Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

Research on the Application of Pension Insurance Actuarial Model Based on Stochastic Interest Rate and Improved APC Mortality Rate

Hao LI[1], Haixia ZHANG[2*]
[1]School of Mathematics and Statistics, Suzhou University, Suzhou, Anhui, China
[2]School of Music, Suzhou University, Suzhou, Anhui, China
*Corresponding Author.

Abstract:

In this paper, the representative retirement annuity in the old-age insurance is taken as the research object, the retirement age is described by the (a,b,0) distribution class, and the interest rate is described by the combination of the origin reflection Brownian motion and the Quasi-Gaussian distribution, the second-order moment present value and equilibrium net premium expression of retirement annuity are obtained by choosing the extended APC model which can fit the mortality rate of China.

***Keywords**: Endowment insurance, Retirement annuity, Stochastic interest rate, Extended APC model.*

I. INTRODUCTION

The old-age insurance is a kind of social insurance that refers to the old people who withdraw from the social work life completely or basically within the legal scope and receive the old-age pension from the social insurance department, to provide them with a stable and reliable source of livelihood. Old-age insurance is a kind of social security system which is widely practiced in the world. The source of the social old-age Insurance Fund is generally shared by the state, the unit and the individual, or by both the unit and the individual, and a wide range of mutual social benefits is realized, the expenses are huge, so we must set up special agencies to carry out the unified planning and management of modernization, specialization and socialization. In addition, due to the wide coverage and large number of participants of the pension insurance, it can raise a large number of pension insurance funds in its operation and provide a huge source of funds for the capital market, especially the pension insurance mode of the fund system, the accumulation of funds in individual accounts over decades has made pension funds larger, provided more funds for the market, and, through the operation and utilization of funds of scale, contributed to the state's Macroeconomic regulation and control of the national economy. China's old-age insurance consists of three parts, namely, basic old-age insurance, enterprise supplement and personal savings. In the Commercial Endowment Insurance, annuity insurance is the most common. Annuity insurance with contributions paid within a time limit, that is, an insured person starts to receive an old-age pension upon the payment of contributions within a specified period of time, and if the recipient of the annuity dies

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

before the age of receipt, the insurance company or the insurance company refunds the higher of the premiums paid and the cash value, or pay the premium according to the stipulated insurance amount. China's legal retirement age is 55 for women and 60 for men, and social pension benefits are paid according to these two age groups. By contrast, commercial pension plans are much more flexible, offer a variety of options and can be changed before they begin. The starting time of annuity payment is usually concentrated in the four age groups of 50, 55,60 and 65 years of age of the insured, and there are other age settings.

This paper takes the retirement annuity insurance under the flexible retirement system as the research object and constructs its pricing model. Retirement annuity insurance is a kind of deferred insurance that annuity recipients begin to pay when they reach retirement age. At present, China is in a rapid population ageing, when according to incomplete statistics, the elderly population will account for about 23% of the total population, reaching more than 300 million. In addition, the average life expectancy of the Chinese population has increased to a certain extent due to factors such as economic development, improvement in the quality of life and advances in medical technology. Before founding ceremony of the People's Republic of China, the average life expectancy was 35 years. By 1978, it had reached 68.2 years, in 2015, the average life expectancy of the Chinese population will rise to 76.3 years. By 2020, the life expectancy of the Chinese population will increase by one year from the previous five years. On top of that, the sheer number of aging people and their extended life expectancy in retirement are bound to put pressure on state pension payments and expose insurance companies to huge financial risks. In order to solve the problem of economic pressure caused by the population ageing, China is actively promoting the establishment of the flexible retirement system by learning from the foreign policy of postponing the retirement age. The so-called flexibility is characterized by the principle of voluntary choice of the retirement age of citizens within the retirement system, with the possibility of retiring at the minimum prescribed age or of postponing retirement in accordance with one's own circumstances. The advantages of flexible retirement system lie in saving human capital waste, improving the efficiency of using human capital, reforming the structure of labor market, reducing the pressure of pension payment and alleviating the negative effect of aging. In a study on the feasibility and difficulties of implementing a flexible retirement system in China, Zhou Xu [1](2014) learned that the retirement age has been extended from 60 to 65 in France since 2004, and from 60 to 64 in Sweden every month, the earlier the pension, the smaller it is, and the later the pension, the greater it is in each month between the ages of 65 and 70. In Britain, the retirement age is 65, with full entitlement to a pension after 44 years of contributions. In the United States, the retirement age is 62, with a limit of 62 years, early retirement reduced, late retirement increased. At present, the retirement age of Chinese citizens is 60 years for males and 55 years for females. In order to effectively deal with the population ageing problem, China's retirement age must be reformed, and the flexible retirement system should be an important option for the reform of China's social pension system.

