

Research on Ecological Compensation Mechanism of Transboundary Water Pollution Control on Left and Right Banks Based on Differential Game

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Abstract:

Only the contradiction between upstream and downstream is focused in the problem of transboundary water pollution for river basins and the "tragedy of Commons" between left and right banks is ignored. It is the fundamental reason to cause externality that the market mechanism can not play a role in transboundary water pollution control on the left and right banks of the river basin. The external effect of transboundary water pollution control on the left and right banks of the river basin can be internalized by using the ecological compensation mechanism under the leadership of the government, which is an effective means to solve the problem of transboundary water pollution on the left and right banks. Differential game theory is applied in this paper. The effects of ecological compensation mechanism on the government's pollution control efforts for the left and right banks of the transboundary river basin are investigated. The game equilibrium strategies about the government's pollution control efforts on the left and right banks under the three governance modes are analyzed separately. The three governance modes are no ecological compensation, ecological compensation and central government intervention respectively. The three governance modes are compared. The effects of the transboundary ecological compensation mechanism in the river basin are analyzed. The effectiveness of the conclusions is verified by numerical examples. The corresponding countermeasures are proposed. The theory basis for the ecological compensation mechanism of transboundary water pollution control is provided. The results show that when the problem of transboundary water pollution control in the river basin is appeared, it is inadvisable to fight alone for left and right bank governments under non-cooperative governance model. The overall benefits of the basin can be improved effectively under the central government intervention. However, when the right government provides enough funds for the left bank government to carry out ecological compensation, the initiative of the left bank government pollution control will be effectively stimulated and it is promoted to achieve the optimal overall benefits of the river basin.

Keywords: *Transboundary water pollution on the left and right banks, River basin ecological compensation, Government-led, Feedback equilibrium, Differential game.*

I. INTRODUCTION

When the ecological environment is under increasing pressure and cannot bear the social and economic activities of human beings, environmental problems will hinder the development of all countries. As a relatively complete ecological geographical unit, the river basin inevitably overlaps with different boundaries or administrative divisions. With the increasing contradiction between economic development and ecological environment, transboundary pollution problems occur frequently. Water resource has the property of public goods, and its externality, non-competition and non-exclusivity are the fundamental cause of "tragedy of the commons"[1]. Most of the transboundary water pollution problems in river basins focus on the contradiction between upstream and downstream, while ignoring the perspective of left and right banks. Due to the transboundary mobility of water resources, the left and right banks of the river are distributed in different regions, affect each other due to pollutant discharge, and are more likely to fall into the dilemma of "public pond" due to unclear responsibility attribution, which has gradually become the focus of pollution, the focus of conflict and the difficulty of governance[2-4]. Therefore, the pollution of any part of the water resources in the basin may cause damage to the circulation system of the whole basin. The contradiction and conflict between the left and right banks has a long history. The reason is due to the long-term accumulation of ecological environmental pollution and economic development level constraints that have led to high water environmental governance costs. Under the background of the government chasing political achievements and enterprises chasing benefits, the internal power of public land treatment is insufficient[5]. It is difficult to accurately judge the discharge standard and pollution contribution rate of the left and right banks. According to the obtained water quality monitoring data of the provincial boundary section, it is also impossible to define the pollution responsibility[6] and the amount of rewards and punishments[7], which has an impact on the implementation of the external restraint mechanism. In order to make up for the deficiency of transboundary water pollution control on the left and right banks, it is necessary to explore the external mechanism that forms the driving force for the treatment of endogenous pollution on the left and right banks.

There are few studies on the treatment of transboundary water pollution on the left and right banks at home and abroad, mainly focusing on how to divide the responsibility of pollution. The adjustment of concentration accounting method mainly highlights the contribution of provincial pollutant discharge to the water quality of each monitoring section. The representative section judgment method mainly divides the representative sections according to the differences in the setting of sewage outlets and the different discharge characteristics of provinces, selects the representative sections of provinces from the monitoring sections, takes the arithmetic mean of the water quality monitoring values of the representative sections as the pollutant concentration value of the province in the buffer zone, and then by comparing the representative water quality value with the water quality target value, the compliance status of each province can be judged[8]; According to the proportion of annual average emission of pollutants and per capita GDP level, the amount of reward, punishment and compensation is allocated[9]. Due to the high requirements for pollution source data or sewage outlet location, the above methods are rarely used. In practice, some regions try to adopt the method of equal sharing between the two sides. A few scholars have conducted relevant research from the perspective of cooperation and game[10], and from a micro point of view, studied the water resources allocation scheme based on the relative utility function and the

asymmetric Nash bargaining method[11]. However, due to the attention of the left and right banks to individual relative benefits, Negotiation and cooperation often lead to conflicts[12], environmental letters and visits and superior supervision can promote the left and right bank governments to jointly control pollution and eliminate illegal emissions[13]. The above research clarified the necessity of cooperation between the left and right banks and the possibility of joint pollution control. However, the specific regulatory mechanism to promote the control of transboundary water pollution on the left and right banks needs to be further deepened.

Although the research results of ecological compensation for transboundary water pollution control on the left and right banks of the basin are very limited, the research results of ecological compensation for transboundary water pollution in the upstream and downstream of the basin and ecological compensation for transboundary air pollution in adjacent areas are abundant[14,15]. As for the ecological compensation for transboundary water pollution in the upstream and downstream of the basin, scholars mainly focus on the methods of ecological compensation[16-25], compensation standards[26-31], compensation mechanism[32-36], multi-body cooperation and consultation[37-42], the balanced sharing of interests[43], the phenomenon of collusion between government and enterprise[44,45]. As for the ecological compensation of regional transboundary air pollution, scholars mainly focus on the compensation payment principle[46], compensation mechanism[47-53], compensation standard[54,55], compensation mode[56,57], and collaborative governance [58-60]. From the perspective of game methods and policy choices, the above research provides a good idea for solving the problem of ecological compensation for transboundary water pollution on the left and right banks, but the applicability of the above policy mechanism transplantation needs to be further explored.

The transboundary cooperation between the left and right banks of a river basin is based on the dynamic change process of continuous time. Differential game is a dynamic game to study the competition and cooperation of multiple participants in a time continuous system. Therefore, based on previous studies and differential game theory, this paper starts from the new perspective of transboundary water pollution ecological compensation on the left and right banks, cuts into the governance pain points and blind spots, which is a useful supplement to the previous transboundary governance research on the upstream and downstream, constructs a differential game model between the left and right banks of the transboundary basin under the constraint mechanism of ecological compensation, and studies the feedback equilibrium strategy of the left and right banks, It is expected to provide theoretical reference for improving the ecological compensation mechanism for transboundary water pollution control on the left and right banks.

II. MODEL CONSTRUCTION AND BASIC ASSUMPTIONS

This article takes the treatment of transboundary water pollution on the left and right banks as the research object, and studies the impact of the transboundary ecological compensation mechanism of the river basin on the coordinated control of water pollution on the left and right banks. According to the situation that the junction between the left and right banks in the transboundary section of the basin belongs to different regions, the left bank area and the right bank area are set, hereinafter referred to as "left bank" and "right bank".

Hypothesis 1: A river belongs to different jurisdictions, and there are two game parties, the left bank government and the right bank government, both of which are rational subjects.

Hypothesis 2: Under the concept of xi Jinping's ecological civilization thought that "clear water and green mountains are gold and silver mountains", both the left and right bank governments will control the water pollution in the transboundary basin. The cost of pollution control investment by the left and right bank governments in the transboundary basin is related to the investment level of pollution control efforts, and increases with the increase of the investment level of pollution control efforts. Referring to the hypothesis of effort cost in literature [35], the effort cost of the governments on the left and right banks of the basin at time t is expressed as $C_z(t) = \frac{1}{2}k_zE_z^2(t)$ and $C_y(t) = \frac{1}{2}k_yE_y^2(t)$ respectively, where $E_z(t), E_y(t) \geq 0$ represents the investment level of the governments on the left and right banks of the basin at time t ; $k_z, k_y > 0$ represents the cost coefficient of pollution control efforts of the governments on the left and right banks of the basin respectively.

