

An Improved Inventory Model for Decaying Items with Quadratic Demand and Trade Credits

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ABSTRACT

The present research work shows a deterministic inventory model for decaying products. Most of these works consider the constant parameters with the inventory model. Here we assume the quadratic decreasing demand rate and assume the contractor provides a credit period to the buyer. After the completion of the period, the contractor will charge some interest. Shortages are permitted till the next replenishment. Numerical examples are also used to illustrate the performance of the model. The obtained results are expected to provide improve the accuracy of the objective function for the developed model.

Keywords: Quadratic Demand, Variable Deterioration, Trade credit, Partial Backlogging.

1. INTRODUCTION:

In practice, the contractor frequently offers a waiting period to his customers for settling the amount. Generally, in a business scenario, no interest is charged by the contractor if the outstanding amount is paid within the permissible period. However, if the full dues are unpaid by the end of the permissible delay period, then interest is charged on the outstanding amount. Therefore, it is clear that a customer will hold the sum up to the last minute of the permissible period given by the supplier. First discussed the term “permissible delay in payment” with the inventory model for economic order quantity [7]. After that [1] extended the [7] model to allow for deteriorating items with ordering policies in a specified manner. Some important work done by the researchers for motivating in associated to trade credits such as [4, 5, 28, 41, 59, 62], etc. For some recent and important inventory work done by [9, 16-20, 22, 24, 27, 29]. Demand and price of products are an integral part of any business organization. The current research work focuses on the quadratic demand rate with trade credit policies. Some recent contribution to inventory modelling with [30, 32, 33, 35-39, 50, 52, 58, 66].

Many researchers took the constant demand rates in their inventory models, but in a realistic situation the assumptions of constant demand do not meet up. Assumed the linear trend in demand in various inventory models [8, 11]. But, the demand for some items increases very rapidly which has been investigated by the quadratic function of time [13, 26, 61]. Even if, the shortage occurs, some buyers wait for backorder and some of them would go to buy from another seller. Most researchers took a constant rate of partial backlogging during the stock level when a shortage occurs [12, 42]. Some important work on stock dependent demand done by [10, 14, 15, 34, 46, 53, 54, 63]. In some cases, the customers are conditioned in

such a way that they agree for continuing to wait for a short period of time during the delay in some shipment to get their first choice.

For some fashionable commodities, the length of the waiting period for the next replenishment would be accepted only when the backlogging is fulfilled. Therefore, the researchers have started to take variable backlogging rates and decrease the function of the waiting time. Nowadays many investigators have adopted inventory policies allowing for a time proportional partial backlogging rate. Some inventory models with time relative backlogging rate are discussed by [2, 3, 6, 23, 31, 51, 56, 57, 60, 64, 65]. Development of hybrid price and stock dependent inventory model with perishable products and the advance payment under discount facilities [43]. Some important work related to inflation with the inventory model considers by [21, 25, 40, 45, 47-49, 55]. The discussion with a production inventory model for deteriorating items with ramp type demand allows inflation and shortage under fuzziness [44]. In most of the above-mentioned papers, authors have taken a constant rate of deterioration. In practice, it can be observed that a constant rate of deterioration occurs rarely. Most of the items deteriorate as fast as the time passes. Therefore it is much more realistic to consider deterioration rate as a variable.

2. ASSUMPTIONS AND NOTATIONS:

To develop an inventory model with quadratic demand and partial backlogging, the following notations and assumptions are used

- (i) $N(t)$ be the inventory level at time t .
- (ii) θt is the variable rate of defective units out of on hand inventory at time t , $0 < \theta \ll 1$.
- (iii) C_o is the inventory ordering cost per order.
- (iv) h_c , p_c , s_c and l_c are the holding cost per unit per unit time, unit purchase cost per unit, shortage cost per unit per unit time and lost sales per unit respectively.
- (v) I_e , I_c are the interest earned per unit time and the interest charge which invested in inventory per unit time, $I_r \geq I_e$.
- (vi) M is the permissible delay period for settling accounts in time units.
- (vii) t_0 is the time at which shortage starts and T is the length of replenishment cycle. $0 \leq t_0 \leq T$.
- (viii) The variable demand rate is $d(t) = \alpha - \beta t - \gamma t^2$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$.
- (ix) Unsatisfied demand is backlogged at a rate $\exp(-\delta x)$, where x is the time up to next replenishment. The backlogging parameter δ is a positive constant.
- (x) Replenishment rate is infinite.
- (xi) A single item is considered over the prescribed interval.
- (xii) There is no repair or replenishment of deteriorated units.

