

Research on Mobile Payment Selection Based on Multiple Logistic Model

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Abstract:

With the deepening of the informatization degree of social life, the utilization rate of communication equipment is also increasing. Mobile phones have become an indispensable tool in People's Daily life, study and work. Although the improper use of mobile phones will have many negative effects, the convenience it brings to people is beyond doubt. Based on this situation, communication companies, in order to expand their customer base, will fully study the psychology and actual situation of different types of personnel, according to these research results to formulate different packages for people to choose. On the other hand, how to choose the appropriate tariff according to their own needs, it needs scientific method to guide. In order to expand their customer base, these mobile communication companies are offering various billing services based on people's desire. People will choose the right packaging according to their needs. In order to guide people to make more scientific choices, some reasonable models are needed as a reference. As the Logistic selection model is a common and simple one among many decision models, this paper studies how to establish the Logistic selection model to guide people to make scientific and reasonable decisions. In order to establish the model objectively, this paper collected the sales data of a mobile company. Through the detailed analysis of the data and statistical model analysis, according to the characteristics of the data, the final choice of the Logistic model. This model is widely used in solving all kinds of decision-making problems, and it is more accurate, so this paper chooses this model is more reasonable. In this paper, based on the sales data of a mobile company, a Logistic selection model is established through the analysis of the data, so that people can use this model to choose the required tariff package scientifically and reasonably. At the same time, the parameter values in the model are determined by parameter estimation, and the final model is obtained. Firstly, a Logistic selection model is established by analyzing the data information of a mobile company's customers' payment choice. Because this model is used in all kinds of decision-making problems, the model established in this paper has certain reference and guiding significance. Because there are parameters to be estimated in the model, in order to determine the value of parameters, this paper uses maximum likelihood estimation method to estimate the parameters in the model. Finally, the rationality of the estimated parameters is proved by conformity test.

Keywords: *Logistic model, Parameter estimation, Consistent estimates, Psychological perspective.*

I. INTRODUCTION

With the development of information technology, mobile phones have become an essential communication tool for people. In order to expand their business and attract users to choose their own business, communication companies will launch a variety of mobile phone packages for people to choose every year. In the setting of these mobile phone rates, communication companies also fully grasp people's psychology. The introduction of mobile phone plans to meet the needs of different types of people. How people choose their own business according to the mobile phone tariff packages launched by communication companies is a problem worth studying. According to the theoretical and empirical studies of domestic scholars over the years, the Logistic model [1, 2] has a very credible ability of identification, prediction and promotion, and it has the following probability density:

$$p(Y=j | X=x) = \frac{e^{\beta_j^T x}}{\sum_{t=1}^k e^{\beta_t^T x}} \quad (j=1, 2, \dots, k), \quad (1)$$

X_1, X_2, \dots, X_k are independent of each other.

X_j is the column vector of R^q . $X_j \sim N(\mu_j, \sigma_j)$, $\mu_j, \sigma_j > 0$ are unknown,

$j=1, 2, \dots, k$. Y is the random variable whose value is $1, 2, \dots, k$.

$X = (X_1, X_2, \dots, X_k)$, The value of X is x .

$\beta \in R^q$ is an unknown parameter, ($\|\beta\| > 0$), $k > 1, q > 1$.

Multinomial Logistic model is an effective tool for credit risk identification of commercial banks in China, and it is widely used in reality. For example, there are k stores, and all stores have q features. In order to study how customers choose stores, the multinomial Logistic selection model can be considered.

II. MODEL BUILDING

With the development of communication technology, mobile phones have become an indispensable tool in People's Daily life. It is very important to choose a suitable tariff type among the numerous options offered by mobile companies. This paper analyzes the data information of a mobile company's previous customers' tariff selection.

The following multinomial Logistic selection model is constructed:

$$p(Y = j | X = x) = \frac{\exp(\beta^T x_j)}{\sum_{i=1}^{10} \exp(\beta^T x_i)} \quad (j = 1, 2, \dots, 10) \quad (2)$$

X_1, X_2, \dots, X_{10} are independent of each other. X_j It's a 3-dimensional column vector, $X_j \sim N(\mu_j, \sigma_j)$, $j = 1, 2, \dots, 10$. Y Is the random variable whose value is $1, 2, \dots, 10$. $X = (X_1, X_2, \dots, X_{10})$, The value of X is x . $\beta \in R^3$ is an unknown parameter, and $\|\beta\| > 0$.