Hypothesis 3: Considering that the pollution control efforts of the government on the left bank will have a direct impact on the water quality and quantity of the whole basin, the pollution control efforts of the government on the right bank are difficult to affect the left bank, so the right bank government faces greater pressure to control pollution. In order to encourage the left bank government to actively control water pollution and ensure the water quality and quantity required for economic development, the right bank government gives the left bank government an ecological compensation subsidy of $\eta(t)$ proportion to the investment cost of the left bank government's pollution control efforts, and $0 < \eta(t) < 1$, that is, the right bank government only gives part of the left bank government's pollution control ecological compensation subsidy.

Hypothesis 4: Pollutant emission reduction of rivers in the transboundary basin is closely related to the investment level of pollution control efforts of the governments of the left and right banks, and it is a dynamic process. Then, the random differential equation of pollutant emission reduction $P(t)$ change in transboundary rivers can be expressed as:

$$\dot{P}(t) = \varphi_z E_z(t) + \varphi_y E_y(t) - \delta P(t), \quad P(0) = P_0 \quad (1)$$

In the formula, P_0 is the emission reduction of the left and right banks of the transboundary basin at the initial time; $P(t)$ represents the emission reduction on the left and right banks of the transboundary basin at time t , φ_z and φ_y are the influence coefficients of the government's pollution control efforts on pollutant emission reduction of the left and right banks respectively, δ represents the attenuation rate of pollutant emission reduction.

Hypothesis 5: Water pollution control in transboundary river basins is an inevitable requirement for clear water and green mountains to be golden mountains and silver mountains, and it is also a necessary meaning to realize coordinated development of economy, society and ecological environment, which can

seek social welfare effects for the transboundary basin. Promoting energy conservation and emission reduction, advocating green development, supporting strategic emerging industries and promoting the transformation of extensive economic development mode are conducive to accelerating economic structure adjustment and are of great significance to the sustainable development of regional economy on the left and right banks. At the same time, the implementation of transboundary water pollution control can not only effectively improve the living environment of residents, but also improve the satisfaction of residents' life; it can also attract foreign investment to change the current regional investment environment and reduce the policy capital cost of regional development. It is assumed that the social welfare effect brought by the government's pollution control and emission reduction on the left and right banks of the transboundary basin is:

$$R(t) = R_0 + \alpha E_z(t) + \beta E_y(t) + \mu P(t) \tag{2}$$

In the formula, R_0 is the transboundary welfare of the basin at the initial time, α and β respectively represent the impact coefficient of pollution control effort investment of the left bank government and the right bank government on social welfare effect, $\alpha, \beta > 0$; μ represents the influence coefficient of pollutant emission reduction on the social welfare effect of transboundary basins, $\mu > 0$.

Hypothesis 6: the governments on the left and right banks of the transboundary basin have the same positive discount rate R , $R > 0$. They are in infinite interval to pursue their social welfare maximization, due to the particularity of their functions, the governments on the left and right banks not only pursue economic benefits, but also consider the social and ecological benefits brought by pollution control, that is, the social welfare impact brought by the efforts of the governments on the left and right banks. The pollution control efforts of the governments on the left and right banks are the source of social welfare effects. Therefore, this paper uses $\omega_z R(t)$ and $\omega_y R(t)$ to represent the social welfare benefits brought by the pollution control efforts of the governments on the left and right banks respectively. In the process of transboundary water pollution control, as the pressure party for pollution control, the right bank government determines the investment in pollution control efforts and the proportion of cost subsidies to the government on the left bank at every moment; as the key party of pollution control, the left bank government determines its own investment in pollution control at every moment. The goals of both are constrained by the change process (1) of the emission reduction of transboundary river pollutants. Therefore, the objective functions of the governments on the left and right banks are respectively for:

$$\max_{E_z} J_z = \int_0^{\infty} e^{-rt} [\omega_z R(t) - (1 - \eta(t)) C_z(t)] dt \tag{3}$$

$$\max_{E_y, \eta} J_y = \int_0^{\infty} e^{-rt} [\omega_y R(t) - C_y(t) - \eta(t) C_z(t)] dt \tag{4}$$

Where, ω_z and ω_y respectively represent the impact coefficients of the social welfare effect of transboundary basins brought about by investment in pollution control efforts on the income of the governments on the left and right banks, $\omega_z, \omega_y > 0$.

This paper defines the differential game between the left bank government and the right bank government. There are three control variables $E_z(t) \geq 0$, $E_y(t) \geq 0$, $0 \leq \eta(t) < 1$ and a state variable $C_z(t) \geq 0$. However, in the left and right bank government non ecological compensation non cooperative governance model, there are only two control variables $E_z(t)$ and $E_y(t)$, and $\eta(t) = 0$. The parameters in the model are assumed to be time independent constants.

III. MODEL ANALYSIS

In order to better analyze the impact of the transboundary ecological compensation mechanism of the basin on the collaborative control of water pollution by the governments on the left and right banks, this section further discusses the optimal pollution control decisions of the left and right bank governments and the overall benefits of the basin under the left and right bank government non cooperative governance model without ecological compensation (A), the left and right bank government with ecological compensation governance model (B) and the left and right bank government collaborative governance model under the intervention of the central government (C). In order to improve readability, superscript i is used to represent three governance modes, $i \in \{A, B, C\}$; The subscript j is used to represent the governance subject of the left and right banks, $j \in \{Z, Y\}$.

3.1 The Left and Right Bank Governments without Ecological Compensation and Non Cooperative Governance Mode A

Under governance mode A, when the government on the right bank does not provide ecological compensation to the government on the left bank, that is, $\eta(t) = 0$. Therefore, the objective functions of the governments on the left and right banks of the transboundary basin are respectively:

$$\max_{E_Z} J_Z^A = \int_0^{\infty} e^{-\rho t} [\omega_z R(t) - C_z(t)] dt \quad (5)$$

$$\max_{E_Y} J_Y^A = \int_0^{\infty} e^{-\rho t} [\omega_y R(t) - C_y(t)] dt \quad (6)$$

Theorem 1: under the non cooperative governance mode without ecological compensation, the static feedback equilibrium strategies of the governments on the left and right banks of the basin are respectively:

$$E_Z^{A*} = \frac{(r\alpha + \delta\alpha + \varphi_z\mu)\omega_z}{(r + \delta)k_z} \quad (7)$$

$$E_Y^{A*} = \frac{(r\beta + \delta\beta + \varphi_y\mu)\omega_y}{(r + \delta)k_y} \quad (8)$$

Prove: In order to obtain the static feedback equilibrium, the continuous bounded differential functions

$V_j(P)$, $j \in (Z, Y)$ exist HJB(Hamilton-Jacobi-Bellman) equations for any $P \geq 0$:

$$rV_Z^A = \underset{E_Z}{\text{mar}} \left\{ \omega_Z [R_0 + \alpha E_Z(t) + \beta E_Y(t) + \mu P(t)] - \frac{1}{2} k_Z E_Z^2(t) + \frac{\partial V_Z^A}{\partial P} [\varphi_Z E_Z(t) + \varphi_Y E_Y(t) - \delta P(t)] \right\} \quad (9)$$

$$rV_Y^A = \underset{E_Y}{\text{mar}} \left\{ \omega_Y [R_0 + \alpha E_Z(t) + \beta E_Y(t) + \mu P(t)] - \frac{1}{2} k_Y E_Y^2(t) + \frac{\partial V_Y^A}{\partial P} [\varphi_Z E_Z(t) + \varphi_Y E_Y(t) - \delta P(t)] \right\} \quad (10)$$

