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Research on Regional Comprehensive Energy Market Equilibrium Based on Complex Network and Evolutionary Game Theory

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Abstract:

All kinds of complex systems in real life can be modeled as complex networks, and the behavior characteristics of complex systems can be studied through the properties of networks. Evolutionary game combines game theory and dynamic evolution process, and emphasizes a dynamic equilibrium. This paper mainly combines the theoretical knowledge of complex network and game theory to transform the regional integrated energy market into a complex network, that is, the power generators, sellers and large users are abstracted into nodes in the network, and there is an interactive relationship between them. The evolutionary game method is used to study the strategy evolution of each market entity. According to the Lyapunov stability theory, the stability of equilibrium point is studied, and the influence of some external factors on the equilibrium point is discussed, and a numerical example is given to verify it.

***Keywords**: Complex network, Evolutionary game theory, Integrated energy market, Equilibrium stability.*

I. INTRODUCTION

With the gradual liberalization of the integrated energy market, the market participants are becoming more complex and diverse[1], and includes both traditional power generation companies and power consumption of large users, also appeared the new power supply in the electricity market entities (mainly sell electricity service providers and load aggregators) and emerging interest subjects (mainly distributed power and electric cars). They all influence the power market, leading to the power transaction and decision-making behavior become more and more complex. However, the traditional optimization theory is usually dominated by single agent decision making, and its theoretical system has been unable to solve the increasingly complex multi-agent behavior decision making At present, the research on integrated energy market mainly focuses on the game bidding of the power generation side or the cooperation between the power generation side and large users, but there is little content about the use of complex network to study the market. Therefore, it is a good method to study integrated energy market through evolutionary game theory based on complex network.

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Evolutionary game theory is undoubtedly an effective tool to study the interactive decision making among participants. Literature[2] combined the classical results of multi-population evolutionary game theory with graph theory and proposed a new mathematical equation of network population evolutionary game dynamics, extending the standard replication dynamic equation to any connected network with a limited number of participants. In addition, the stability of the extended equation is studied when the equilibrium point is reached in evolutionary games. Literature[3] Outlines many deterministic dynamical systems driven by evolutionary game theory, including ordinary differential equations, differential inclusion, difference equations, and reaction-diffusion systems, It is shown that a static, equity-based view cannot in principle forever explain the long-term behavior of participants adjusting their behavior to maximize their benefits. Literature[4] studied Nash equilibrium and Pareto optimal Nash equilibrium of finite horizon non-cooperative dynamic game, studied the existence of understanding, and proved that this game is a potential game. Literature[5] proposed a Bayesian sequential game model to determine the types of attackers, and based on the sequential game tree, the equilibrium strategy of both sides of the game was obtained, and the optimal protection strategy was selected, so as to provide reference for related studies on smart grid information security. Literature[6] proves that Darwinian dynamics based on mutation and selection is the core of mathematical models of adaptation and coevolution of biological populations. The result of evolution is often not a fitness maximized equilibrium, but can include oscillations and chaos. Literature[7] analyzes the equilibrium stability of multi-group asymmetric evolutionary game in today's increasingly complex energy market in various scenarios, and analyzes the influence of some external factors on the equilibrium point. Stackelberg model was established in literature[8] and the real-time pricing strategy interaction among multiple power retailers was analyzed, and the equilibrium solution of the game was obtained. On this basis, the author carries out simulation experiments and the simulation results explain the real-time pricing problem in smart grid well.

In addition, due to the increasing complexity of integrated energy market, more and more market players participate in the game, so complex network can be introduced to study the evolutionary game of integrated energy market based on complex network. Literature[9-13] introduces the definition and properties of random network, small-world network and static model, and explains the generation mode, degree distribution, cluster coefficient and other contents of these models. Literature[14] proposed a community division algorithm based on incremental clustering of core graphs, and gave the calculation formula of similarity between nodes and communities. The existing algorithm was used to divide core communities for the core network composed of a small number of highly numbered nodes. Literature[15] reviews the latest advances in the field of complex networks, focusing on the statistical mechanics of network topology and dynamics, and discussing the main models and analysis tools, including random graphs, small world and scale-free networks, as well as the interaction between topology and network robustness against failure and attack. Literature[16] summarizes concepts such as small-world effect, degree distribution, clustering, network correlation, random graph model, network growth and priority connection model, and dynamic processes occurring on the network. Based on complex network theory, literature[17] proposed the weighted line spacing as the line vulnerability index, defined the sum of current passing through the line as the shortest electrical path between generator nodes and load nodes, and conducted vulnerability analysis on IEEE 39-node system and The Central Sichuan-Chongqing power grid.

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Literature[18] explained the large difference in complex networks due to the length of the path connecting control input to the target node and the redundancy of the shortest length path, and provided an upper bound on the control energy as a function of the path length between the drive node and the target node on the infinite path graph for a single target node.

Based on the above analysis, most of the current researches on integrated energy market only focus on the two-sided game model and its bidding strategy, and there are few researches on the asymmetric game with three groups or more and its stability. However, the comprehensive energy market is increasingly complex, and the multi-subject game is a complex process of dynamic evolution with more complex economic behavior characteristics[19]. Based on this, this paper combines the theory of complex network and evolutionary game, and makes an in-depth discussion and research on the trend of increasingly complex integrated energy market. Firstly, according to the actual physical background of the comprehensive energy market, a three-way game model between the power generator, the seller and the large user is established, and the number of equilibrium points is discussed and the stability conditions of the equilibrium point are studied by using Lyapunov stability theory. On this basis, the stability conditions of equilibrium point are simulated and verified by matlab. Then, based on the research of three-agent two-strategy model, complex network theory is introduced to discuss the 30-node two-strategy game. The adjacency matrix and return matrix of the model are defined, and the differential equation of the strategy evolution of the market entities is established. Finally, the influence of some external parameters on the equilibrium point of the game is discussed.