3. FORMULATION AND SOLUTION OF THE MODEL:

The depletion of inventory during the interval $(0, t_0)$ is due to joint effect of demand and deterioration of items and the demand is partially backlogged in the interval (t_0, T) . The differential equations describing the inventory level $I(t)$ in the interval $(0, T)$ are given by

$$N'(t) + \theta t I(t) = -d(t), \quad 0 \leq t \leq t_0 \quad \dots (1)$$

$$N'(t) = -d(t) e^{-\delta t}, \quad t_1 \leq t \leq T \quad \dots (2)$$

With the conditions, $N(t_0) = 0$ and $N(0) = S$... (3)

The solutions of equations (1) and (2) can be obtained as

$$N(t) = \alpha(t_0 - t) - \frac{\beta}{2}(t_0^2 - t^2) - \frac{\gamma}{3}(t_0^3 - t^3) + \frac{\alpha\theta}{6}(t_0^3 - 3t_0t^2 + 2t^3) - \frac{\beta\theta}{8}(t_0^2 - t^2)^2 - \frac{\gamma\theta}{30}(3t_0^5 - 5t_0^3t^2 + 2t^5), \quad 0 \leq t \leq t_0 \quad \dots (4)$$

and $N(t) = \left\{ \alpha\delta^2 - \beta\delta(\delta t + 1) - \gamma(\delta^2 t^2 + 2\delta t + 2) \right\} \frac{e^{-\delta t}}{\delta^3} - \left\{ \alpha\delta^2 + \beta\delta(\delta t_1 + 1) + \gamma(\delta^2 t_1^2 + 2\delta t_1 + 2) \right\} \frac{e^{-\delta t_1}}{\delta^3}, \quad t_0 \leq t \leq T \quad \dots (5)$

Also the initial inventory level

$$S = \alpha t_1 - \frac{\beta}{2} t_1^2 + \left(-\gamma + \frac{\alpha\theta}{2} \right) \frac{t_0^3}{3} - \frac{\beta\theta t_0^4}{8} - \frac{\gamma\theta t_0^5}{10} \quad \dots (6)$$

The inventory holding cost (C_H) per cycle is given by

$$C_H = C_h \int_0^{t_1} I(t) dt = C_h \left(\frac{\alpha t_1^2}{2} - \frac{\beta t_1^3}{3} - \frac{\gamma t_1^4}{4} + \frac{\alpha\theta t_1^4}{12} - \frac{\beta\theta t_1^5}{15} - \frac{\gamma\theta t_1^6}{18} \right) \quad \dots (7)$$

The deterioration cost (C_D) per cycle is given by

$$C_D = p_c \left\{ N(0) - \int_0^{t_0} d(t) dt \right\} = p_c \left\{ \frac{\alpha\theta t_0^3}{6} - \frac{\beta\theta t_0^4}{8} - \frac{\gamma\theta t_0^5}{10} \right\} \quad \dots (8)$$

The shortage cost (C_S) per cycle due to backlog is given by

$$C_S = -s_c \int_{t_0}^T N(t) dt = \frac{s_c}{\delta^4} \left\{ \alpha\delta^2 - \beta\delta(2 + \delta T) - \gamma(6 + 4\delta T + \delta^2 T^2) \right\} e^{-\delta T} - \frac{s_c}{\delta^4} \left[\alpha\delta^2 \{1 - \delta(T - t_1)\} - \beta\delta \{ (2 - \delta T)(1 + \delta t_1) + \delta^2 t_1^2 \} - \gamma \{ 6 + 6\delta t_1 + \delta^2 t_1^2 - 2\delta T - \delta^2(T - t_1)(2t_1 + \delta t_1^2) \} \right] e^{-\delta t_1} \quad \dots (9)$$

and Lost sales cost (C_L) per cycle due to lost sales is given by

$$C_L = l_c \int_{t_0}^T (1 - e^{-\delta t})(\alpha - \beta t - \gamma t^2) dt = l_c \left[\alpha(T - t_0) - \frac{\beta}{2}(T^2 - t_0^2) - \frac{\gamma}{3}(T^3 - t_0^3) + \frac{1}{\delta^3} \{ \alpha\delta^2 - \beta\delta(1 + \delta T) \} \right]$$

$$\begin{aligned}
 & -\gamma(2 + 2\delta T + \delta^2 T^2) \} e^{-\delta T} - \frac{1}{\delta^3} \{ \alpha \delta^2 - \beta \delta (1 + \delta t_0) \\
 & - \gamma(2 + 2\delta t_0 + \delta^2 t_0^2) \} e^{-\delta t_0}] \dots(10)
 \end{aligned}$$

Now regarding the permissible delay period M for settling the accounts, there arise two cases $M \leq t_0$ or $M > t_0$.

