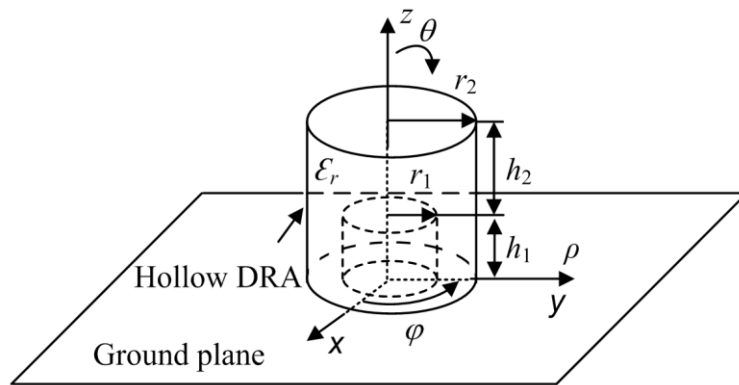
where

$$Z_U = Z_{02} \frac{Z_{03} + jZ_{01} \tan[k_{z2}(h_2)]}{Z_{01} + jZ_{03} \tan[k_{z2}(h_2)]} \tag{5}$$

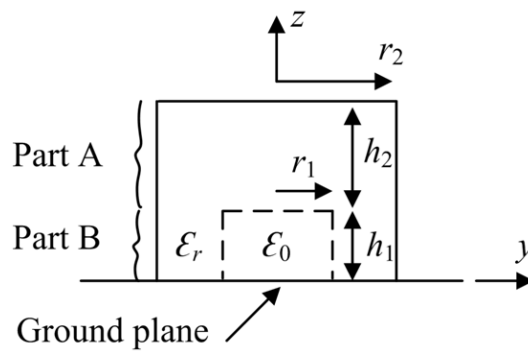
and

$$Z_L = jZ_{01} \tan[k_{z1}h_1] \tag{6}$$

In the following work we need to find the characteristic equations of k_{z1} and k_{z2} , so as to solve the resonance frequency of the CHDRA by using (4).



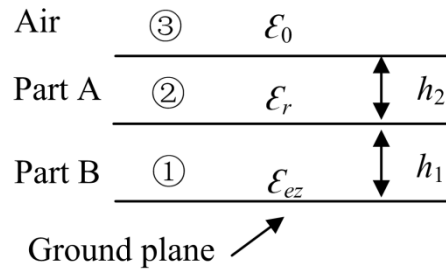
(a)



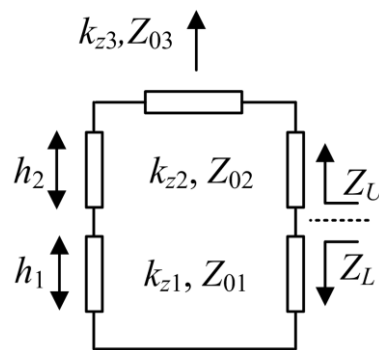
(b)

Fig. 1. Configuration of the CHDRA resting on a ground plane.

(a) Perspective view. (b) Front view (x-z plane).



(a)



(b)

Fig. 2. (a) Horizontal slab guide model of Fig. 1. and (b) its equivalent transmission line networks

First, we assume that Part A is an isolated solid cylindrical DR with radius r_2 , height h_2 , and dielectric constant ϵ_r . Following the analysis of [16], k_{z2} can be obtained as follows:

$$k_{\rho}^2 + k_{z2}^2 = \epsilon_r k_0^2 \tag{7}$$

where k_{ρ} is the dielectric wavenumber along the radial and

$$\left(\frac{1}{k_{\rho}} \frac{J_1'(k_{\rho} r_2)}{J_1(k_{\rho} r_2)} + \frac{1}{k'_{\rho}} \frac{K_1'(k'_{\rho} r_2)}{K_1(k'_{\rho} r_2)} \right) \times \left(\frac{\epsilon_r J_1'(k_{\rho} r_2)}{k_{\rho} J_1(k_{\rho} r_2)} + \frac{1}{k'_{\rho}} \frac{K_1'(k'_{\rho} r_2)}{K_1(k'_{\rho} r_2)} \right) = \frac{(k_{\rho}^2 + k'_{\rho}{}^2)(k_{\rho}^2 + \epsilon_r k'_{\rho}{}^2)}{(k_{\rho} k'_{\rho})^4 r_2^2} \tag{8}$$

Where

$$k'_{\rho} = \sqrt{k_0^2(\epsilon_r - 1) - k_{\rho}^2} \tag{9}$$

is the radial wavenumber outside the dielectric rod.

Next, we assume the Part B is an isolated ring DR [17,18], with inner radius r_1 , outer radius r_2 and height h_1 , and dielectric constant ϵ_r .