II. EVOLUTIONARY GAME AND EQUILIBRIUM STABILITY ON COMPLEX NETWORKS

2.1 Theoretical Basis of Complex Networks

In mathematics, networks are represented by graphs. The Vertex of a graph refers to a certain object or object, and the Edge refers to the relationship between things. A more basic graph is an undirected graph, where edges are gndirected. The directed graph, on the other hand, is a directed graph, where edges are directional and there may be edges from A to B but no edges from B to A. The Weight of an edge is the value assigned to each edge, which can be used to represent a number of physical meanings. For example, weights can be used to indicate the distance between two points, the connectivity between network nodes, etc.

Now, remember that the number of nodes in the network is *N*, the set of nodes is $V = \{v_i, i \in N\}$, the set of edges is $E = \{e_{ij}, i \neq j, i, j \in N\}$. Then a graph with a node set *V* and an edge set *E* can be denoted as $G(V, E)$. The structure of the graph can be fully expressed by adjacency matrix, namely:

$$A = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1N} \\ a_{21} & 0 & \cdots & a_{2N} \\ \cdots & \cdots & 0 & \cdots \\ a_{N1} & a_{N2} & \cdots & 0 \end{bmatrix} \tag{1}$$

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Where, a_{ij} represents the connection strength (weight) between nodes. If the structure of the network is undirected, then $a_{ij} = a_{ji}$, otherwise directed. We assume that there are no self loops on the graph, then $a_{ii} = 0(\forall i = 1, \cdots, N)$. If all $a_{ij} = 1$, the network is powerless, otherwise it is weighted.

2.2 Evolutionary Game Theory

Game theory is a mathematical theory and method of competition or struggle phenomena. Game theory considers the prediction and actual behavior of individuals in the game, in order to study their game strategies. The player refers to the decision-making body in a game, while the strategy refers to the individual's response to the different behaviors of other players, such as competition or cooperation. Gain refers to the gain or loss gained by an individual after making certain decisions in the game with other players. Equilibrium is a strategy combination, which is the optimal response of all participants to the strategies of other participants at the same time, that is, the optimal strategy. But equilibrium is not necessarily the best outcome, it's just the most stable outcome, or the most likely outcome.

By applying the knowledge of complex network to evolutionary game, a new research method is obtained. First, it is necessary to establish the corresponding complex network model for specific problems. Among them, each node in the network is the participant of the game, and the edge between nodes represents their game relationship. If player v wants to interact with player w, there will be A directed edge pointing from v to w, and the value $a_{v,w}$ in the adjacent moment A is the weight given by v in the game with w. When $a_{v,w} > 0$ and $a_{w,v} = 0$, there is interaction between v and w, but only v can benefit. $a_{v,w} = 0$ and $a_{w,v} = 0$ indicate that there is no interaction between the two participants.

There is a policy set $S = \{1, \cdots, M\}$, which represents the different behaviors of the participants in the face of the different behaviors of other participants. For example, for populations in nature, different strategies can represent different ways of life, and these different ways of life often bring different benefits. Here, we assume that all participants choose only the pure strategy (single strategy) $s \epsilon S = \{1, \cdots, M\}$, and the probability of choosing this strategy is denoted as x_s. The payoff matrix $B_v \in \mathbb{R}^{M \times N}$ represents the payoff corresponding to different strategies adopted by player v and other players in the game.

$$B_v = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \tag{2}$$

For example, $b_{1,2}$ represents the benefit of participant v when he adopts strategy 1 and other participants adopt strategy 2. As time goes by, the interaction between participants keeps happening, dynamic selection will change the probability of participant v choosing strategy s, which depends on the relationship between the payoff p_s of strategy s and the average payoff $\Psi = \sum_{s=1}^{M} x_s p_s$. Known as the replicator equation[20] :

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$$\frac{dx_s}{dt} = x_s(p_s - \Psi) \quad (3)$$

When $p_s > \Psi$, the probability of choosing strategy x_s will increase as time goes by, indicating that the return of choosing strategy is better than the average value. When $p_s < \Psi$, x_s will decrease, and when $p_s = \Psi$, the system game will be in a stable state. By definition $0 \leq x_s \leq 1$ and $\sum_{s=1}^{M} x_s = 1$. In addition, payoff p_s and mean payoff Ψ depend on the population's strategy distribution $x = [x_1, \cdots, x_M]$.

2.3 Nash Equilibrium and Equilibrium Point

Nash Equilibrium, also known as non-cooperative game Equilibrium, is an important concept in game theory. Nash equilibrium covers a wide range of fields and can be found in economics, biology, artificial intelligence and other disciplines. In the course of a game, if one of the players chooses a certain strategy regardless of the strategy choice of the game object, the strategy is called dominant strategy. If any participant chooses the optimal strategy when the strategies of all other participants are determined, the strategy combination is defined as Nash equilibrium[21].

For example, in the famous Prisoner's Dilemma game, the distribution of earnings is as follows:

TABLE I. Revenue distribution of Prisoner's dilemma game

Prisoner B / **Prisoner A**	**confess**	**reject**
confess	2 years, 2 years	0 year, 5 years
reject	5 years, 0 year	0.5 years, 0.5 years

As can be seen from TABLE I, if the other party confesses, his refusal will be sentenced to 5 years, while if he confesses, he will be sentenced to 2 years. If the other party refuses to confess, both parties will be sentenced to 0.5 years, while if they confess, they will be released directly. Therefore, in this situation of no communication, each prisoner will choose to confess, and confession will bring the greatest benefit. This is the prisoner's dilemma. This result is called the Nash equilibrium of the game.