Considering the current population ageing of China and the flexible retirement system, this paper studies the pricing actuarial model of pension insurance, which is one of the most important kinds of pension insurance in China. The research is divided into three parts: The assumption of retirement age, the stochastic description of interest rate, and the fitting of mortality rate. Through the above three parts, this

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

paper constructs the pricing model of retirement annuity insurance under the flexible retirement system. The situation is similar for women, assuming that the insured person in the retirement annuity insurance under the flexible retirement system is male.

II. ASSUMPTION AND MODELS

2.1 Assumption of Retirement Age

Set $T_x^*(x)$ is the actual retirement age of a x old citizen in line with the retirement policy, two assumptions are made:(1) Under the flexible retirement system, the retirement age of male citizens is a random variable with integer value which on a scale of $[60,65]$; (2) $T_x^*(x)$ is subject to the $(a,b,0)$ distribution class.

So for a positive integer discrete random variable $T_x^*(x)$, which distribution function is $p_k = P(T_x^*(x)=k)$, and

$$p_k = p_{k+1}(a+\frac{b}{k}),\ k=60,61,\cdots,65. \tag{1}$$

Where a and b is a parameter. Except for the degenerate distribution, the $(a,b,0)$ distribution class only includes Poisson distribution, Binomial distribution and Negative Binomial distribution.

TABLE I. The values of parameters a and b in the $(a,b,0)$ distribution class

the $(a,b,0)$ distribution class	the values of parameters a and b
Poisson distribution $P(\lambda)$	$a=0,\ b=\lambda$
Binomial distribution $B(k,p)$	$a=-\frac{p}{1-p},\ b=\frac{(k+1)p}{1-p}$
Negative Binomial distribution $NB(k,p)$	$a=\frac{p}{1+p},\ b=\frac{(k-1)p}{1+p}$

2.2 Stochastic Description of Interest Rates

With the continuous development of interest rate research, people find that the real interest rate is not constant but with randomness and volatility. In theory, there are two kinds of stochastic modeling methods for interest rate: discrete modeling [2-4] and continuous modeling [5-7]. The idea of the discrete modeling method is to assume that the annual interest rate is the same value and that the annual interest rate is expressed as a sequence of random variables, or is characterized only by Markov's property, although the interest rate dependence of different years is taken into account, but this is far from the case. In the early 1990s, the continuous modeling method was proposed in the actuarial field, which is called stochastic perturbation interest rate modeling method. In this kind of method, it is emphasized to use different stochastic processes to model the accumulative interest force. The advantage of this method is to consider the continuous stochastic fluctuation of interest rate, which is more consistent with the actual situation, therefore, the continuous interest rate modeling methods are more reasonable.

In this paper, a continuous-time interest rate model is established by using reflection Brownian motion and Poisson process for the interest force accumulation function δ_t, assuming that the interest force accumulation function satisfies

$$\delta_t = \delta t + \beta |W(t)| + \gamma N(t), \tag{2}$$

Where δ is the risk-free interest rate, β is the adjusting parameter, γ is the range of each jump, $|W(t)|$ is the reflecting Brownian motion at the origin, and $N(t)$ is the Poisson process with $\lambda(\lambda > 0)$ velocity, and let δ, β and $|W(t)|$ be independent of each other. The combination of reflective Brownian motion with random perturbation and Poisson process can well reflect the mean reversion and time fluctuation of interest rate in China. The discount function $v(t)$ can be expressed as

$$v(t) = e^{-\delta t - \beta |W(t)| - \gamma N(t)}. \tag{3}$$

Since $|W(t)|$ is a reflection of Brownian motion at the origin, then

$$E(e^{-\beta |w(t)|}) = 2e^{\frac{1}{2}\beta^2 t}[1 - \phi(\beta(t))], \tag{4}$$

Where $\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ is the probability density function of the Normal distribution.