By solving the first-order optimal conditions of pollution control effort investment levels E_Z^A and E_Y^A of the governments on the left and right banks of the basin in equation (9) and (10), letting the first-order partial derivative equal to zero, and the following equation can be obtained:

$$E_Z = \frac{\alpha \omega_Z + \varphi_Z V_Z^{A'}}{k_Z} \quad (11)$$

$$E_Y = \frac{\beta \omega_Y + \varphi_Y V_Y^{A'}}{k_Y} \quad (12)$$

Substituting equation (11) and (12) into HJB equation (9) and (10), it can be obtained:

$$rV_Z^A = \omega_Z R_0 + \frac{(\alpha \omega_Z + \varphi_Z V_Z^{A'})^2}{2k_Z} + \frac{(\beta \omega_Z + \varphi_Y V_Z^{A'}) (\beta \omega_Y + \varphi_Y V_Y^{A'})}{k_Y} + (\mu \omega_Z - \delta V_Z^{A'}) P(t) \quad (13)$$

$$rV_Y^A = \omega_Y R_0 + \frac{(\beta \omega_Y + \varphi_Y V_Y^{A'})^2}{2k_Y} + \frac{(\alpha \omega_Z + \varphi_Z V_Z^{A'}) (\alpha \omega_Y + \varphi_Z V_Y^{A'})}{k_Z} + (\mu \omega_Y - \delta V_Y^{A'}) P(t) \quad (14)$$

According to the structural characteristics of equation (13) and (14), it is assumed that the specific expression of function $V_j(P)$ is:

$$V_Z^A(P) = m_1 P + m_2, \quad V_Y^A(P) = l_1 P + l_2 \quad (15)$$

Among them, m_1 , m_2 and l_1 , l_2 are constants, then there are:

$$V_Z^{A'}(P) = m_1, \quad V_Y^{A'}(P) = l_1 \quad (16)$$

Substituting equation (15) and (16) into equation (13) and (14), it can be obtained:

$$rV_Z^A = \omega_Z R_0 + \frac{(\alpha \omega_Z + \varphi_Z m_1)^2}{2k_Z} + \frac{(\beta \omega_Z + \varphi_Y m_1) (\beta \omega_Y + \varphi_Y l_1)}{k_Y} + (\mu \omega_Z - \delta m_1) P(t) \quad (17)$$

$$rV_Y^A = \omega_Y R_0 + \frac{(\beta\omega_Y + \varphi_Y l_1)^2}{2k_Y} + \frac{(\alpha\omega_Z + \varphi_Z m_1)(\alpha\omega_Y + \varphi_Z l_1)}{k_Z} + (\mu\omega_Y - \delta l_1)P(t) \quad (18)$$

Equation (17) and (18) can satisfy any $P \geq 0$, through calculation, the parameters of the optimal return function can be obtained as follows:

$$\begin{cases} m_1 = \frac{\mu\omega_Z}{r + \delta} \\ m_2 = \frac{\omega_Z R_0}{r} + \frac{(\alpha\omega_Z + \varphi_Z m_1)^2}{2rk_Z} + \frac{(\beta\omega_Z + \varphi_Y m_1)(\beta\omega_Y + \varphi_Y l_1)}{rk_Y} \\ l_1 = \frac{\omega_Y \mu}{r + \delta} \\ l_2 = \frac{\omega_Y R_0}{r} + \frac{(\beta\omega_Y + \varphi_Y l_1)^2}{2rk_Y} + \frac{(\alpha\omega_Z + \varphi_Z m_1)(\alpha\omega_Y + \varphi_Z l_1)}{rk_Z} \end{cases} \quad (19)$$

By substituting m_1, m_2 and l_1, l_2 into equation (13) and (14), it can be obtained that the optimal return function of the government on the left and right banks is:

$$V_Z^{A*}(P) = \frac{\omega_Z R_0}{r} + \frac{\mu\omega_Z}{r + \delta} P + \frac{(\alpha\omega_Z + \varphi_Z m_1)^2}{2rk_Z} + \frac{(\beta\omega_Z + \varphi_Y m_1)(\beta\omega_Y + \varphi_Y l_1)}{rk_Y} \quad (20)$$

$$V_Y^{A*}(P) = \frac{\omega_Y R_0}{r} + \frac{\omega_Y \mu}{r + \delta} P + \frac{(\beta\omega_Y + \varphi_Y l_1)^2}{2rk_Y} + \frac{(\alpha\omega_Z + \varphi_Z m_1)(\alpha\omega_Y + \varphi_Z l_1)}{rk_Z} \quad (21)$$

By substituting equation (19) into equation (11) (12) (15), theorem 1 can be obtained.

Inference 1: Under the left and right bank governments without ecological compensation and non cooperative governance mode, the optimal pollution control effort investment levels $E_Z(t)$ and $E_Y(t)$ of the governments on the left and right banks, the impact coefficients φ_Z and φ_Y of the pollution control effort investment of the left and right governments on pollutant emission reduction, the impact coefficients α and β of the pollution control effort investment of the left and right governments on social welfare effect, and the impact coefficient μ of river basin emission reduction on social welfare effect, And the impact coefficient ω_Z and ω_Y of the social welfare effect of the transboundary basin brought by the investment in pollution control efforts on the government revenue on the left and right banks are positively correlated; they are negatively correlated with the decay rate δ of pollutant emission reduction, the discount rate r , and the government pollution control effort cost coefficients k_Z and k_Y on the left and right banks of the river basin.

Proof: Find the first partial derivatives for $\varphi_Z, \varphi_Y, \alpha, \beta, \mu, \delta, r, \omega_Z, \omega_Y, k_Z$ and k_Y respectively,

$$\text{and get } \frac{\partial E_Z^{A*}}{\partial \varphi_Z} = \frac{\mu \omega_Z}{(r + \delta)k_Z} > 0, \quad \frac{\partial E_Y^{A*}}{\partial \varphi_Y} = \frac{\mu \omega_Y}{(r + \delta)k_Y} > 0, \quad \frac{\partial E_Z^{A*}}{\partial \alpha} = \frac{\omega_Z}{k_Z} > 0, \quad \frac{\partial E_Y^{A*}}{\partial \beta} = \frac{\omega_Y}{k_Y} > 0,$$

$$\frac{\partial E_Z^{A*}}{\partial \mu} = \frac{\varphi \omega_Z}{(r + \delta)k_Z} > 0, \quad \frac{\partial E_Y^{A*}}{\partial \mu} = \frac{\varphi_Z \omega_Y}{(r + \delta)k_Y} > 0, \quad \frac{\partial E_Z^{A*}}{\partial \omega_Z} = \frac{\alpha \delta + \alpha r + \mu \omega_Z}{(r + \delta)k_Z} > 0, \quad \frac{\partial E_Y^{A*}}{\partial \omega_Y} = \frac{\beta \delta + \beta r + \mu \omega_Y}{(r + \delta)k_Y} > 0,$$

$$\frac{\partial E_Z^{A*}}{\partial \delta} = \frac{-(\mu \varphi_Z \omega_Z)}{(r + \delta)^2 k_Z} < 0, \quad \frac{\partial E_Y^{A*}}{\partial \delta} = \frac{-(\mu \varphi_Y \omega_Y)}{(r + \delta)^2 k_Y} < 0, \quad \frac{\partial E_Z^{A*}}{\partial r} = \frac{-(\mu \varphi_Z \omega_Z)}{(r + \delta)^2 k_Z} < 0, \quad \frac{\partial E_Y^{A*}}{\partial r} = \frac{-(\mu \varphi_Y \omega_Y)}{(r + \delta)^2 k_Y} < 0,$$

$$\frac{\partial E_Z^{A*}}{\partial k_Z} = \frac{-[\omega_Z(\alpha \delta + \alpha r + \mu \varphi_Z)]}{(r + \delta)(k_Z)^2} < 0, \quad \frac{\partial E_Y^{A*}}{\partial k_Y} = \frac{-[\omega_Y(\beta \delta + \beta r + \mu \varphi_Y)]}{(r + \delta)(k_Y)^2} < 0.$$

The certificate is completed.

From inference 1, it can be seen that when the left and right bank governments carry out water pollution control, they should comprehensively consider their own pollution control efforts and investment costs, contribution of pollution control efforts to promoting pollutant emission reduction in river basin, the social welfare effect brought by basin emission reduction and its own benefits and other factors, so as to determine the optimal investment level of pollution control efforts. It can be seen that both the left bank government and the right bank government make decisions based on the perspective of maximizing their own benefits, without considering the overall benefits of the basin.