In this paper, we define the Nash equilibrium of the policy set as follows:

$$\Phi^{NE} = \left\{ \boldsymbol{X}: \ \forall v, \forall x_{v,s} > 0 \ \ p_{v,s}^{G} = p_{v,s'}^{G} \ \ \forall x_{v,s'} > 0 \cap p_{v,s}^{G} \geq p_{v,s'}^{G}, \forall x_{v,s'} = 0 \right\} \quad (4)$$

And strict Nash equilibrium is defined as:

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$$\Phi^{NES} = \left\{\boldsymbol{X}: \ \forall v, \forall x_{v,s} > 0 \ p_{v,s}^{\mathcal{G}} = p_{v,s'}^{\mathcal{G}}, \ \ \forall x_{v,s'} = 0\right\} \tag{5}$$

Nash's theorem[21] proves that all games, $\Phi^{NE} \neq 0$.

According to the definition of Nash equilibrium strategy set Φ^{NE}, for any player, the strategy choice is optimal when reaching the stable state of the game. If there are other different equilibria, the benefits of these two different equilibria must be the same. While Φ^{NES} states that the payoff of the equilibrium satisfying the strict Nash equilibrium must be higher than that of all other equilibria.

2.4 Stability of Equilibrium Point

Before considering the stability of equilibrium point, we need to further derive the replication dynamic equation (3).

Let $s_w \in S$ be the pure strategy adopted by the general actor w, and $e_{s_v} \in \mathbb{R}^M$ represent the M-dimensional column vector with position s_v 1 and position 0 elsewhere. Then, the effective payoff of vertex participant v is expressed as $\varphi_v^G(s_1, \cdots, s_N)$, i.e

$$\varphi_v^G(s_1, \cdots, s_N) = \sum_{w=1}^{N} a_{v,w}\, e_{s_v}^T \boldsymbol{B_v} \boldsymbol{e_{s_w}} = \boldsymbol{e_{s_v}^T} \boldsymbol{B_v} \boldsymbol{k_v}(s_1, \cdots, s_N)$$

The $\boldsymbol{k}_v(s_1, \cdots, s_N) = \sum_{w=1}^{N} a_{v,w}\, \boldsymbol{e}_{\boldsymbol{s_w}}$. Let the state variable of the policy be: $\boldsymbol{x}_v = \left[x_{v,1} \ \ 1 - x_{v,1}\right]^{\mathrm{T}}$, for simplicity, Let $\mathcal{Y}_v = x_{v,1}$ and $\boldsymbol{\mathcal{Y}} = \left[\mathcal{Y}_1 \cdots \mathcal{Y}_{\mathrm{N}}\right]^{\mathrm{T}}$, and the return matrix is as follows:

$$B_v = \begin{bmatrix} b_{v,1} & b_{v,2} \\ b_{v,3} & b_{v,4} \end{bmatrix}$$

Make

$$\sigma_{v,1} = b_{v,1} - b_{v,3}, \sigma_{v,2} = b_{v,4} - b_{v,2}$$

And

$$\sigma_{v,1} = b_{v,1} - b_{v,3}, \sigma_{v,2} = b_{v,4} - b_{v,2}$$

Thus, the replication dynamic equation (3) can be written

$$\dot{\mathcal{Y}}_{\mathrm{v}} = \mathcal{Y}_{\mathrm{v}}(1 - \mathcal{Y}_{\mathrm{v}}) f_v(\boldsymbol{\mathcal{Y}}) \tag{6}$$

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Among them, $f_v(\boldsymbol{\mathcal{Y}}) = \sigma_{v,1}k_{v,1}(\boldsymbol{\mathcal{Y}}) - \sigma_{v,2}k_{v,2}(\boldsymbol{\mathcal{Y}})$.

In order to analyze the stability of (6), we introduce Jacobi matrix:

$$\boldsymbol{J}(\mathcal{Y}^*) = \left(\frac{\partial \dot{\mathcal{Y}}}{\partial \mathcal{Y}}\right)\Big|_{\mathcal{Y}=\mathcal{Y}^*} \tag{7}$$

The stability of equilibrium $\mathcal{Y}^*$ depends on the eigenvalue $\lambda(\boldsymbol{J}(\mathcal{Y}^*))$ of $\boldsymbol{J}(\mathcal{Y}^*)$.

Theorem 1[2]: Let $\boldsymbol{\mathcal{Y}}^* \in \Phi^{P}$, then the following description is equivalent:

(a) $((\sigma_{v,1}k_{v,1}(\boldsymbol{\mathcal{Y}}^*) \leq (\sigma_{v,2}k_{v,2}(\boldsymbol{\mathcal{Y}}^*) \cap \mathcal{Y}_v^* = 0) \cup (\sigma_{v,1}k_{v,1}(\boldsymbol{\mathcal{Y}}^*) \geq \left(\sigma_{v,2}k_{v,2}(\boldsymbol{\mathcal{Y}}^*) \cap \mathcal{Y}_v^* = 1\right))$

(b) $\lambda_v(\boldsymbol{J}(\boldsymbol{\mathcal{Y}}^*)) \leq 0$

(c) $\boldsymbol{\mathcal{Y}}^* \in \phi^{NE}$

Lemma 1[2]: let $\boldsymbol{\mathcal{Y}}^* \in \Phi^{P}$, then the following description is equivalent:

(a) $((\sigma_{v,1}k_{v,1}(\boldsymbol{\mathcal{Y}}^*) < (\sigma_{v,2}k_{v,2}(\boldsymbol{\mathcal{Y}}^*) \cap \mathcal{Y}_v^* = 0) \cup (\sigma_{v,1}k_{v,1}(\boldsymbol{\mathcal{Y}}^*) > \left(\sigma_{v,2}k_{v,2}(\boldsymbol{\mathcal{Y}}^*) \cap \mathcal{Y}_v^* = 1\right))$

(b) $\lambda_v(\boldsymbol{J}(\boldsymbol{\mathcal{Y}}^*)) < 0$

(c) $\boldsymbol{\mathcal{Y}}^* \in \phi^{NES}$

In other words, according to Lyapunov's qualitative theory[22], the stability of the equilibrium point can be obtained by analyzing the positive and negative cases of the eigenvalues of its Jacobi matrix. Therefore, when we find the real part R_{λ_k} of all eigenvalues corresponding to each equilibrium point, we can analyze the stability of the equilibrium point. There are three cases:

(1) If all $R_\lambda < 0$, the equilibrium is asymptotically stable, and the system can form a stable strategy set (refined Nash equilibrium) after a long-term evolutionary game;

(2) if all $R_\lambda > 0$， , then the equilibrium point is not asymptotically stable and does not satisfy the asymptotic stability condition;

(3) As part $R_\lambda > 0$ and part $R_\lambda < 0$, the saddle point is in a critical stable state.

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III. THREE MARKET MAIN BODY DOUBLE STRATEGY GAME

3.1 Model Establishment

Before establishing a specific differential model, the strategies of each market player should be defined:

(1) For (POW), the probability of executing pure strategy S_{po1} is x, indicating that a higher electricity price p_1 is set; Accordingly, the probability of executing strategy S_{po2} is $1-x$, indicating that a lower price p_2 is specified.

(2) For (SEL), the probability of implementing pure strategy S_{se1} is y, indicating that a higher sales price p_3 is set; Accordingly, the probability of executing strategy S_{se2} is $1-y$, indicating the formulation of a lower selling price p_4.

(3) For large users (USE), the probability of implementing the pure strategy S_{us1} is z, indicating that the electricity price p_1 or p_2 of the power producer is directly selected; Accordingly, the probability of implementing strategy S_{us2} is $1-z$, indicating that the electricity price p_3 or p_4 of the e-commerce supplier is selected.

Then, the distribution of benefits is defined, as shown in TABLE II.

TABLE II. Income distribution of the three market players under different strategies

POW and SEL				USE	
				S_{us1}	S_{us2}
POW	S_{po1}	SEL	S_{se1}	(a_1, a_2, a_3)	(b_1, b_2, b_3)
	S_{po2}		S_{se2}	(c_1, c_2, c_3)	(d_1, d_2, d_3)
	S_{po1}	SEL	S_{se1}	(e_1, e_2, e_3)	(f_1, f_2, f_3)
	S_{po2}		S_{se2}	(g_1, g_2, g_3)	(h_1, h_2, h_3)

Based on the defined revenue distribution, it is assumed that the expected revenue of S_{po1}, S_{se1} and S_{us1} are E_{po1}, E_{se1} and E_{us1} respectively for the power generator, e-commerce seller and large user. Accordingly, the returns of strategies S_{po2}, S_{se2} and S_{us2} are E_{po2}, E_{se2} and E_{us2} respectively.

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Meanwhile, it is assumed that the average expected returns of generators, e-commerce sellers and large users are $E_{p_{av}}$, $E_{s_{av}}$ and $E_{u_{av}}$ respectively. Therefore, We can obtain the benefits of adopting different strategies as follows:

$$\begin{cases} E_{po1} = (1-y)[zc_1 + (1-z)d_1] + y[za_1 + (1-z)b_1] \\ E_{po2} = (1-y)[zg_1 + (1-z)h_1] + y[ze_1 + (1-z)f_1] \\ E_{se1} = (1-z)[xb_2 + (1-x)f_2] + z[xa_2 + (1-x)e_2] \\ E_{se2} = (1-z)[xd_2 + (1-x)h_2] + z[xc_2 + (1-x)g_2] \\ E_{us1} = (1-x)[ye_3 + (1-y)g_3] + x[ya_3 + (1-y)c_3] \\ E_{us2} = (1-x)[yf_3 + (1-y)h_3] + x[yb_3 + (1-y)d_3] \end{cases} \tag{8}$$

The average income of the three entities is:

$$\begin{cases} E_{p_{av}} = x \cdot E_{po1} + (1-x) \cdot E_{po2} \\ E_{s_{av}} = y \cdot E_{se1} + (1-y) \cdot E_{se2} \\ E_{u_{av}} = z \cdot E_{us1} + (1-z) \cdot E_{us2} \end{cases} \tag{9}$$

According to the dynamic replication equation (3), it can be obtained:

$$\begin{cases} \dot{x} = x \cdot (E_{po1} - E_{p_{av}}) \\ \dot{y} = y \cdot (E_{se1} - E_{s_{av}}) \\ \dot{z} = z \cdot (E_{us1} - E_{u_{av}}) \end{cases} \tag{10}$$

Substitute equation (8) and equation (9) into equation (10), and then combine the distribution of income to obtain:

$$\begin{cases} \dot{x} = x(1-x) \cdot (a_1 - b_1 - c_1 + d_1 - e_1 + f_1 + g_1 - h_1)yz + (b_1 - d_1 - f_1 + h_1)y + (c_1 - d_1 - g_1 + h_1)z + d_1 - h_1 \\ \dot{y} = y(1-y) \cdot (a_2 - b_2 - c_2 + d_2 - e_2 + f_2 + g_2 - h_2)zx + (e_2 - f_2 - g_2 + h_2)z + (b_2 - d_2 - f_2 + h_2)x + f_2 - h_2 \\ \dot{z} = z(1-z) \cdot (a_3 - b_3 - c_3 + d_3 - e_3 + f_3 + g_3 - h_3)xy + (c_3 - d_3 - g_3 + h_3)x + (e_3 - f_3 - g_3 + h_3)y + g_3 - h_3 \end{cases} \tag{11}$$