An infinite long ring dielectric rod is considered as shown in Fig. 3. We can divide the rod into three regions. Regions 1 and 3 are the air inside and outside the ring, whereas region 2 is the dielectric ring itself. Let $k_{\rho i}$ ($i=1, 2, 3$) and k_z denote the radial and axial wavenumbers of three regions, respectively. Then we can apply the equation (1)-(4) in [17] and the first root of (4) is taken as the value of $k_{\rho i}$ ($i=1, 2, 3$) for the HEM_{111} mode of the ring DR.

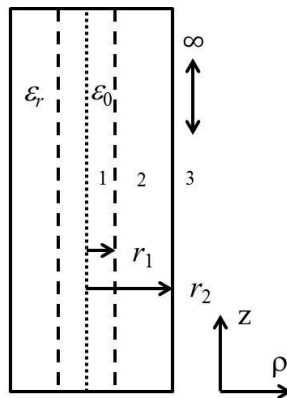


Fig. 3. Vertical tubular guide model for determining radial wavenumber $k_{\rho i}$ ($i=1, 2, 3$).

III. FORMULA

This section is to obtain an engineering formula that determines the operating frequency of the CHDRA of HEM_{111} mode when the dielectric constant and DRA dimensions are given. Let $k_2 = \frac{k_0}{\sqrt{\epsilon_r}}$, it can be express as follows:

$$r_2 k_2 = F_2 \left(\epsilon_r, \frac{h_2}{r_2}, \frac{h_1}{h_2}, \frac{r_1}{r_2} \right) \quad (10)$$

Where F_2 is a function to be determined from the solution of (4) in [17]. For convenience, let $x = \epsilon_r, y = \frac{h_2}{r_2}, z = \frac{h_1}{h_2}$, and $p = \frac{r_1}{r_2}$ then become $t = F_2(x, y, z, p)$. It was observed from the data generated by solving (4) in [17] that F_2 can be approximated as follows:

$$t = \frac{H_S}{x^4} + \sum_{i=1}^4 \frac{1}{x^{4-i}} \left(\frac{A_i}{e^{B_i y + C_i}} + D_i \right) \left(\frac{1}{e^{E_i z + F_i}} \right) \left(\frac{1}{e^{G_i p}} \right) \quad (11)$$

where $F_S, A_i, B_i, C_i, D_i, E_i, F_i$, and, G_i are all constants. The totally number of them is 29. Next, we will determine these parameters using by the method of CMA-ES. To begin, we define a 4×8 matrix M that contains all of the parameters as follows:

$$M = \begin{bmatrix} A_1 & B_1 & C_1 & D_1 & E_1 & F_1 & G_1 & H_5 \\ A_2 & B_2 & C_2 & D_2 & E_2 & F_2 & G_2 & 0 \\ A_3 & B_3 & C_3 & D_3 & E_3 & F_3 & G_3 & 0 \\ A_4 & B_4 & C_4 & D_4 & E_4 & F_4 & G_4 & 0 \end{bmatrix} \quad (12)$$

From M, we can obtain an optimal parameter matrix by minimizing the maximum percentage error. In our study, we repeat the optimization process independently for 100 times and obtain the optimal parameters matrix M_0 as follows:

$$M_0 = \begin{bmatrix} -5.8810 & 1.46339 & -1.08690 & -5.57449 & 5.71275 & 2.27361 & -1.029936 & 8.19663 \\ 7.20216 & 1.11604 & 6.51077 & -0.51108 & -0.74128 & -0.78439 & -3.81169 & 0 \\ 0.70606 & 1.42304 & -0.92597 & 0.81025 & 0.022947 & 1.60377 & -0.14970 & 0 \\ 0.45170 & 0.0029793 & 1.47840 & -0.18099 & -1.35065 & -1.63340 & -3.84002 & 0 \end{bmatrix} \quad (13)$$

It gives the optimal values of $F_S, A_i, B_i, C_i, D_i, E_i, F_i$ and G_i . We can insert these values into (11) to obtain t or $r_2 k_2$. Using the original data as the reference, it was found that the maximum error of the CMA-ES formulas is 2.48% over the range of $9 \leq \square, r \leq 27, 0.4 \leq h_2/r_2 \leq 2, 0.3 \leq h_1/h_2 \leq 0.8, \text{ and } 0.3 \leq r_1/r_2 \leq 0.8$.

VI. CONCLUSION

In this paper, the CMA-ES method is applied to obtain a design formula of the CHDRA. The DWN and TTL are used in the analysis to derive characteristic equations of the hollow structure. By solving the equations, the resonant frequency of the CHDRA is determined. Then the design formula of the CHDRA is given with an error less than 2.48 %, which is good enough for DRA design. This formula can reduce the difficulty of DRA design for engineers and promote the development of DRA technology in the industrial field.

ACKNOWLEDGEMENTS

This work was supported by Tianjin Science and Technology Support Key Project, under Grant 20YFZCGX00700, the Project of Tianjin Enterprise Science and Technology Commissioner, under Grant 21YDTPJC00210, Research Program of Tianjin University of Technology and Education, China, under Grant KJ1914, and Scientific Research Fund of Tianjin Education Commission, under Grant 2021KJ005.

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