In addition, since $N(t)$ follows the Poisson distribution with parameter λt, and the probability distribution of $N(t)$ is

$$P(N(t) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \tag{5}$$

Then

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

$$E(e^{-\gamma N(t)}) = \sum_{k=0}^{\infty} e^{-\gamma k} \frac{(\lambda t)^k}{k!} e^{-\lambda t} = e^{-\lambda t} \sum_{k=0}^{\infty} \frac{(\lambda t e^{-\gamma})^k}{k!} = e^{-\lambda t} \cdot e^{\lambda t e^{-\gamma}} = e^{-\lambda t(1-e^{-\gamma})}, \quad (6)$$

Combined (3)-(6), we can get

$$E(v(t)) = E(e^{-\delta t - \beta|W(t)| - \gamma N(t)}) = 2[1-\phi(\beta(t))]e^{-[\delta + \lambda t(1-e^{-\gamma}) - \frac{1}{2}\beta^2]t}. \quad (7)$$

2.3 Fitting Models of Population Mortality

In Life Insurance actuarial theory and practice, there are many factors that affect the pricing of life insurance products, the most important factor besides interest rate is mortality rate. In recent years, with the change of material living conditions, the improvement of medical technology and the improvement of natural environment, people's life expectancy have been raised to a certain extent. According to the world's historical demographic data, we can see that the human mortality rate is changing with time, with a strong random and intermittent jump. In recent years, domestic and foreign scholars gradually attach importance to the study of mortality. Because the deterministic mortality model considers the future mortality to be constant, that does not correspond to the actual situation, so scholars focus on mortality randomness. At present, there are two types of stochastic mortality models, one is the use of the Stochastic differential equation method, which gives the Stochastic differential equation of mortality dynamics in continuous time, and the other is the relationship between mortality dynamics and mortality, the second is to establish a multi-factor linear structure model with age effect, time effect and queue effect for the central mortality rate, and to predict the mortality rate by using the discrete parameter estimation method, the main advantage of these models is that they take full account of Age, time Period, birth year (Cohort) and other factors, called APC models.

Haberman and Renshaw [8](2006) first proposed the RH model, which form is

$$\log m(x,t) = \alpha_x + \beta_x^{(1)} k_t + \beta_x^{(2)} \gamma_{t-x}. \quad (8)$$

On the basis of the RH model, Currie [9](2006) made a simplified model, the APC model, which reflects the trend $\beta_x^{(1)} = 1$ and $\beta_x^{(2)} = 1$ of the logarithmic change of mortality rate by age. It takes the form of

$$\log m(x,t) = \alpha_x + k_t^* + \gamma_{t-x}, \quad (9)$$

where, $m(x,t)$ represents the central mortality of x-year-olds at time t;and $\alpha_x, \beta_x^{(1)}$ and $\beta_x^{(2)}$ are the bases of logarithmic changes in age-related mortality; k_t^* is a time effect variable, it is a random walk process or an ARIMA process, its function is to reflect the changes in the level of mortality in time; γ_{t-x} is

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

the effect of birth year, reflecting the effect of birth year $t-x$ on mortality. The advantage of the APC model is that the stability problem of the RH model in parameter estimation is avoided.

Based on the APC model, Ma H F [10](2020) proposed an extended model-the EAPC model, which is as follows

$$\log m(x,t)=\alpha_x+k_t^*+\gamma_{t-x}(x-\bar{x}), \tag{10}$$

Where x is the age of the insured, $\bar{x}$ is the average value of age, and $(x-\bar{x})$ represents the age factor associated with γ_{t-x}. The empirical results show that the EAPC model is better than the APC model in fitting Chinese population data.

Cairns, Blake and Dowd [11-12](2006) proposed a mortality model based on a Logistic transformation for the elderly (60-89 years) , known as the CBD model The model can describe the characteristics of the mortality rate of the elderly at retirement age well, and its expression is

$$\log itq(x,t)=\log\frac{q(x,t)}{1-q(x,t)}=k_t^{(1)}+k_t^{(2)}(x-\bar{x}), \tag{11}$$

Where $q(x,t)=1-\exp(-m(x,t))$, $q(x,t)$ is the probability of a b-year-old policyholder dying within t-year.