3.2 Stability Analysis

On the basis of the established mathematical model (11), we study the steady-state stability of the game. For the convenience of writing, we remember $L_{pow}(x) = \dot{x}$, $L_{sel}(y) = \dot{y}$ and $L_{use}(z) = \dot{z}$. The asymptotic stability of the three-way game system can be judged by the eigenvalues of the corresponding Jacobi matrix (denoted as $\boldsymbol{J}$). In this problem, $\boldsymbol{J} \in \mathbb{R}^{3\times3}$}, where the row elements of the matrix are the partial derivatives of $L_{POW}(x)$, $L_{SEL}(x$ and $L_{USE}(x)$ with respect to x, y and z respectively. $\boldsymbol{J}$ has three eigenvalues, denoted as λ_1, λ_2 and λ_3 respectively. The Jacobi matrix is of the following form:

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$$\boldsymbol{J}=\begin{bmatrix}\frac{\partial L_{pow}(x)}{\partial x} & \frac{\partial L_{pow}(x)}{\partial y} & \frac{\partial L_{pow}(x)}{\partial z}\\ \frac{\partial L_{sel}(y)}{\partial x} & \frac{\partial L_{sel}(y)}{\partial y} & \frac{\partial L_{sel}(y)}{\partial z}\\ \frac{\partial L_{use}(z)}{\partial x} & \frac{\partial L_{use}(z)}{\partial y} & \frac{\partial L_{use}(z)}{\partial y}\end{bmatrix} \tag{12}$$

According to (11), we make $x(1-x)=0$, $y(1-y)=0$ and $z(1-z)=0$, and it is easy to get that there are at least eight equilibrium points in the system of this three-way game and their corresponding eigenvalues, as shown in TABLE III.

TABLE III. Different balance of eigenvalues and the corresponding stability conditions

Eigenvalue\equilibrium point	(0,0,0)	(1,0,0)	(0,1,0)	(0,0,1)	(1,1,0)	(1,0,1)	(0,1,1)	(1,1,1)
λ_1	d_1-h_1	h_1-d_1	b_1-f_1	c_1-g_1	f_1-b_1	g_1-c_1	a_1-e_1	e_1-a_1
λ_2	f_2-h_2	b_2-d_2	h_2-f_2	e_2-g_2	d_2-b_2	a_2-c_2	g_2-e_2	c_2-a_2
λ_3	g_3-h_3	c_3-d_3	e_3-f_3	h_3-g_3	a_3-b_3	d_3-c_3	f_3-e_3	b_3-a_3

An equilibrium point is asymptotically stable ($MESS$) when the eigenvalue corresponding to it is negative or the real part of the complex root is negative (the left half plane of the complex plane), otherwise it is unstable ($ASEP$). At the equilibrium point of gradual stability, the final strategy combination of the three market players is stable. The following eight equilibrium points meet the conditions of asymptotic stability as shown in TABLE IV.

TABLE IV. Equilibrium point and corresponding asymptotic stability conditions

Equilibrium point	MESS condition
(0,0,0)	$d_1<h_1$, $f_2<h_2$, $g_3<h_3$
(1,0,0)	$h_1<d_1$, $b_2<d_2$, $c_3<d_3$
(0,1,0)	$b_1<f_1$, $h_2<f_2$, $e_3<f_3$
(0,0,1)	$c_1<g_1$, $e_2<g_2$, $h_3<g_3$
(1,1,0)	$f_1<b_1$, $d_2<b_2$, $a_3<b_3$
(1,0,1)	$g_1<c_1$, $a_2<c_2$, $d_3<c_3$
(0,1,1)	$a_1<e_1$, $g_2<e_2$, $f_3<e_3$
(1,1,1)	$e_1<a_1$, $c_2<a_2$, $b_3<a_3$

It has been proved in literature[7] that under certain parameters, the system has and only has the above eight equilibrium points. In these 8 cases, after long-term evolution, the system can form $MESS$ and achieve evolutionary stable equilibrium, while the equilibrium formed in any other case is unstable.

IV. MULTI-AGENT DOUBLE-STRATEGY GAME BASED ON COMPLEX NETWORK

4.1 Model Establishment

We consider a double-strategy game between 10 power generators, 10 sellers and 10 large users. The generation providers are divided into two categories: class A (POW_A) and class B (POW_B). Sellers are also divided into two categories, which are classified as class A sellers (SEL_A) and class B sellers (SEL_B) They have their own pricing strategies and corresponding benefits. Define the adjacency matrix A_s of the model and the corresponding payoff matrix of the game. Among them, the revenue matrix includes: two kinds of internal game revenue matrix, two kinds of internal game revenue matrix, the game revenue matrix between the power generators and the sellers, the game revenue matrix between the power generators and large users, and the game revenue matrix between the sellers and large users. The specific definition is as follows.

The adjacency matrix $A_s \in \mathbb{R}^{30\times 30}$ is defined as shown in (13).

$$A_s = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{13}$$

Among them, $A_{ij} \in \mathbb{R}^{10\times 10}$, $(i,j = 1 \cdots 3)$. They represent power generators $A_1 - A_5$ (1-5 columns/row), power generators $B_1 - B_5$ (6-10 columns/row), sellers $A_1 - A_5$ (11-15 columns/row), sellers $B_1 - B_5$ (16-20 columns/row) and large user $1 - 10$ (21-30 columns/row), respectively. The value of the corresponding position represents the importance (weight) of both sides to the game.