3.2 Left and Right Bank Governments Have Ecological Compensation Governance Mode B

Under governance mode B, the right bank government first determines its own investment in pollution control efforts. At the same time, in order to encourage the left bank government to control pollution, the right bank government will provide a certain proportion of ecological compensation for the pollution control cost of the left bank government. Under the pollution control incentive, the left bank government determines its investment in pollution control efforts. The objective functions of government decisions on the left and right banks of the basin are equation (3) and equation (4) respectively. In the ecological compensation model, the left bank government often determines its own pollution control effort level according to the pollution control effort investment strategy of the right bank government and the proportion of ecological compensation to it, and the left and right bank governments take maximizing their own income as the optimal strategy. From the perspective of long-term dynamics, The investment decision of pollution control efforts between the left and right bank governments constitutes a Stackelberg master-slave game In this case, the objective functions of government decision-making on the left and right banks of the basin are:

$$\max_{E_Z^B} J_Z^B = \int_0^\infty e^{-\eta t} [\omega_Z R(t) - (1 - \eta(t)) C_Z(t)] dt \quad (22)$$

$$\max_{E_Y^B, \eta} J_Y^B = \int_0^\infty e^{-\eta t} [\omega_Y R(t) - C_Y(t) - \eta(t) C_Z(t)] dt \quad (23)$$

Theorem 2: Under the ecological compensation governance mode of the left and right bank governments, the equilibrium strategies of the Stackelberg game of the left and right bank governments are as follows:

$$E_z^{B*} = \frac{(r\alpha + \delta\alpha + \varphi_z\mu)\omega_z}{(1 - \eta(t)^*)(r + \delta)k_z} \quad (24)$$

$$E_y^{B*} = \frac{(r\beta + \delta\beta + \varphi_y\mu)\omega_y}{(r + \delta)k_y}, \quad (25)$$

The optimal cost sharing ratio of the right bank government to the left bank government when the left and right bank governments make cooperative decisions

$$\eta(t) = \frac{\omega_y - \omega_z}{\omega_y + \omega_z}, (\omega_y > \omega_z) \quad (26)$$

Proof: In order to obtain the equilibrium solution of Stackelberg game, the reverse solution method is adopted to solve the decision-making problem of the left bank government first, and the HJB equation of its objective function is:

$$rV_z^B = \max_{E_z} \{ \omega_z [R_0 + \alpha E_z(t) + \beta E_y(t) + \mu P(t)] - (1 - \eta(t)) \frac{1}{2} k_z E_z^2(t) + \frac{\partial V_z^B}{\partial P} [\varphi_z E_z(t) + \varphi_y E_y(t) - \delta P(t)] \} \quad (27)$$

Similarly, the first-order condition of equation (27) for E_z is:

$$E_z = \frac{\alpha\omega_z + \varphi_z V_z^B}{(1 - \eta(t)) k_z}. \quad (28)$$

The rational right bank government can predict that the left bank government can determine its own strategy according to Equation (28), so the HJB equation of the right bank government is:

$$rV_y^B = \max_{E_y, \eta} \{ \omega_y [R_0 + \alpha E_z(t) + \beta E_y(t) + \mu P(t)] - \frac{1}{2} k_y E_y^2(t) - \eta(t) \frac{1}{2} k_z E_z^2(t) + \frac{\partial V_y^B}{\partial P} [\varphi_z E_z(t) + \varphi_y E_y(t) - \delta P(t)] \} \quad (29)$$

Thus, by substituting equation (28) into equation (29) to find the first-order conditions about E_y and $\eta(t)$ B respectively, it can be obtained:

$$E_Y = \frac{\beta\omega_Y + \varphi_Y V_Y^{B^*}}{k_Y} \quad (30)$$

$$\eta(t) = \frac{\alpha\omega_Y + \varphi_Z V_Y^{B^*} - \alpha\omega_Z - \varphi_Z V_Z^{B^*}}{\alpha\omega_Y + \varphi_Z V_Y^{B^*} + \alpha\omega_Z - \varphi_Z V_Z^{B^*}} \quad (31)$$

Similarly, assume that the specific expression of the function $V_j(P)$ is:

$$V_Z^B(P) = f_1 P + f_2, \quad V_Y^B(P) = g_1 P + g_2 \quad (32)$$

Among them, f_1, f_2 and g_1, g_2 are constants, then there are:

$$V_Z^{B^*}(P) = f_1, \quad V_Y^{B^*}(P) = g_1 \quad (33)$$

Substituting equations (28), (30)~ (33) into equations (27) and (29), the parameters of the optimal return function can be obtained:

$$\begin{cases} f_1 = \frac{\mu\omega_Z}{r + \delta} \\ f_2 = \frac{\omega_Z R_0}{r} + \frac{(\alpha\omega_Z + \varphi_Z f_1)^2}{2(1 - \eta(t))rk_Z} + \frac{(\beta\omega_Z + \varphi_Y f_1)(\beta\omega_Y + \varphi_Y g_1)}{rk_Y} \\ g_1 = \frac{\omega_Y \mu}{r + \delta} \\ g_2 = \frac{\omega_Y R_0}{r} + \frac{(\beta\omega_Y + \varphi_Y g_1)^2}{2(1 - \eta(t))rk_Y} + \frac{(\alpha\omega_Z + \varphi_Y f_1)(\alpha\omega_Y + \varphi_Y g_1)}{rk_Y} \end{cases} \quad (34)$$

By substituting f_1, f_2 and g_1, g_2 into equation (27) and (29), it can be obtained that the optimal revenue functions of the left and right bank governments respectively:

$$V_Z^{B^*}(P) = \frac{\omega_Z R_0}{r} + \frac{\mu\omega_Z}{r + \delta} P + \frac{(\alpha\omega_Z + \varphi_Z f_1)^2}{2(1 - \eta(t))rk_Z} + \frac{(\beta\omega_Z + \varphi_Y f_1)(\beta\omega_Y + \varphi_Y g_1)}{rk_Y} \quad (35)$$

$$V_Y^{B^*}(P) = \frac{\omega_Y R_0}{r} + \frac{\omega_Y \mu}{r + \delta} P + \frac{(\beta\omega_Y + \varphi_Y g_1)^2}{2(1 - \eta(t))rk_Y} + \frac{(\alpha\omega_Z + \varphi_Y f_1)(\alpha\omega_Y + \varphi_Y g_1)}{rk_Z} \quad (36)$$

Substituting equations (35)(36) into equations (28)(30)(31), theorem 2 can be obtained.

Inference 2: It can be seen from theorem 2 that only when $a < B$, the right bank government will make ecological compensation to the left bank government. Under the ecological compensation governance mode of the left bank government, the optimal pollution control effort investment level of the left bank

government is positively correlated with φ_z , φ_y , α , β , μ , ω_z and ω_y ; negatively correlated with δ , r , k_z and k_y ; Meanwhile, it is positively correlated with the right bank government of $\eta(t)$.

Prove: $\eta(t) = \frac{\omega_y - \omega_z}{\omega_y + \omega_z}$, it can be seen from this formula that when $\omega_z < \omega_y$, $\eta(t) > 0$;

$$\frac{\partial E_z^{B*}}{\partial \varphi_z} = \frac{[\mu(\omega_y + \omega_z)]}{(r + \delta)2k_z} > 0, \quad \frac{\partial E_y^{B*}}{\partial \varphi_y} = \frac{\mu\omega_y}{(r + \delta)k_y} > 0,$$

$$\frac{\partial E_z^{B*}}{\partial \alpha} = \frac{\omega_y + \omega_z}{2k_z} > 0, \quad \frac{\partial E_y^{B*}}{\partial \beta} = \frac{\omega_y}{k_y} > 0, \quad \frac{\partial E_z^{B*}}{\partial \mu} = \frac{[\varphi_z(\omega_y + \omega_z)]}{2k_z(r + \delta)} > 0, \quad \frac{\partial E_y^{B*}}{\partial \mu} = \frac{\varphi_y\omega_y}{k_y(r + \delta)} > 0,$$

$$\frac{\partial E_z^{B*}}{\partial k_z} = \frac{-[(\omega_y + \omega_z)(\alpha\delta + \alpha r + \mu\varphi_z)]}{2(k_z)k_z^2(r + \delta)} > 0, \quad \frac{\partial E_y^{B*}}{\partial \omega_y} = \frac{(\beta\delta + r\beta + \mu\varphi_y)}{k_y(r + \delta)} > 0, \quad \frac{\partial E_z^{B*}}{\partial \delta} = \frac{-[\mu\varphi_z(\omega_y + \omega_z)]}{(r + \delta)^2 2k_z} > 0,$$