Based on the APC model, this paper presents an improved mortality model

$$\log m(x,t)=\alpha_x+k_t^*(x-\bar{x})+\gamma_{t-x}(x-\bar{x}), \tag{12}$$

Then, a concise expression of survival probability $p(x,t)$ can be obtained by using Logistic transformation, it can be expressed as

$$p(x,t)=1-q(x,t)=\exp(-m(x,t))=\exp[-\exp(\alpha_x+k_t^*(x-\bar{x})+\gamma_{t-x}(x-\bar{x}))]. \tag{13}$$

This model not only gets the exponential relation between survival probability $p(x,t)$ and central mortality rate $m(x,t)$, but also retains the excellent effect of EAPC model in fitting the mortality rate of Chinese population.

2.4 Pricing Model of Elastic Retirement Annuity Insurance

Under a flexible retirement system, the present value of the retirement annuity paid at the beginning of

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

the year by an insured person [13] of age x can be expressed as

$$B=\begin{cases} 0, T_x < T_x^* - x \\ \sum_{k=T_x^*-x}^{\omega-(T_x^*-x+1)} v(k), T_x \geq T_x^* - x \end{cases}, \quad (14)$$

Where, ω is the ultimate age, $v(t)$ is the discount factor, T_x^* is the retirement age of the x-year-old and T_x is the remaining life of the x-year-old. In the above formula (14), when $T_x \geq T_x^* - x$ shows that the-year-old policyholder is still alive before retirement, otherwise the present value of the retirement annuity is 0.

Thus, the actuarial present value of the retirement annuity paid one at the beginning of the year by x-year-old policyholder is

$$\ddot{a}_{x:\overline{T_x^*-x}|} = E(B) = E(\sum_{k=T_x^*-x}^{\omega-(T_x^*-x+1)} v(k) I_{(T_x \geq T_x^*-x)})$$

$$= \sum_{k=60}^{65} [\sum_{t=(T_x^*-x)}^{\omega-(T_x^*-x)} E[p(x,t)v(t)I_{(T_x^*=k)}]]$$

$$= \sum_{k=60}^{65} [\sum_{t=(k-x)}^{\omega-(k-x)} [E[v(t)] \cdot E[p(x,t)] \cdot P(T_x^* = k)]]$$

$$= \sum_{k=60}^{65} [\sum_{t=(k-x)}^{\omega-(k-x)} [2\exp(-\exp(\alpha_x + k_t^*(x-\bar{x}) + \gamma_{t-x}(x-\bar{x})))(1-\phi(\beta(t))]$$

$$\cdot e^{-[\delta+\lambda t(1-e^{-\gamma})-\frac{1}{2}\beta^2]t} \cdot p_{k-1}(a+\frac{b}{k})]$$

$$= 2\sum_{k=60}^{65} [\sum_{t=(k-x)}^{\omega-(k-x)} \exp(-\exp(\alpha_x + k_t^*(x-\bar{x}) + \gamma_{t-x}(x-\bar{x}))(1-\phi(\beta(t))$$

$$\cdot e^{-[\delta+\lambda t(1-e^{-\gamma})-\frac{1}{2}\beta^2]t} \cdot p_{k-1}(a+\frac{b}{k})] \quad . \quad (15)$$

Accordingly, under the flexible retirement system, the present value of x-year-old policyholder's periodic life annuity payment one at the beginning of the year can be expressed as

$$C = \sum_{t=x}^{T^*-x-1} I_{(t \leq T_x < t+1)} \sum_{j=0}^{t} v(j) . \quad (16)$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