The respective returns matrix of different market players in the game is shown in TABLE V:

TABLE V. Definition of income matrix of both parties in game of different entities

payoff matrix	former payoff	latter payoff
$POW_A - POW_B$	B_{10}	B_{11}
$SEL_A - SEL_B$	B_{20}	B_{22}
$POW -$ SEL	B_{12}	B_{21}
$POW -$ USE	B_{13}	B_{31}
$SEL - USE$	B_{23}	B_{32}
$POW_A - POW_A$	E	E
$POW_B - POW_B$	E	E
$SEL_A - SEL_A$	E	E
$SEL_B - SEL_B$	E	E
$USE - USE$	E	E

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Since it is a two-strategy game, each payoff matrix is 2×2.

Here, the game model of generator A_1 is shown in Fig 1. Since generator A_1 only involves B_{10}, B_{12} and B_{13} when participating in the game (the identity matrix E can be ignored), only the definitions of these income matrices are given:

$$B_{10} = \begin{bmatrix} b_{11} & b_{12} \\ b_{13} & b_{14} \end{bmatrix},\ B_{12} = \begin{bmatrix} b_{21} & b_{22} \\ b_{23} & b_{24} \end{bmatrix},\ B_{13} = \begin{bmatrix} b_{31} & b_{32} \\ b_{33} & b_{34} \end{bmatrix}$$

And, $\sigma_{11} = b_{13} - b_{11}$, $\sigma_{12} = b_{12} - b_{14}$, $\sigma_{21} = b_{23} - b_{21}$, $\sigma_{22} = b_{22} - b_{24}$, $\sigma_{31} = b_{33} - b_{31}$, $\sigma_{32} = b_{32} - b_{34}$, according to the adjacency matrix A_s defined above and Equation (6), we can establish the strategy evolution differential equation model of the generator, namely:

$$\dot{x_1} = x_1(1 - x_1) f_v(\boldsymbol{x})$$

$$f_v(\boldsymbol{x}) = [\sigma_{11} \cdot x \cdot \sum_{j=6}^{10} A(1,j) + \sigma_{21} \cdot y \cdot \sum_{j=11}^{20} A(1,j) + \sigma_{31} \cdot z \cdot \sum_{j=21}^{30} A(1,j)] - [\sigma_{12} \cdot (1 - x) \cdot \sum_{j=6}^{10} A(1,j) + \sigma_{22} \cdot (1 - y) \cdot \sum_{j=11}^{20} A(1,j) + \sigma_{32} \cdot (1 - z) \cdot \sum_{j=21}^{30} A(1,j)]$$

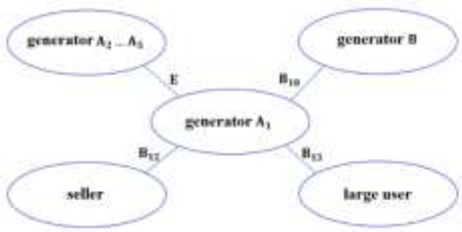

Fig 1: Game situation of generator A1

V. EXAMPLE SIMULATION

In all the simulation contents in this section, the step size $h = 0.1$ of the given differential iteration format, and the uniform convergence judgment condition (equilibrium) : the average error of the two iteration strategies of all market players is less than 10^{-5} and the number of iterations k is less than 10000.

5.1 Simulation of Three-Agent Two-Strategy Game Equilibrium Point

According to the established mathematical model (11), the differential iteration equation corresponding to the strategy of each market entity is established, and the stability conditions in 4 are simulated and verified by matlab. Fig 2 and 3 verify the stability conditions of equilibrium points (0,0,0) and (1,0,1) respectively, and simulate the two equilibrium points twice.

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The two simulation results of each equilibrium point are shown in (a) and (b) of the corresponding figure respectively. Each simulation takes the random initial value condition, that is, the probability of the generator, the seller and the large user choosing their respective strategy 1 at the initial moment. According to the simulation results, it can be found that in Fig 2, when the power generator, seller and large user reach the game equilibrium state, they all choose their own strategy 2, that is, reach the equilibrium point (0,0,0). In Fig 3, power generator and large users choose strategy 1, while sellers choose strategy 2, reaching the equilibrium point (1,0,1). That is to say, under the condition of given the asymptotic stability of the equilibrium point, power generators, sellers and large user of strategy selection results and the theoretical research results conform to the well, and it has nothing to do with the strategy choosing of initial time (excluding the initial moments of the probability of selection strategy 1 to 0 or 1), the nature of the equilibrium is asymptotically stable.

In order to prevent accidental errors, 500 simulations were conducted for each case. Except for extreme conditions (all 0 or all 1), all the results were consistent with the theory, and these results could be reflected in the study in the next section.