III. ESTIMATION OF PARAMETER UNIT VECTOR

The overall observed value is shown in Table I, which represents the monthly function fee (yuan), domestic data traffic (G), and call duration (min) of the tariff in sale:

TABLE I. Data of test

Y	1	2	3	4	5	6	7	8	9	10
X	$\begin{pmatrix} 58 \\ 0.15 \\ 150 \end{pmatrix}$	$\begin{pmatrix} 88 \\ 0.3 \\ 350 \end{pmatrix}$	$\begin{pmatrix} 128 \\ 0.6 \\ 650 \end{pmatrix}$	$\begin{pmatrix} 158 \\ 0.6 \\ 900 \end{pmatrix}$	$\begin{pmatrix} 188 \\ 0.6 \\ 1200 \end{pmatrix}$	$\begin{pmatrix} 58 \\ 0.5 \\ 50 \end{pmatrix}$	$\begin{pmatrix} 88 \\ 0.7 \\ 200 \end{pmatrix}$	$\begin{pmatrix} 128 \\ 1 \\ 420 \end{pmatrix}$	$\begin{pmatrix} 158 \\ 2 \\ 510 \end{pmatrix}$	$\begin{pmatrix} 188 \\ 2.5 \\ 600 \end{pmatrix}$
sales	9440 1	5889 9	1888 7	4457	5798	4671 3	2036 9	4799	1952	1426

Label samples $(X^{(i)}, Y^{(i)})$ ($i = 1, 2, \dots, 257701$) in order of sale.

$S_d = \{i | Y^{(i)} = d\}$, N_d is the number of elements $S_d, d = 1, 2, \dots, 10$.

$$\bar{X} = \frac{1}{257701} \sum_{i=1}^{257701} X^{(i)} = \begin{pmatrix} 79.793 \\ 0.385 \\ 265.022 \end{pmatrix}. \quad (3)$$

$\hat{\mu}_d = X^{(i)} - \bar{X}$. The value of $\hat{\mu}_d$ are shown in Table II and Table III:

TABLE II. Data of test

d	1	2	3	4	5
$\hat{\mu}_d$	$\begin{pmatrix} -21.793 \\ -0.235 \\ -115.022 \end{pmatrix}$	$\begin{pmatrix} 8.207 \\ -0.085 \\ 84.978 \end{pmatrix}$	$\begin{pmatrix} 48.207 \\ 0.215 \\ 384.978 \end{pmatrix}$	$\begin{pmatrix} 78.207 \\ 0.215 \\ 634.978 \end{pmatrix}$	$\begin{pmatrix} 108.207 \\ 0.215 \\ 934.978 \end{pmatrix}$

TABLE III. Data of test

d	6	7	8	9	10
$\hat{\mu}_d$	$\begin{pmatrix} -21.793 \\ 0.115 \\ -215.022 \end{pmatrix}$	$\begin{pmatrix} 8.207 \\ 0.315 \\ -65.022 \end{pmatrix}$	$\begin{pmatrix} 48.207 \\ 0.615 \\ 154.978 \end{pmatrix}$	$\begin{pmatrix} 78.207 \\ 1.615 \\ 244.978 \end{pmatrix}$	$\begin{pmatrix} 108.207 \\ 2.115 \\ 334.978 \end{pmatrix}$

$$\hat{V}_d = \sum_{d=1}^{10} \frac{N_d}{100} \hat{\mu}_d \hat{\mu}_d^T ; \hat{V}_d = \begin{pmatrix} 975 & 6 & 6865 \\ 6 & 0 & 24 \\ 6865 & 24 & 54239 \end{pmatrix}, \hat{V} = \hat{\Sigma}^{-1} \hat{V}_d \hat{\Sigma}^{-1}. \hat{\Sigma} = \frac{1}{10} \sum_{d=1}^{10} \hat{\mu}_d \hat{\mu}_d^T.$$

$$\hat{\Sigma} = \begin{pmatrix} 4140 & 40 & 23960 \\ 40 & 0 & 160 \\ 23960 & 160 & 169270 \end{pmatrix}, \hat{\Sigma}^{-1} = \begin{pmatrix} 0.0643 & -2.1961 & -0.007 \\ -2.1961 & 76.5520 & 0.2388 \\ -0.007 & 0.2388 & 0.0008 \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} 0.025 & -0.8479 & -0.0027 \\ -0.8479 & 28.9425 & 0.091 \\ -0.0027 & 0.091 & 0.0003 \end{pmatrix}.$$

The maximum characteristic root of \hat{V} is =28.9676, and its corresponding characteristic vector is

$$\hat{\eta} = \begin{pmatrix} 0.0293 \\ -0.9996 \\ -0.0031 \end{pmatrix}.$$

Estimation of unit vector $\frac{\beta}{\|\beta\|}$ of parameter β is $\hat{\beta}_d = \hat{\eta} = \begin{pmatrix} 0.0293 \\ -0.9996 \\ -0.0031 \end{pmatrix}$.