Thus, the actuarial present value of x-year-old's fixed-term annuity after the first year of payment of one to year T_x^* is

$$\ddot{a}_{x:\overline{T_x^*-x}|} = E(C) = E(\sum_{k=0}^{(T_x^*-x+1)} I_{(t\le T_x<t+1)} \sum_{j=0}^{k} v(j))$$

$$= \sum_{k=60}^{65}[\sum_{t=0}^{k-x-1} E[q(x,t)\sum_{j=0}^{t} v(j) I_{(T_x^*=k)}]]$$

$$= \sum_{k=60}^{65}[\sum_{t=0}^{k-x-1} E[q(x,t)]E[\sum_{j=0}^{t} v(j)]E[I_{(T_x^*=k)}]]$$

$$= \sum_{k=60}^{65}[\sum_{t=0}^{k-x-1} [1-{}_{t+1}p_x+{}_tp_x]E[\sum_{j=0}^{t} v(j)]E[I_{(T_x^*=k)}]]$$

$$= 2\sum_{k=60}^{65}[\sum_{t=(k-x)}^{\omega-(k-x)}[[1-\phi(\beta(t))]e^{-[\delta+\lambda t(1-e^{-\gamma})-\frac{1}{2}\beta^2]t}$$

$$\cdot(1-\exp(-\exp(\alpha_x+k_{t+1}^*(x-\bar{x})+\gamma_{t+1-x}(x-\bar{x}))+\exp(-\exp(\alpha_x+k_t^*(x-\bar{x})+\gamma_{t-x}(x-\bar{x})) \quad \cdot p_{k-1}(a+\frac{b}{k})]] \quad .$$

(17)

In addition, the second-order moment of a retirement annuity may be obtained

$$E(B^2) = E(\sum_{k=T_x^*-x}^{\omega-(T_x^*-x+1)} v(k) I_{(T_x\ge T_x^*-x)})^2$$

$$= \sum_{k=60}^{65}[\sum_{t=(T_x^*-x)}^{\omega-(T_x^*-x)} E[p(x,t)(v(t))^2 I_{(T_x^*=k)}]]$$

$$= \sum_{k=60}^{65}[\sum_{t=(k-x)}^{\omega-(k-x)}[E[v(t)]^2\cdot E[p(x,t)]\cdot P(T_x^*=k)]]$$

$$= 2\sum_{k=60}^{65}[\sum_{t=(k-x)}^{\omega-(k-x)}[\exp(-\exp(\alpha_x+k_t^*(x-\bar{x})+\gamma_{t-x}(x-\bar{x})))[1-\phi(\beta(t))]]\cdot e^{-[2\delta+2\lambda t(1-e^{-\gamma})-\beta^2]t}\cdot p_{k-1}(a+\frac{b}{k})]] \quad ,$$

(18)

And the general expression for the equilibrium net premium is

$$P = \frac{E(Z_1)}{E(Z_2)} = \frac{\ddot{a}_{x:|T_x^*-x}}{\ddot{a}_{x:\overline{T_x^*-x}|}} \quad . \qquad (19)$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
May-June 2022 Page No. 112-122
Article History: Received: 24 February 2022, Revised: 05 April 2022, Accepted: 08 May 2022, Publication: 30 June 2022

2.5 Example

Consider 30-year-old Chinese men who are insured for retirement between the ages of 32 and 62 and begin receiving a pension after retirement. Since there are many parameters involved in the pricing model of retirement pension, it is necessary to set the parameters and make some assumptions. First of all, the retirement age T^* is assumed to follow a Negative Binomial distribution $NB(k, p)$, the related actuarial function parameter setting table is as follows

TABLE II. Actuarial function setting table of retirement pension insurance

T_x^*	δ	β	γ	λ	ω
62	0.04	0.006	0.001	2	100

The original data required for the calculation of the central mortality rate $m(x,t)$ are from the 1996-2018 China Demographic Year book [14], with an upper age limit ω of 100 years, and the mortality model parameters [8] are fitted by the maximum likelihood method [15].

According to the parameter setting table and the fitting value of the mortality model, the actuarial present value of the pension when the expected retirement age is delayed can be obtained by simulating calculation with R software, we can get TABLE III.