Case I: $M \leq t_0$ In this case, since the credit period (M) is smaller than the length of period with positive inventory stock of the item, therefore the buyer can use the sale revenue to earn the interest with the rate I_c per unit time. The interest earned (I_E) is given by

$$\begin{aligned}
 I_E &= p_c I_e \int_0^{t_0} (t_0 - t)(\alpha - \beta t - \gamma t^2) dt \\
 &= p_c I_e \left(\frac{\alpha t_1^2}{2} - \frac{\beta t_1^3}{6} - \frac{\gamma t_1^4}{12} \right) \dots (11)
 \end{aligned}$$

and the interest payable (I_p) is given by

$$\begin{aligned}
 I_p &= p_c I_c \int_M^{t_0} N(t) dt \\
 &= p_c I_c \left\{ \frac{\alpha}{2} (t_0 - M)^2 - \frac{\beta}{6} (2t_0^3 - 3t_0^2 M + M^3) - \frac{\gamma}{12} (3t_0^4 - 4t_0^3 M + M^4) \right. \\
 &\quad + \frac{\alpha \theta}{12} (t_0^4 - 2t_0^3 M + 2t_0 M^3 - M^4) - \frac{\beta \theta}{120} (8t_0^5 - 15t_0^4 M + 10t_0^2 M^3 - 3M^5) \\
 &\quad \left. - \frac{\gamma \theta}{90} (5t_0^6 - 9t_0^5 M + 5t_0^3 M^3 - M^6) \right\} \dots (12)
 \end{aligned}$$

Hence, the total average cost of the system is given by

$$TAC_1 = \frac{1}{T} (C_o + C_H + C_D + C_S + C_L + I_p - I_E) = \frac{R_1}{T} \dots (13)$$

To minimize total average cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial TAC_1}{\partial t_0} = 0 \dots (14)$$

and
$$\frac{\partial TAC_1}{\partial T} = 0 \dots (15)$$

provided they satisfy the following conditions

$$\left. \begin{aligned}
 & \frac{\partial^2 TAC_1}{\partial t_0^2} > 0, \frac{\partial^2 TAC_1}{\partial T^2} > 0 \\
 & \left(\frac{\partial^2 TAC_1}{\partial t_0^2} \right) \left(\frac{\partial^2 TAC_1}{\partial T^2} \right) - \left(\frac{\partial^2 TAC_1}{\partial t_0 \partial T} \right)^2 > 0
 \end{aligned} \right\} \dots (16)$$

and

The equations (13) and (14) are equivalent to the following equations respectively

$$\left[h_c \left(t_0 + \frac{\theta t_0^3}{3} \right) + p_c \frac{\theta t_0^2}{2} - s_c (T - t_0) e^{-\delta t_0} + l_c (e^{-\delta t_0} - 1) \right. \\ \left. + p_c I_r \left\{ (t_0 - M) + \frac{\theta}{6} (2t_0^3 - 3t_0^2 M + M^3) \right\} \right] (\alpha - \beta t_0 - \gamma t_0^2) \\ - p_c I_e \left(\alpha t_1 - \frac{\beta t_1^2}{2} - \frac{\gamma t_1^3}{3} \right) = 0 \quad \dots (17)$$

and $\frac{s_c}{\delta^3} \left[\alpha \delta^2 - \beta \delta (1 + \delta t_1) - \gamma (2 + 2\delta t_0 + \delta^2 t_0^2) \right] e^{-\delta t_0} \\ - \left[\alpha \delta^2 - \beta \delta (1 + \delta T) - \gamma (2 + 2\delta T + \delta^2 T^2) \right] e^{-\delta T} \\ + l_c (\alpha - \beta T - \gamma T^2) (1 - e^{-\delta T}) - \frac{R_1}{T} = 0 \quad \dots (18)$

The numerical solution of these equations can be obtained by using some suitable computational numerical method.