IV. ESTIMATION OF PARAMETER VECTOR MODULUS

Using maximum likelihood method [3, 4] to estimate the modulus α of parameter β , the multinomial Logistic selection model constructed in this paper can be rewritten as:

$$p(Y = y | X = x) = \frac{\exp(\|\beta\| \beta_d^T x_y)}{\sum_{t=1}^{10} \exp(\|\beta\| \beta_d^T x_t)} \quad (4)$$

Replace the estimator $\hat{\beta}_d$ of the parameter unit vector β_d in the above equation, then:

$$p(Y = y | X = x) = \frac{\exp(\alpha \hat{\beta}_d^T x_y)}{\sum_{t=1}^{10} \exp(\alpha \hat{\beta}_d^T x_t)}$$

Based on the above formula, the joint density of (X, Y) is:

$$\begin{aligned} p(x, y) &= p_{Y|X}(y) p_X(x) \\ &= \frac{\exp(\alpha \hat{\beta}_d^T x_y)}{\sum_{t=1}^{10} \exp(\alpha \hat{\beta}_d^T x_t)} \prod_{j=1}^{257701} \frac{1}{(\sqrt{2\pi})^4} |\Sigma_j|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(x_j - \mu_j)^T \Sigma_j^{-1} (x_j - \mu_j)\right] \end{aligned} \quad (5)$$

Omit the irrelevant part of α , and get the likelihood function:

$$\begin{aligned} L(\alpha; y^{(1)}, \dots, y^{(257701)}) &= \prod_{i=1}^{257701} \frac{\exp(\alpha \hat{\beta}_d^T x_{y_i})}{\sum_{t=1}^{10} \exp(\alpha \hat{\beta}_d^T x_t)} \\ &= \frac{\exp\left(94401 \alpha \hat{\beta}_d^T x_1 + 58899 \alpha \hat{\beta}_d^T x_2 + 18887 \alpha \hat{\beta}_d^T x_3 + 4457 \alpha \hat{\beta}_d^T x_4 + 5798 \alpha \hat{\beta}_d^T x_5 \right. \\ &\quad \left. + 46713 \alpha \hat{\beta}_d^T x_6 + 20369 \alpha \hat{\beta}_d^T x_7 + 4799 \alpha \hat{\beta}_d^T x_8 + 1952 \alpha \hat{\beta}_d^T x_9 + 1426 \alpha \hat{\beta}_d^T x_{10}\right)}{\left[\sum_{t=1}^{10} \exp(\alpha \hat{\beta}_d^T x_t)\right]^{257701}} \end{aligned} \quad (6)$$

The logarithmic likelihood function is:

$$\begin{aligned}
 l(\alpha; y^{(1)}, \dots, y^{(257701)}) &= 94401\alpha\hat{\beta}_d^T x_1 + 58899\alpha\hat{\beta}_d^T x_2 + 18887\alpha\hat{\beta}_d^T x_3 \\
 &+ 4457\alpha\hat{\beta}_d^T x_4 + 5798\alpha\hat{\beta}_d^T x_5 + 46713\alpha\hat{\beta}_d^T x_6 + 20369\alpha\hat{\beta}_d^T x_7 \\
 &+ 4799\alpha\hat{\beta}_d^T x_8 + 1952\alpha\hat{\beta}_d^T x_9 + 1426\alpha\hat{\beta}_d^T x_{10} - 257701 \ln \sum_{t=1}^{10} \exp(\alpha\hat{\beta}_d^T x_t)
 \end{aligned} \tag{7}$$

The likelihood equation is:

$$\sum_{i=1}^{257701} \hat{\beta}_d^T x^{(i)} - 257701 \frac{\sum_{t=1}^{10} \hat{\beta}_d^T x_t \exp(\alpha\hat{\beta}_d^T x_t)}{\sum_{t=1}^{10} \exp(\alpha\hat{\beta}_d^T x_t)} = 0 \tag{8}$$