$$\frac{\partial E_y^{B*}}{\partial \delta} = \frac{-(\mu\varphi_z\omega_y)}{(r + \delta)^2 k_y} > 0, \quad \frac{\partial E_z^{B*}}{\partial r} = \frac{-[\mu\varphi_z(\omega_y + \omega_z)]}{(r + \delta)^2 2k_z} > 0, \quad \frac{\partial E_y^{B*}}{\partial r} = \frac{-(\mu\varphi_y\omega_y)}{(r + \delta)^2 k_y} > 0,$$

$$\frac{\partial E_z^{B*}}{\partial k_z} = \frac{-[(\omega_y + \omega_z)(\alpha\delta + \alpha r + \mu\varphi_z)]}{2(k_z)^2(r + \delta)} > 0, \quad \frac{\partial E_y^{B*}}{\partial k_y} = \frac{-[\omega_y(\beta\delta + \beta r + \mu\varphi_y)]}{(k_z)^2(r + \delta)} > 0,$$

$$\frac{\partial E_z^{B*}}{\partial \eta(t)} = \frac{[\omega_z(\alpha\delta + \alpha r + \mu\varphi_z)]}{k_z(r + \delta)(\eta(t) - 1)^2} > 0. \text{ The certificate is completed.}$$

From inference 2, it can be seen that the left and right bank governments still aim at maximizing their own benefits when making decisions, but the ecological compensation of the right bank government can stimulate the pollution control efforts of the left bank government.

3.3 Collaborative Governance Mode of Left and Right Bank Governments under the Intervention of Central Government C

It can be seen from the above that no matter what kind of governance model, the decision-making of the governments on the left and right banks of the transboundary basin is based on the goal of maximizing their own benefits, without considering the goal of maximizing the overall benefits of the basin. In order to facilitate comparative analysis, it is considered that under the intervention of the central government, the governments on the left and right banks of the basin should establish a long-term cooperative relationship and form a binding cooperative governance agreement to maximize the overall benefits of the basin. The objective function in this case is:

$$\max_{E_z, E_y} J_T^C = \int_0^\infty e^{-nt} [(\omega_z + \omega_y)R - C_z - C_y] dt \quad (37)$$

Theorem 3: In the collaborative governance mode under the intervention of the central government, the static feedback equilibrium solutions of the governments on the left and right banks of the transboundary basin are respectively:

$$E_Z^{C*} = \frac{(r\alpha + \delta\alpha + \varphi_Z\mu)(\omega_Z + \omega_Y)}{(r + \delta)k_Z} \quad (38)$$

$$E_Y^{C*} = \frac{(r\beta + \delta\beta + \varphi_Y\mu)(\omega_Z + \omega_Y)}{(r + \delta)k_Y} \quad (39)$$

Prove: There exists an optimal return function V_T^C , with the HJB equation for all $P \geq 0$:

$$rV_T^C = \max_{E_Z, E_Y} \left\{ (\omega_Z + \omega_Y)[R_0 + \alpha E_Z + \beta E_Y + \mu P] - \frac{1}{2}k_Y E_Y^2 - \frac{1}{2}k_Z E_Z^2 + \frac{\partial V_T^C}{\partial P}[\varphi_Z E_Z + \varphi_Y E_Y - \delta P] \right\} \quad (40)$$

Find the maximization first-order condition of the above equations on E_Z and E_Y respectively, it can be obtained:

$$E_Z = \frac{\alpha(\omega_Z + \omega_Y) + \alpha V_T^C(P)}{k_Z} \quad (41)$$

$$E_Y = \frac{\beta(\omega_Z + \omega_Y) + \beta V_T^C(P)}{k_Y} \quad (42)$$

Similarly, assume that the specific expression of the function is

$$V_T^C(P) = q_1 P + p_2 \quad (43)$$

Among them, q_1 and p_2 are constants, then there are

$$V_T^C(P) = q_1 \quad (44)$$

Substituting equations (41)~ (44) into equation (40), the parameter of the optimal return function can be obtained

$$\begin{cases} q_1 = \frac{(\omega_Z + \omega_Y)\mu}{r + \delta} \\ p_2 = \frac{(\omega_Z + \omega_Y)R_0}{r} + \frac{[\beta(\omega_Z + \omega_Y) + \varphi_Y q_1]^2}{2rk_Y} + \frac{[\alpha(\omega_Z + \omega_Y) + \varphi_Z q_1]^2}{2rk_Z} \end{cases} \quad (45)$$

By substituting q_1 and p_2 into equation (40), the optimal return function of the system can be obtained as

$$V_T^{C^*}(P) = \frac{(\omega_z + \omega_y)R_0}{r} + \frac{(\omega_z + \omega_y)\mu}{r + \delta} P + \frac{[\beta(\omega_z + \omega_y) + \varphi_y q_1]^2}{2rk_y} + \frac{[\alpha(\omega_z + \omega_y) + \varphi_z q_1]^2}{2rk_z} \quad (46)$$

The optimal return functions of the left and right bank governments are respectively

$$V_z^C = \omega_z V_T^C \quad (47)$$

$$V_y^C = \omega_y V_T^C \quad (48)$$

Inference 3: It can be seen from Theorem 3 that under the collaborative governance mode with central government intervention, the optimal investment level of pollution control efforts of the left and right banks governments is positively correlated with φ_z , φ_y , α , β and μ , and negatively correlated with δ , r , k_z and k_y . It is worth noting that the optimal pollution control effort investment level of the left and right bank governments is positively correlated with: $(\omega_z + \omega_y)$.

Prove: $\frac{\partial E_z^{C^*}}{\partial \varphi_z} = \frac{\mu(\omega_y + \omega_z)}{(r + \delta)k_z} > 0$, $\frac{\partial E_y^{C^*}}{\partial \varphi_y} = \frac{\mu(\omega_y + \omega_z)}{(r + \delta)k_y} > 0$, $\frac{\partial E_z^{C^*}}{\partial \alpha} = \frac{\omega_y + \omega_z}{k_z} > 0$,

$\frac{\partial E_y^{C^*}}{\partial \beta} = \frac{\omega_y + \omega_z}{k_y} > 0$, $\frac{\partial E_z^{C^*}}{\partial \mu} = \frac{\varphi_z(\omega_y + \omega_z)}{(r + \delta)k_z} > 0$, $\frac{\partial E_y^{C^*}}{\partial \mu} = \frac{\varphi_y(\omega_y + \omega_z)}{(r + \delta)k_y} > 0$, $\frac{\partial E_z^{C^*}}{\partial \delta} = \frac{-[\mu\varphi_z(\omega_y + \omega_z)]}{k_z(r + \delta)^2} < 0$,

$\frac{\partial E_y^{C^*}}{\partial \delta} = \frac{-[\mu\varphi_y(\omega_y + \omega_z)]}{k_y(r + \delta)^2} < 0$,

$\frac{\partial E_z^{C^*}}{\partial r} = \frac{-[\mu\varphi_z(\omega_y + \omega_z)]}{k_z(r + \delta)^2} < 0$, $\frac{\partial E_y^{C^*}}{\partial r} = \frac{-[\mu\varphi_y(\omega_y + \omega_z)]}{k_y(r + \delta)^2} < 0$,

$\frac{\partial E_z^{C^*}}{\partial k_z} = \frac{-[(\omega_y + \omega_z)(\alpha\delta + \alpha r + \mu\varphi_z)]}{(k_z)^2(r + \delta)} < 0$, $\frac{\partial E_y^{C^*}}{\partial k_y} = \frac{-[(\omega_y + \omega_z)(\beta\delta + \beta r + \mu\varphi_y)]}{(k_y)^2(r + \delta)} < 0$,

$\frac{\partial E_z^{C^*}}{\partial(\omega_y + \omega_z)} = \frac{(\alpha\delta + \alpha r + \mu\varphi_z)}{k_z(r + \delta)} > 0$, $\frac{\partial E_y^{C^*}}{\partial(\omega_y + \omega_z)} = \frac{(\beta\delta + \beta r + \mu\varphi_y)}{k_y(r + \delta)} > 0$, It can be seen that $(\omega_y + \omega_z) = \omega_y + \omega_z$, the partial derivative is greater than 0, and both are positively correlated with $(\omega_y + \omega_z)$.