Case II: $M > t_0$

In this case, the buyer earns the interest during the period (0, M) and pays no interest. The interest earned (I'_E) in this case is given by

$$I'_E = p_c I_e \left[\int_0^{t_0} (t_0 - t) (\alpha - \beta t - \gamma t^2) dt + (M - t_0) \int_0^{t_0} (\alpha - \beta t - \gamma t^2) dt \right] \\ = p_c I_e \left[M \left(\alpha t_0 - \frac{\beta t_0^2}{2} - \frac{\gamma t_0^3}{3} \right) - \left(\frac{\alpha t_0^2}{2} - \frac{\beta t_0^3}{3} - \frac{\gamma t_0^4}{4} \right) \right] \quad \dots (19)$$

Therefore, the total average cost in this case is given by

$$TAC_2 = \frac{1}{T} (O_c + C_H + C_D + C_S + C_L - I'_E) = \frac{R_2}{T} \quad \dots (20)$$

Now, for minimization of total average cost per unit of time, the optimal values of t_0 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial TAC_2}{\partial t_0} = 0 \quad \dots (21)$$

and $\frac{\partial TAC_2}{\partial T} = 0 \quad \dots (22)$

provided, they satisfy the following conditions

$$\frac{\partial^2 TAC_2}{\partial t_0^2} > 0, \quad \frac{\partial^2 TAC_2}{\partial T^2} > 0$$

and $\left(\frac{\partial^2 TAC_2}{\partial t_0^2} \right) \left(\frac{\partial^2 TAC_2}{\partial T^2} \right) - \left(\frac{\partial^2 TAC_2}{\partial t_0 \partial T} \right)^2 > 0 \quad \dots (23)$

The equations (20) and (21) are equivalent to the following equations respectively

$$\left[h_c \left(t_0 + \frac{\theta t_0^3}{3} \right) + p_c \frac{\theta t_0^2}{2} - s_c (T - t_0) e^{-\delta t_0} + l_c (e^{-\delta t_0} - 1) - p_c I_e (M - t_0) \right] (\alpha - \beta t_1 - \gamma t_1^2) = 0 \quad \dots (24)$$

and

$$\frac{s_c}{\delta^3} \left[\{ \alpha \delta^2 - \beta \delta (1 + \delta t_0) - \gamma (2 + 2\delta t_0 + \delta^2 t_0^2) \} e^{-\delta t_0} - \{ \alpha \delta^2 - \beta \delta (1 + \delta T) - \gamma (2 + 2\delta T + \delta^2 T^2) \} e^{-\delta T} \right] + l_c (1 - e^{-\delta T}) (\alpha - \beta T - \gamma T^2) - \frac{R_2}{T} = 0 \quad \dots (25)$$

These equations can be solved numerically by using any computational method.

4. NUMERICAL ILLUSTRATION:

To illustrate the model numerically the following parameter values are considered.

$$\begin{aligned} \alpha &= 22 \text{ units} , & O_c &= \text{Rs} .250 \text{ per order} \\ \beta &= 14 \text{ units} , & h_c &= \text{Rs} .2. \text{per unit per year} \\ \beta &= 2 \text{ units} , & p_c &= \text{Rs} .9.0 \text{ per unit} \\ \theta &= 0.025 \text{ units} , & s_c &= \text{Rs} .11 .0 \text{ per unit per year} \\ I_c &= \text{Rs} .0.25 \text{ per year} , & l_c &= \text{Rs} .4.0 \text{ per unit} \\ I_e &= \text{Rs} .0.18 \text{ per year} \end{aligned}$$

If $T =$ one year, then for the minimization of total average cost for the optimal value of t_0 can be obtained by solving the following equations

$$\frac{d TAC_1}{d t_0} = 0 \text{ and } \frac{d TAC_2}{d t_0} = 0 \text{ for both cases respectively.}$$

EXAMPLE. CASE I: When $M < t_0$ and $M = 0.45$ year, the optimal policy can be obtained such as $t_0 = 0.841$ year, $S = 20.86$ Units and $TAC_1 = \text{Rs} . 245.60$ per year.

CASE II: When $M > t_0$ and $M = 0.95$ year, the optimal policy can be obtained such as $t_0 = 0.840$ year $S = 20.82$ Units, and $TAC_1 = \text{Rs} . 235.87$ per year.

5. CONCLUSION:

In this paper, we have developed a lot size inventory model for a single deteriorated item with time-varying quadratic demand under the conditions of permissible delay in payments. Shortages are allowed and the backlogging rate is dependent on the waiting time for the next replenishment. We have discussed the effect of variation in various parameters of the system. Another strand of literature has contributed to the advent of methodology. To fine knowledge, the optimal ordering policy has now no longer evolved for a marketplace with a crisis, an opening that this paper attempt to fill.

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