According to the calculated results, the likelihood equation can be written as:

$$\begin{aligned}
 &291610 - \frac{257701 \left(\begin{aligned} &1.0845 \exp(1.0845\alpha) + 1.1935 \exp(1.1935\alpha) \\ &+ 1.1356 \exp(1.1356\alpha) + 1.2396 \exp(1.2396\alpha) \\ &+ 1.1886 \exp(1.1886\alpha) + 1.0446 \exp(1.0446\alpha) \\ &+ 1.2587 \exp(1.2587\alpha) + 1.4488 \exp(1.4488\alpha) \\ &+ 1.0492 \exp(1.0492\alpha) + 1.1494 \exp(1.1494\alpha) \end{aligned} \right)}{\begin{aligned} &\exp(1.0845\alpha) + \exp(1.1935\alpha) + \exp(1.1356\alpha) + \exp(1.2396\alpha) \\ &+ \exp(1.1886\alpha) + \exp(1.0446\alpha) + \exp(1.2587\alpha) + \exp(1.4488\alpha) \\ &+ \exp(1.0492\alpha) + \exp(1.1494\alpha) \end{aligned}} = 0
 \end{aligned} \tag{9}$$

$$\begin{aligned}
 &12133.2655 \exp(1.0845\alpha) - 15956.1435 \exp(1.1935\alpha) - 1035.2556 \exp(1.1356\alpha) \\
 &- 27836.1596 \exp(1.2396\alpha) - 14693.4086 \exp(1.1886\alpha) + 22415.5354 \exp(1.0446\alpha) \\
 &- 32758.2487 \exp(1.2587\alpha) - 81747.2088 \exp(1.4488\alpha) + 21230.1108 \exp(1.0492\alpha) \\
 &- 4591.5294 \exp(1.1494\alpha) = 0
 \end{aligned} \tag{10}$$

Let the above formula $A = e^\alpha$ be transformed into:

$$\begin{aligned}
 &12133.2655A^{1.0845} - 15956.1435A^{1.1935} - 1035.2556A^{1.1356} - 27836.1596A^{1.2396} \\
 &- 14693.4086A^{1.1886} + 22415.5354A^{1.0446} - 32758.2487A^{1.2587} \\
 &- 81747.2088A^{1.4488} + 21230.1108A^{1.0492} - 4591.5294A^{1.1494} = 0
 \end{aligned} \tag{11}$$

Multiply both ends of the equation by $A^{-0.4488}$:

$$\begin{aligned}
 &12133.2655A^{1.0845} - 15956.1435A^{0.7447} - 1035.2556A^{0.6868} - 27836.1596A^{0.7908} \\
 &- 14693.4086A^{0.7398} + 22415.5354A^{0.5958} - 32758.2487A^{0.8099} \\
 &+ 21230.1108A^{0.6004} - 4591.5294A^{0.7006} = 81747.2088A
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 &0.1484A^{1.0845} - 0.1952A^{0.7447} - 0.0127A^{0.6868} - 0.3405A^{0.7908} - 0.1797A^{0.7398} \\
 &+ 0.2742A^{0.5958} - 0.4007A^{0.8099} + 0.2597A^{0.6004} - 0.0562A^{0.7006} = A
 \end{aligned} \tag{13}$$

$$A=1.0320, e^\alpha=1.0320, \hat{\alpha}=0.0315$$

V. CONSISTENCY ESTIMATION OF PARAMETER VECTORS

According to Slutsky's theorem, the estimation of parameter vector β is consistent estimation:

$$\hat{\beta} = \hat{\alpha} \cdot \hat{\beta}_d = 0.0315 \cdot \begin{pmatrix} 0.0293 \\ -0.9996 \\ -0.0031 \end{pmatrix} = \begin{pmatrix} 0.00092295 \\ -0.0314874 \\ -0.00009765 \end{pmatrix} \tag{14}$$

VI. CONCLUSIONS

Based on the analysis of the sales data of the tariff package launched by a mobile company, this paper chooses to use the logistics model to help consumers make a more reasonable choice. When estimating the parameters in the model, the maximum likelihood estimation method is selected to estimate the parameters. Thus, a reasonable model can be obtained, which can help consumers make scientific and reasonable decisions when choosing tariff packages, and has great theoretical and practical significance.

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