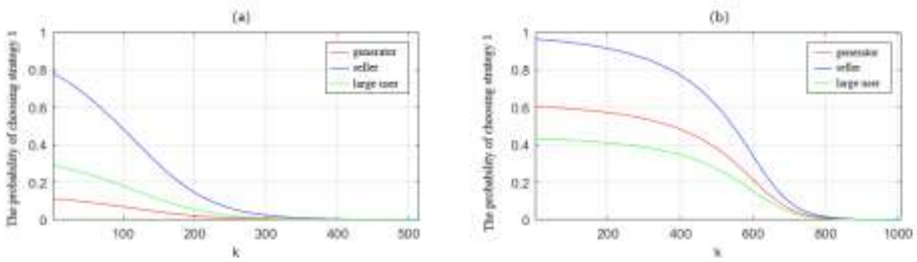

Fig 2: Asymptotically stable condition of (0,0,0): $d_1 < h_1$, $f_2 < h_2$, $g_3 < h_3$

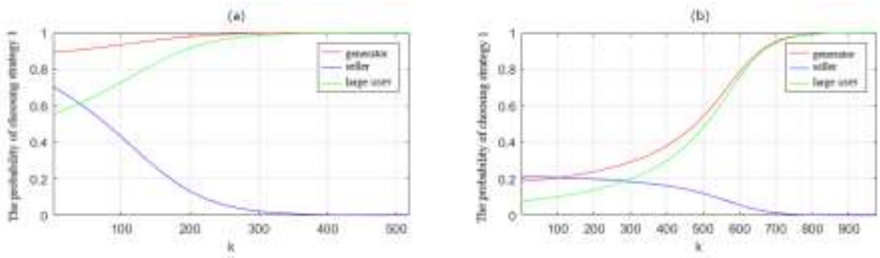

Fig 3: Asymptotically stable condition of (1,0,1): $g_1 < c_1$, $a_2 < c_2$, $d_3 < c_3$

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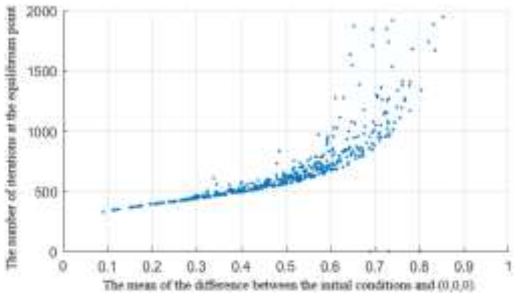

Fig 4: The relationship between the number of iterations of equilibrium points (0,0,0) and the initial value

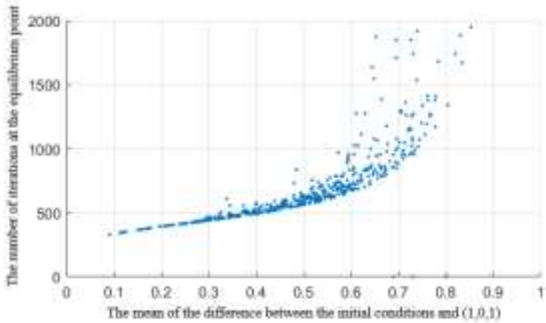

Fig 5: The relationship between the number of iterations of equilibrium points (1,0,1) and the initial value

5.1.1 Research on evolutionary game speed

In the process of verifying the stability conditions, it can be found that the number of iterations is not the same every time the game equilibrium is reached. The conjecture is related to the initial value condition, i.e. the strategy distribution at the initial moment. Therefore, for the case of $(0,0,0)$ and $(1,0,1)$, the relationship between the initial value condition and the number of iterations k when the equilibrium is reached is studied. Take the mean of the difference between the initial condition and the equilibrium point as the independent variable (called x_m). For example, for $(1,0,1)$, $x_m = [(1-x) + y + (1-z)]/3$, and the corresponding iteration number k is taken as the dependent variable. After simple calculation, it is found that the smaller x_m is, the smaller the number of iterations k is. In order to prevent accidental error, 500 simulation experiments were carried out for each of the two equilibrium points. The termination conditions and profit distribution Settings of the equilibrium points were unchanged in each experiment, but the initial values were different. The results are shown in Fig 4 and 5. The experimental results also verify the stability conditions of the equilibrium point.

According to the simulation results, it can be found that for these three equilibrium points, when x_m increases, the number of iterations k tends to rise. In addition, when $x_m \epsilon [0,0.5]$, the relation of k to x_m is close to linear: while when $x_m > 0.5$, the increase rate of k becomes faster, showing an exponential increase trend.

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In addition, during the simulation, a small part of x_m is found to be very small but the number of iterations k is very large. After further study, the reason is that the initial value conditions of these simulations are relatively poor, such as $x = 0.001$ and $x = 1$ at the equilibrium point. Although the total x_m is small, a large number of iterations are still needed to meet the convergence conditions.

5.2 Double Strategy Game Simulation of 30 Entities

Fig 6: Network structure of 30 market entities

According to the model established in Fig 6, matlab is used to simulate the strategy evolution of thirty market entities, including 5 class A power generators, 5 class B power generators, 5 class A sellers, 5 class B sellers and 10 large users. Firstly, the adjacency matrix, payoff matrix and initial strategy selection of the network are randomly given, and a simulation is carried out to simulate the strategy evolution of market subjects.

Under the simulation conditions, the network is a complete graph of 30 nodes, and the initial values of the strategy are random. The results of the game are as follows: Class A power generators chooses strategy 2 while class B power generators chooses strategy 1. Both class A and class B sellers choose strategy 2. The big users have opted for strategy 1, which is to work with generators to make them more profitable. The simulation results are shown in the Fig 7,8 and 9.