The certificate is completed.

It can be seen from inference 3 that the optimal investment decision of pollution control efforts of the governments on the left and right banks considers not only their respective benefits, but also the overall benefits of the basin.

IV. COMPARATIVE ANALYSIS

By comparing the optimal investment strategy and optimal benefits of the left and right bank

governments under the modes of without ecological compensation, and non cooperation, ecological compensation governance and central government intervention, the following relevant inferences can be obtained.

Inference 4: It can be seen from theorems 1 ~ 3, $E_Z^{C^*} > E_Z^{B^*} > E_Z^{A^*}$, $E_Y^{C^*} > E_Y^{B^*} = E_Y^{A^*}$.

Prove: $E_Y^{B^*} = E_Y^{A^*}$, $E_Z^{B^*} - E_Z^{A^*} = \frac{[(\omega_Y - \omega_Z)(\alpha\delta + \alpha r + \mu\phi z)]}{2kz(r + \delta)}$, It can be seen from the front that $\omega_Y > \omega_Z$,

so $E_Z^{B^*} - E_Z^{A^*} > 0$; $E_Y^{C^*} - E_Y^{B^*} = \frac{[\omega_Z(\beta\delta + \beta r + \mu\phi_Y)]}{k_Y(r + \delta)}$,

$E_Z^{C^*} - E_Z^{B^*} = \frac{[(\omega_Y + \omega_Z)(\alpha\delta + \alpha r + \mu\phi z)]}{2kz(r + \delta)}$; Since all parameters are greater than 0; So

$E_Z^{B^*}$, $E_Z^{C^*}$, $E_Y^{B^*}$, $E_Y^{C^*}$ are all greater than 0; It is concluded that $E_Z^{C^*} > E_Z^{B^*} > E_Z^{A^*}$, $E_Y^{C^*} > E_Y^{B^*} = E_Y^{A^*}$.

The certificate is completed.

It can be seen from inference 4 that the investment level of pollution control efforts of the left bank government is the lowest under the non cooperative mode without ecological compensation. This shows that in the absence of ecological compensation and state subsidies from the right bank government, the mode of consciously controlling water pollution by relying on the left bank government cannot be sustained for a long time. It also confirms that the situation of protecting the left bank and benefiting the right bank will weaken the will of water pollution control in the left bank area. Ecological compensation provided by the right bank government to the left bank government can stimulate the investment level of the left bank government's pollution control efforts without reducing the investment level of the right bank government. This also shows that the right bank government has realized that the investment of the left bank government's pollution control efforts can bring benefits to itself, and is willing to provide certain ecological compensation to the left bank government.

Inference 5: It can be known from equations (1) and (19), $m_1=l_1=f_1=g_1, m_2<f_2, l_2<g_2$, according to equations (15) and (32), $V_Z^{B^*}(P) > V_Z^{A^*}(P)$, $V_Y^{B^*}(P) > V_Y^{A^*}(P)$ can be obtained.

Prove: $V_Z^{B^*} - E_Z^{A^*} = \frac{[\omega_Z(\omega_Y - \omega_Z)(\beta\delta + \beta r + \mu\phi z)^2]}{4kzr(r + \delta)^2}$,

$V_Y^{B^*} - E_Y^{A^*} = \frac{[(\omega_Y)^2(\omega_Y - \omega_Z)(\beta\delta + \beta r + \mu\phi_Y)^2]}{4k_Yr\omega_Z(r + \delta)^2}$, When $\omega_Y > \omega_Z$, $\eta(t) > 0$ is meaningful; According to the

formula, $V_Z^{B^*}$, $E_Z^{A^*}$, $V_Y^{B^*}$, $E_Y^{A^*}$ are always greater than 0, so $V_Z^{B^*} > E_Z^{A^*}$, $V_Y^{B^*} > E_Y^{A^*}$.

The certificate is completed.

Obviously, it can be seen from inference 5 that the overall benefit in the transboundary basin when ecological compensation is implemented is greater than that in the basin without ecological compensation and non cooperation.

From equations (34) and (45), it can be known that $f_1 + g_1 = q_1$, and $g_1 > f_1$, it can be known from equations (26) and (34) that:

$$f_2 + g_2 = \frac{(\omega_z + \omega_y)R_0}{r} + \frac{(\beta\omega_y + g_1\varphi_y)^2(\omega_z + \omega_y)}{4rk_y\omega_z} + \frac{(\alpha\omega_z + f_1\varphi_z)^2(\omega_z + \omega_y)}{4rk_z\omega_z} + \frac{(\beta\omega_z + f_1\varphi_z)(\beta\omega_y + g_1\varphi_y)}{rk_y} \quad (49)$$

Then according to equation (45), inference 6 can be obtained.

Inference 6: When $3\omega_z > \omega_y > \omega_z$, there is $V_T^{C^*}(P) > V_Z^{B^*}(P) + V_Y^{B^*}(P)$; when $3\omega_z < \omega_y$, there is $V_T^{C^*}(P) < V_Z^{B^*}(P) + V_Y^{B^*}(P)$.

Prove: From the previous formula, it can be known that

$$\omega_y > \omega_z, \eta(t) = \frac{\omega_y - \omega_z}{\omega_y + \omega_z}, f_1 = \frac{\mu\omega_z}{r + \delta}, g_1 = \frac{\mu\omega_y}{r + \delta}, f_2 = \frac{R_0\omega_z}{r} + \frac{(\alpha\omega_z + \varphi_z fl)^2}{[2rk_z(1 - \eta(t))]} + \frac{(\beta\omega_z + \varphi_z fl)(\beta\omega_y + \varphi_y gl)}{rk_y},$$

$$g_2 = \frac{R_0\omega_y}{r} + \frac{(\beta\omega_y + \varphi_y gl)^2}{[2rk_y(1 - \eta(t))]} + \frac{(\alpha\omega_z + \varphi_z fl)(\alpha\omega_y + \varphi_z gl)}{rk_z}, q_1 = \frac{\mu(\omega_y + \omega_z)}{r + \delta},$$

$$p_2 = \frac{R_0(\omega_y + \omega_z)}{r} + \frac{[\beta(\omega_y + \omega_z)\varphi_y q_1]^2}{2rk_y} + \frac{[\alpha(\omega_y + \omega_z)\varphi_z q_1]^2}{2rk_z}, p_2 - f_2 - g_2 \text{ is brought into a group of}$$

values for analysis, and the factor is decomposed into $\left[\frac{-4}{45}, \omega_y - 3\omega_z, (\omega_y)^2 + (\omega_z)^2, \frac{1}{\omega_z} \right]$. It can be known

that when $3\omega_z > \omega_y > \omega_z$, there is $V_T^{C^*}(P) > V_Z^{B^*}(P) + V_Y^{B^*}(P)$; when $3\omega_z < \omega_y$, there is $V_T^{C^*}(P) < V_Z^{B^*}(P) + V_Y^{B^*}(P)$.

The certificate is completed.

From inference 6, it can be seen that collaborative governance decisions based on the overall interests of transboundary basins under the intervention of the central government are not always able to maximize the overall interests of the basin. When $3\omega_z < \omega_y$, the overall benefit of the basin under the ecological compensation governance mode of the left and right bank governments is greater than that under the collaborative governance mode of the left and right governments under the intervention of the central government. Specifically, when $3\omega_z > \omega_y > \omega_z$, it can be seen from equation (26) that $0 < \eta(t) < \frac{1}{2}$, that is, when the ecological compensation provided by the right bank government to the left bank government is less than 50% of its cost proportion, the overall benefit of the basin is the largest under the intervention of

the central government; When $3\omega_z < \omega_r$, it can be seen from equation (26) that $\eta(t) > \frac{1}{2}$, that is, when the ecological compensation provided by the right bank government to the left bank government accounts for more than 50% of its cost proportion, the overall benefit of the governments on the left and right banks of the basin under the cooperative governance with ecological compensation is the largest.