TABLE III. Simulation table of actuarial present value of retirement pension insurance

p	x	$\ddot{a}_{x:\overline{T_x^*-x}\rvert}$	$\ddot{a}_{x:\overline{T_x^*-x}\rvert}$	P
0. 6	30	4. 6629	16. 8976	0. 2759

III. CONCLUSION

In order to better cope with the pressure brought by the aging fiscal deficit, the country urgently needs to put forward the corresponding measures and feasible plans from the policy level, as the progress of our society entering the population ageing is speeding up, therefore, flexible retirement system has been put on the agenda, referring to the retirement system in developed countries, China's statutory retirement age in adults will be adjusted. In addition, the mortality rate of the elderly population will have a certain degree of discontinuity with the improvement of medical technology, quality of life and living environment. The population ageing is one of the country's most important tools for alleviating the problem. Its pricing depends on factors such as interest rates, mortality rates and retirement age; these three factors existing randomness and volatility in theory and practice, and have some discontinuity in the long run, which will cause certain financial risks to the industry. Therefore, it is necessary and urgent to measure the pricing of endowment insurance effectively through reasonable hypothesis. In this paper, we take retirement annuity as an example, assume the distribution of retirement age, and establish an interest rate model based on the volatility, taking the male population of our country as an example, this paper improves the existing population mortality model on the basis of predecessors. The model can get the survival probability by using the central mortality rate through Logistic transformation, the advantage of this method is that it provides an analytical formula for calculating the actuarial present value of retirement annuity, and accordingly obtains the expression of the second moment and the equilibrium net premium of retirement annuity. The pension pricing model discussed in this paper not only provides a theoretical reference for pension pricing, but also provides a practical application for insurance companies and government agencies in pension risk measurement.

ACKNOWLEDGEMENTS

This research was Supported by the Key Research Project of Suzhou University (Grant Nos.2017yzd16), Key Course Construction Project of Specialized Creation and Integration of Suzhou University (Grant Nos.szxy2020zckc13), Social Science Innovation and Development Research Project of Anhui Province(Grant Nos.2020CX104), Non-financial co-operation Project of Suzhou University(Grant Nos. 2020xhx118, 2021xhx103)

REFERENCES

[1] Zhou X(2014),The feasibility and difficulty of implementing the flexible retirement system in China, M.S. thesis, Southwestern University of Finance and Economics
[2] Nolde N, Parker G(2014), “Stochastic analysis of life insurance surplus”, in Insurance: Mathematics and Economics 56:1-13
[3] Dufresne D(2007),“Stochastic life annuities”, in North American Actuarial Journal 1:136-157
[4] Gnoatto A, Grasselli M(2014),“An affine multi-currency model with stochastic volatility and stochastic interest rates”, in SIAM Journal on Financial Mathematics 5:493-531
[5] Haastrecht A, Lord R, Pelsser A, Schrager D(2009), “Pricing long-dated insurance contracts with stochastic interest rates and stochastic volatility”, in Insurance: Mathematics and Economics 45:436-448
[6] Leveille G, Franck A(2011), “Covariance of discounted compound renewal sums with a stochastic interest rate”, in Scandinavian Actuarial Journal 2:138-154
[7] Shang Q, Qin X Z(2009),“The analysis on longevity risk of pension annuities under stochastic mortality and interest”, in Systems Engineering 191:56-61
[8] Haberman S, Renshaw A(2011),“A comparative study of parametric mortality projection models”, in Insurance Mathematics &Economics 1:35-55
[9] Currie I D, Durban M, Eilers P H C(2006), “Generalized liner array models with applications to multidimensional smoothing”, in Journal of the Royal Statistical Society 2:259-280
[10] Ma H F, Xiao H M, Zhao H Y(2020),“A New Mortality Model and Comparative Analysis Based on China’s Population Data”, in Journal of Quantitative Economics 3:99-106

[11] Cairns A J G, Blake D, Dowd K(2006),"A Two-Factor Model for Stochastic Mortality with Paramater Uncertainty Theory and Calibration", in Journal of Risk and Insurance 4:687-718

[12] Cairns A J G, Blake D, Dowd K(2006), "Pricing death: frameworks for the valuation and securitization of mortality risk", in ASTIN Bulletin 1:79-120

[13] Sun R(2016), "The Stochastic Actuarial Models and Simulation about Retirement Annuity under Flexible Retirement System", in Chinese Journal of Engineering Mathematics 2:111-120

[14] National Bureau of Statistics of the People's Republic of China (1996-2018).Demographic Year book of China, China Statistics Publishing House, 2020.

[15] Fan Y, Zhang N, Zhang W Y(2017), "Fitting and Prediction of Stochastic Mortality Models: a Comparative Analysis based on China's Male Population Mortality Data", in Insurance Studies 9:15-31