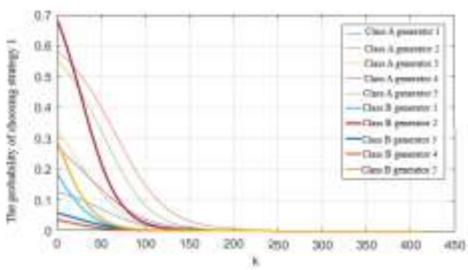

Fig 7: The evolution of generators' strategies

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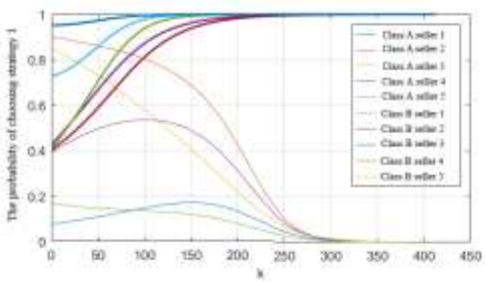

Fig 8: The evolution of sellers' strategies

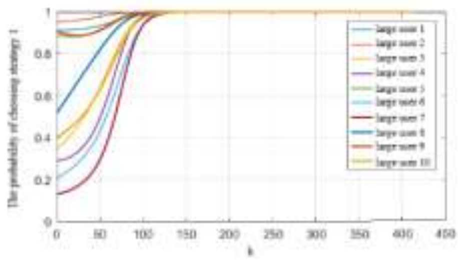

Fig 9: The evolution of large users' strategies

5.2.1 Influence of external factors

The integrated energy market is not invariable, and is often affected by many internal and external environmental conditions, thus causing the strategic choice of market subjects to change. Now consider the shift and evolution of the equilibrium point as a result of changes in the internal and external environment.

(1) If a local policy encourages large users to cooperate directly with power generators, the power generators will set a price p_s that will make large users earn more. At this point, the game weight between the generator and the large user in the adjacency matrix should be appropriately increased, and the parameters in the revenue matrix should be adjusted. The strategy evolution obtained is shown in Fig10 and 11.

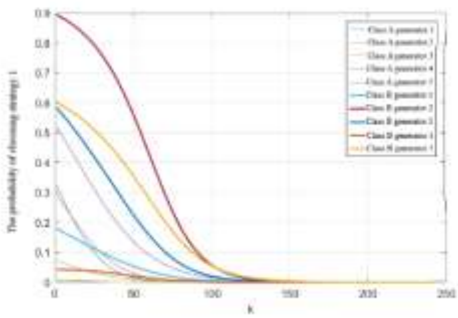

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Fig 10: Generators' strategy evolution under policy guidance

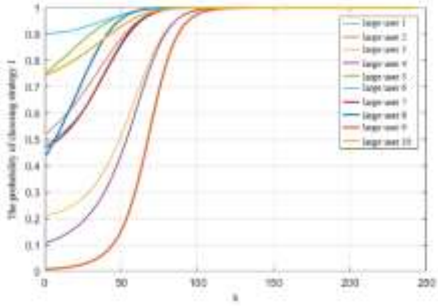

Fig 11: Large users' strategy evolution under policy guidance

According to the simulation results, can be found, in response to a policy, raise the power generators to cooperate with large user and modify the proceeds matrix parameters, guide user to select power suppliers, the simulation results agree well with the expected, the two classes of power generators are selection strategy 2 (low price) to attract users to buy electricity, users won higher yields.

(2) It is assumed that a certain type of sellers in a certain place reduces the sales price in order to increase its market share and attract large users to buy electricity. At this point, the value in the revenue matrix and adjacency matrix of the game between sellers and large users can be appropriately increased, and the strategy evolution obtained is shown in Fig 12 and 13.

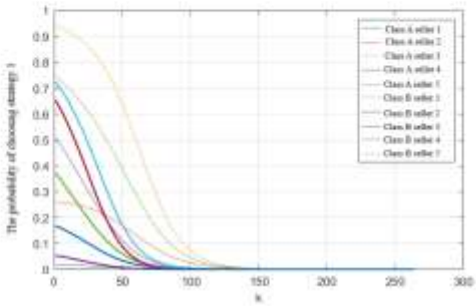

Fig 12: The evolution of sellers' strategy when sellers reduce prices

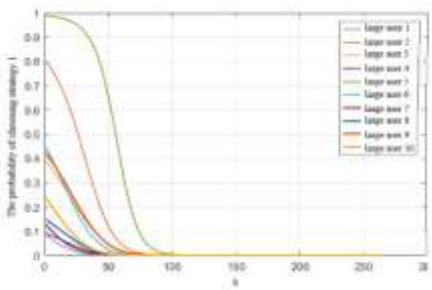

Fig 13: The evolution of large users' strategy when sellers reduce prices

According to the simulation results, sellers choose to reduce the electricity price to increase their market share, increase the interaction weight between sellers and large users, and appropriately modify the parameters in the revenue matrix. The simulation results show that large users will choose the low electricity price of sellers to obtain higher revenue, which is in good agreement with the expected results.

VI. EXAMPLE SIMULATION

Based on the comprehensive analysis of the development status of the comprehensive energy market, this paper creatively establishes the comprehensive energy market evolution model based on the complex network by combining the theoretical knowledge of complex network and game theory, and studies the stability of the model and the evolution of the system. This paper analyzes the two - strategy game involving three market players. There are only 8 stable equilibrium states under. Under certain income conditions, these 8 equilibrium states are evolutionarily stable and strict Nash equilibrium. Moreover, the simulation results show that the faster the initial condition is close to the equilibrium point, the faster the evolution of the system will be. As the distance between them increases, the time for the system to stabilize gradually increases from linear to exponential.

In addition, we find that the government's policy guidance or the decision of a certain department can affect the income and game weight parameters of market entities, and then affect the movement of equilibrium point, so as to make the evolution of the market more reasonable. It also provides some ideas and means for relevant departments to formulate market rules and regulate electric power trading environment.

There are still some shortcomings in this paper, for example, the model does not fully consider the real market returns. In the definition of the revenue matrix, we try to calculate the revenue according to the actual situation through the utility function and cost function of power generation generators, sellers and large users, and carry out simulation analysis, but the result is very different from the expected effect. In addition, when defining the adjacency matrix, it is difficult to quantify the importance of each subject in the market to the game, so there will inevitably be errors in the final income calculation. These are the defects of this study, and I will further discuss and analyze these problems in the future research.

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