To sum up, under the non cooperative governance mode of left and right bank governments without ecological compensation, the optimal decisions of left and right bank governments are based on the maximization of their own benefits. At this time, the self-benefits of left and right bank governments and the overall benefits of the basin are the smallest. Under the intervention of the central government, the overall benefits brought by the collaborative governance decisions of the left and right bank governments are not always optimal. When the ecological compensation provided by the right bank government to the left bank government is sufficient, the enthusiasm of the left bank government to invest in pollution control efforts can be improved and the overall benefits of the basin can be optimized.

V. EXAMPLE ANALYSIS

The optimal decisions and benefits of the left and right bank governments in the transboundary basin depend on the choice of model parameters under three different governance modes. This paper refers to the research results of references [6,37,38], combined with the actual situation, the example parameters are set as shown in TABLE I below.

TABLE I. Specific parameter assignment.

PARAMETER	α	β	μ	k_z	k_y	φ_z	φ_y	δ	r	ω_z	ω_y	P_0
NUMERICAL VALUE	0.5	0.5	0.4	2	2	0.6	0.6	0.1	0.9	0.4	0.5	1

The parameters of each calculation example are substituted into the previous relevant analytical formula to analyze the influence of different parameters on the income and the comparative analysis of the income under different governance modes.

5.1 Impact of Different Parameters on the Overall Income of the Basin

(1) The impact of the cost coefficient of the government's pollution control efforts on the overall benefits of the basin

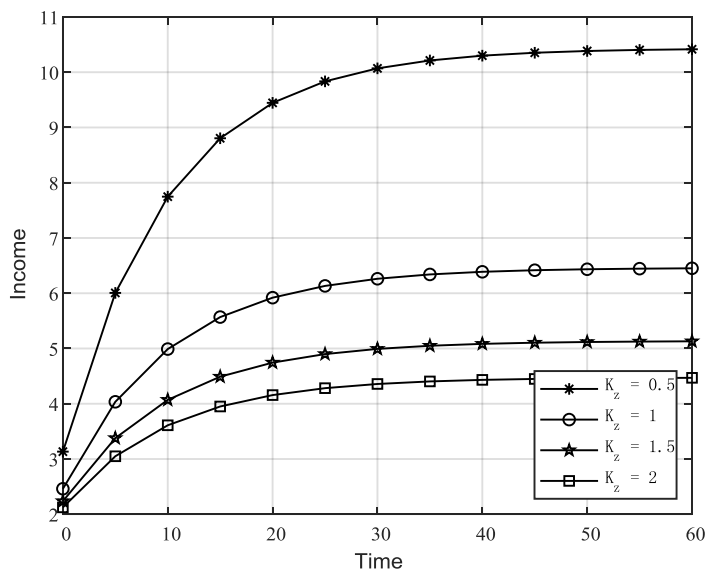


Fig 1: Impact on the overall income of the basin

Through the simulation parameters, it can be seen from Fig 1 that under the premise of ecological compensation by the central government, the impact of the cost coefficient of the left bank government's pollution control efforts on the overall income of the basin is analyzed. It can be seen that the cost coefficient of government pollution control efforts is directly proportional to the investment level of pollution control efforts. With the improvement of the level of pollution control efforts, the cost coefficient of pollution control efforts keeps increasing; with the increase of cost, the overall income of the basin gradually decreases. Therefore, with the increase of pollution control efforts, the overall benefits of the basin gradually decreased. Similarly, it can be concluded that the cost coefficient of the right bank government pollution control efforts has an impact on the overall income of the basin. With the increase of the investment in pollution control efforts, the overall income of the basin gradually decreases. It can be seen that local governments need to be encouraged to actively participate in water pollution control under the premise of certain ecological compensation.

(2) The impact coefficient of government efforts on pollutant emission reduction on the overall income of the basin

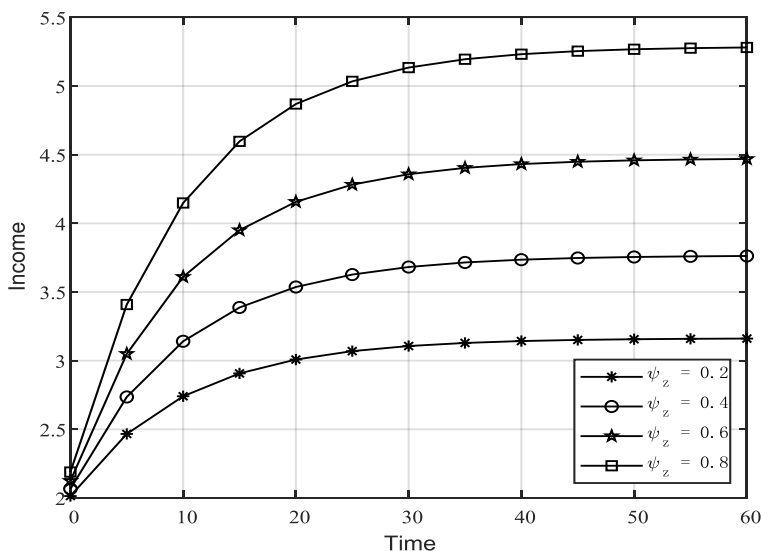


Fig 2: Impact on the overall income of the basin

Through the simulation parameters, it can be seen from Fig 2 that the impact coefficient of the government's pollution control efforts on the pollutant emission reduction on the overall income of the basin is analyzed. It can be seen that with the increase of the impact coefficient, the overall income of the basin increases significantly, and the impact coefficient of the government's pollution control efforts on the pollutant emission reduction is directly proportional to the overall income of the basin. Similarly, the impact coefficient of the right bank government's pollution control efforts invested on pollutant emission reduction increases, and the overall income of the basin gradually increases.

(3) The impact coefficient of the social welfare effect of the government's pollution control efforts on the overall income of the basin

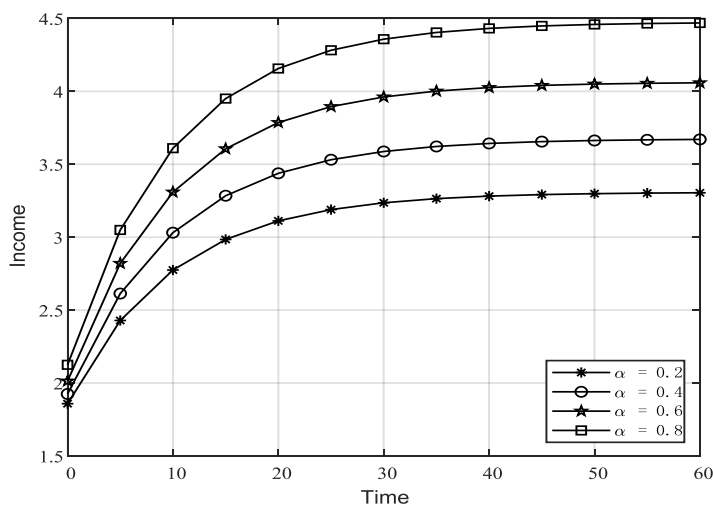


Fig 3: α impact on the overall income of the basin

Through the simulation parameters, it can be seen from Fig 3 that it can be seen from Figure 3 that the

impact coefficient of the government's pollution control effort investment level on the social welfare effect on the overall income of the basin is analyzed. It can be seen that with the increase of the impact coefficient, the change of the overall income of the basin increases significantly, and the impact coefficient of the government's pollution control effort investment level on the social welfare effect is directly proportional to the overall income of the basin. Similarly, the impact coefficient of the right bank government's pollution control efforts on the social welfare effect increases, and the overall income of the basin increases gradually. It can be seen that the more obvious the impact of government's investment in pollution control efforts on social welfare, the better the effect.

5.2 Comparative Analysis of Benefits under Different Governance Modes

By substituting the parameters of each calculation case into the previous relevant analytical equations, and then combining with Equations (15), (32) and (45), we can obtain the comparison diagram of the government income of the left and right banks with or without ecological compensation governance mode, as shown in Figure 4.

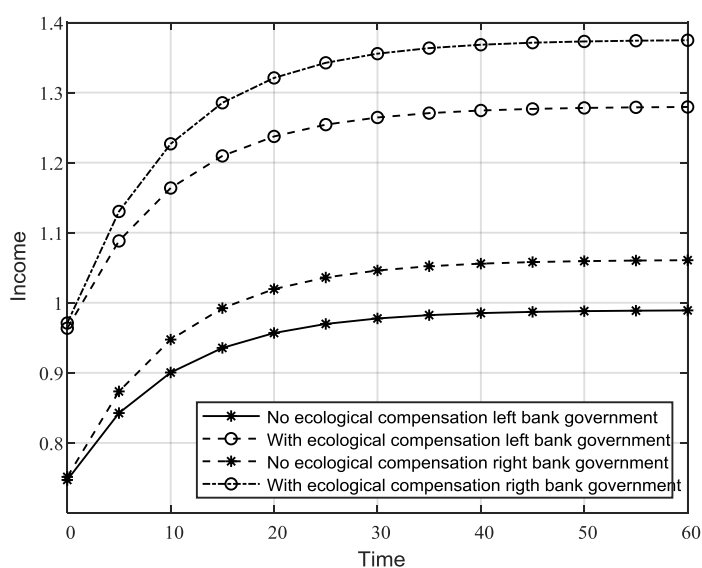


Fig 4: Comparison of the respective benefits of the governments on the left and right banks with or without ecological compensation

As can be seen from Fig 4, cost subsidy can significantly improve the income of the left and right bank governments. Compared with the right bank government, the left bank government has a more obvious income improvement effect under the condition of cost subsidy. This is because the ecological compensation provided by the government of the right bank to the government of the left bank can stimulate the enthusiasm of the government of the left bank in pollution control efforts, so as to reduce the pollution emissions of rivers in the basin and increase the social welfare of the basin, thus increasing the income of the government of the left and right bank. At the same time, because the right bank government provides ecological compensation to the left bank government, the pollution control cost of the left bank government is reduced, but the pollution control cost of the right bank government is increased. Therefore, compared with the right bank government, the left bank government has a more obvious income

improvement effect under the governance mode with ecological compensation.

In addition, when other example parameters remain unchanged, let $\omega_z = 0.2$, $\omega_y = 0.9$, and at this time $3\omega_z < \omega_y$, the total income of the basin under the three governance modes are compared, as shown in Figure 5.

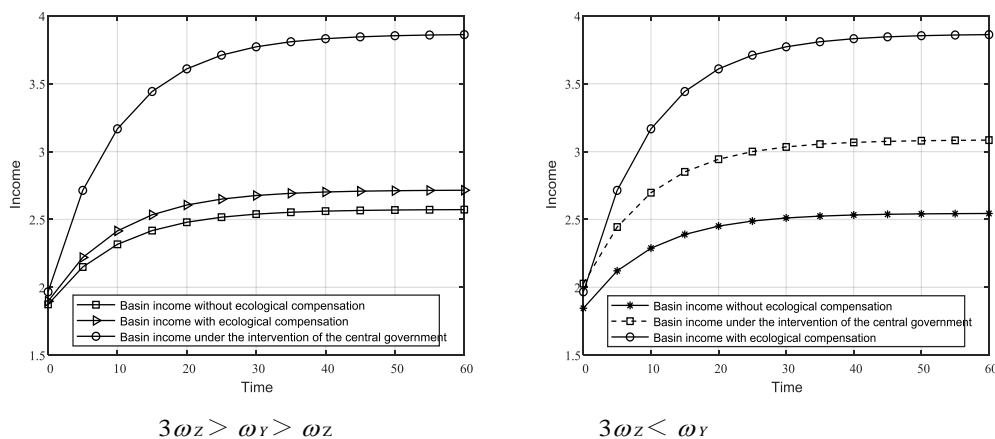


Fig 5: Comparison of the overall benefits of the basin under the three governance modes

As can be seen from Fig 5, when $3\omega_z > \omega_y > \omega_z$, the overall benefits of the left and right bank governments under the intervention of the central government are much greater than those of the left and right bank governments without ecological compensation or with ecological compensation; When $3\omega_z < \omega_y$, the overall benefits of governance decisions made by the governments of the left and right banks with ecological compensation mechanism are much greater than those made by the governments of the left and right banks under the intervention of the central government. The overall benefit of the left and right bank governments in the transboundary basin under the non cooperative governance mode without ecological compensation is much less than that under the other two modes, which once again verifies the results of the inference. Therefore, it can be seen that the governance mode with ecological compensation can promote the improvement of the overall benefits of the basin, and is an effective means to solve the problem of transboundary water pollution control in the basin.

VI. CONCLUSION

Based on the differential game theory, this paper studies the impact of ecological compensation on the control of transboundary water pollution by the governments on the left and right banks. The model results show that when solving the dilemma of water pollution control in transboundary basins, the non cooperative governance model without ecological compensation is absolutely undesirable, and the intervention decision-making model of the central government is not always optimal, and the ecological compensation model of left and right bank cooperation is effective. In particular, the current financial transfer payment will bring great pressure to the national finance, and there will even be insufficient compensation. The ecological compensation mechanism will become a realistic choice to solve the

dilemma of transboundary water pollution control. According to the results of the model analysis, the following aspects should be paid attention to in improving the basin transboundary ecological compensation mechanism.

As a relatively complete eco geographical unit, the decision-making on the left bank often has the most direct impact on the water quality and quantity of the basin. Therefore, the central government and the right bank government should give priority to ensuring the long-term benefits of the left bank government. First, establish a "1 + 2" stepped compensation model of cross basin ecological compensation led by the central government, because the state can coordinate the ecological compensation among provincial functional areas in the region by means of central financial transfer payment, tax adjustment and the establishment of ecological compensation coordination fund; To improve the financial compensation mechanism of the provincial government, the provincial government can coordinate the interests of the local municipal government as an effective supplement to the central ecological compensation, which not only reduces the financial pressure of a single subject to undertake ecological compensation, but also helps to narrow the difference in regional financial revenue, optimize the allocation of resources, and reflect the objectives of fair distribution and stable economic development goals, improve the efficiency of ecological compensation. Local governments can coordinate the interests and ecological compensation within their jurisdiction and the administrative regions below prefectures and cities.

Secondly, establish a binding ecological compensation incentive mechanism between the governments on the left and right banks of the basin. In the process of watershed transboundary ecological compensation, it is often difficult to achieve the optimal and stable equilibrium strategy of the left and right banks if the government only relies on its own consciousness, thus falling into the "prisoner's dilemma" of river basin transboundary. Through the intervention of the central government, establish a reasonable and effective incentive and restraint mechanism. The governments on the left and right banks should sign binding river basin ecological protection compensation agreements, clarify the stakeholders of water pollution control and ecological compensation, standardize the behavior of the subject responsible for compensation, let the defaulting party pay the price, and promote the establishment of river basin transboundary ecological compensation mechanism, so as to optimize the overall income of the basin.

Finally, the practice of river basin transboundary ecological compensation should be combined with the actual situation of the basin and the local conditions to choose the appropriate model of river basin transboundary ecological compensation. For example, the negotiation and transaction between the upstream and downstream of Dawen River Basin, the joint investment between the upstream and downstream governments of Chishui River Basin, the intergovernmental financial transfer payment of Dongjiang River Basin, and the Intergovernmental mandatory withholding based on outbound water quality in Liaohe River Basin and Taihu Lake Basin are all positive practices of the basin's transboundary ecological compensation mechanism at the local level. In addition, due to the lack of special legislation on ecological compensation in China, the local practice of river basin transboundary ecological compensation has fallen into the situation of insufficient legal evidence, and the practical process of local ecological compensation has been affected.

Therefore, it is necessary to speed up the formulation and improvement of laws and regulations on

river basin transboundary ecological compensation, legalize the river basin transboundary ecological compensation mechanism, and promote the river basin transboundary ecological compensation into the track of standardization, institutionalization and legalization.

The research hypothesis of this paper only considers the situation of the government's pollution control efforts on the left and right banks, and does not consider the impact of the participation of enterprises and the public on ecological compensation. In the future, we will further explore the government enterprise strategy interaction under the environment of enterprise and public participation in governance.

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