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Optimal Monitoring and Robust Control of Spatial Diffusion Processes in Uncertain Environments: A Mobile Sensing Approach

Xueming Qian[1,2,*], Baotong Cui[2]
[1]School of Internet of Things, Wuxi Vocational College of Science and Technology, Wuxi, Jiangsu, China
[2]School of Internet of Things Engineering, Jiangnan University, Wuxi, Jiangsu, China
*Corresponding Author.

Abstract:

This paper investigates the optimal monitoring and robust control of an array of forest fire processes modelled as spatial diffusion processes with uncertainty. An optimized framework is proposed for mobile measurement and control of forest fires using mobile actuator-sensor networks with multiple packet losses. By employing Lyapunov functional approach, some sufficient conditions are derived under decentralized output feedback control strategy. The robust stability criterion includes linear operator inequalities and velocity laws for mobile actuation-sensing devices. Also, the case of collocated control and non-collocated control are discussed in the results. Finally, a simulation example is provided to demonstrate the results obtained.

***Keywords**: Spatial diffusion processes, forest fire model, Mobile sensors, Mobile actuators, Uncertain, Multiple loss packets.*

I. INTRODUCTION

In recent years, large forest fires have occurred frequently around the world, causing huge economic losses. How to effectively monitor and control forest fires is becoming a growing concern. The modelling of forest fires is the basis for the study of forest fire monitoring and control. Numerous researchers, starting from statistical methods[1], machine learning[2], gray analysis[3], ecological mathematics[4], and linear systems[5], have conducted in-depth studies on forest fires and given relevant mathematical models, obtaining a series of good results. Further, when considering the spatio-temporal characteristics of fire occurrence in the forest fire model, the spatio-temporal spread-diffusion model of forest fires can be obtained from the physical mechanism, using the temperature field as the basic state to describe the fire and according to the heat conduction, radiation and convection laws[6-7]. These describe the diffusion process of forest fires can be expressed as parabolic partial differential equations[8], which are called distributed parameter systems(DPSs).

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With the rapid development of embedded technology, wireless sensor networks have been widely used due to its low cost, low power, and its sensing and computing ability. Especially in the monitoring of forest fires, wireless sensor networks have played an important role[9-10]. However, for unknown forest fires, the environment is harsh and uncertain, and the wireless sensor network nodes are easily damaged. Moreover, the need to place a large number of sensing nodes in the forest makes the cost and energy consumption increase significantly. Currently, mobile robotics is becoming increasingly sophisticated. Mobile actuation-sensing devices are obtained by adding sensors and actuators to mobile devices. By communicating several mobile actuation-sensing devices, their cooperation forms a mobile actuator-sensor network.

Earlier work on study of mobile actuator and sensor networks can be divided into two cases roughly. One is about the control problems on mobile actuator and sensor networks utilizing multi-agent systems[11-12]. The other is about actuating and sensing device optimization in DPSs. Orlov[13] studied the model of distributed parameter control systems in heat processing using sliding mode control method for the first time. The analysis and control of parabolic partial differential equations with input constraints based on Galerkin approximation is developed in [14]. The study on mobile control of moving actuators and sensors in processes governed by DPSs in [15]. Demetriou[16] provided an optimized framework for the performance improvement of mobile sensors and actuators in DPSs using the Lyapunov stability arguments. So far, a number of useful conclusions about control of DPSs were obtained.

Usually, parameter uncertainties are inevitable in dynamical system due to error of measurement and hardware implementation of system. They are probably one of the main sources contribute to instability of dynamical systems. Also, it is very common in engineering applications that the measurement output of mobile actuators and sensors networks is lost or partially lost due to different causes, i.e. sensor failure and missing measurement caused by the network. Wang and his collaborators consider missing measurements phenomenon in control problem of discrete stochastic nonlinear systems. There have been very few studies dealing with the control of spatial diffusion processes in uncertain environments with multiple missing measurements using mobile actuator and sensor networks, to the best of the authors' knowledge up to now.

In this paper, we will focus upon the optimal mobile monitoring and robust control of a class of diffusion processes in uncertain environments. The uncertainty of the diffusion system and the packet loss of multiple data in mobile sensing make the study more valuable for engineering applications. The sufficient conditions can be found by applying the Lyapunov functional for optimized stability are established including linear operator inequalities(LOIs) and velocity control law.

. And, the simplified stability scheme is also proposed by the velocity law of each moving actuator and sensor if an appropriate Lyapunov functional is introduced.

Notation and Preliminaries. The notation used in the paper is quite standard. Throughout this paper, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space with the norm $|\cdot|$. $f'(\xi)$ means the derivative of the function f with respect to ξ and $\dot{f}(t)$ means f relative to the rate of time change. The notation

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$\mathcal{P} > 0$ where $\mathcal{P}$ is symmetric, means that $\mathcal{P}$ is positive definite, whereas $\lambda_{\max}(\mathcal{P})$ (respectively, $\lambda_{\min}(\mathcal{P})$) denotes its largest (respectively, smallest) eigenvalue. In symmetric block matrices, the star $\star$ is used to denote an ellipsis term that is induced by symmetry.

Let $\mathcal{H}$ be a Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and corresponding induced norm $|\cdot|$. $\mathcal{L}(\mathcal{H})$ denotes all bounded linear operators from $\mathcal{H}$ to $\mathcal{H}$. Given $\mathcal{P}:\mathcal{H}\rightarrow\mathcal{H}$ be a linear operator with dense domain $\mathcal{D}(\mathcal{P})$ in $\mathcal{H}$, whereas $\mathcal{P}^*$ means its adjoint operator. The notation $\mathcal{P} > 0$ means that $\mathcal{P}$ is strictly positive definite, where $\mathcal{P}$ is self-adjoint operator in sense i.e. $\mathcal{P} = \mathcal{P}^*$ and there exists a constant $c > 0$ such that $\langle x, \mathcal{P}x\rangle \geq c\langle x, x\rangle$ and for all $x \in \mathcal{D}(\mathcal{P})$. $\mathcal{P} \geq 0$ means that nonnegative definite operator $\mathcal{P}$ is self-adjoint and $\langle x, \mathcal{P}x\rangle \geq 0$ for all $x \in \mathcal{D}(\mathcal{P})$. In addition, $\mathcal{I}$ is the identity operator. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator.

The domain of operator $\mathcal{A}$ can be defined in the following forms: $(x,y)_{\mathcal{D}(\mathcal{A})} = \langle x,y\rangle + \langle\mathcal{A}x,\mathcal{A}y\rangle, x,y \in \mathcal{D}(\mathcal{A})$ if the operator $\mathcal{A}$ generates a strongly continuous semigroup $T(t)$ on the Hilbert space $\mathcal{H}$. Furthermore, the induced norm $\| T(t) \|$ of the semigroup $T(t)$ satisfies $\| T(t) \| \leq \sigma e^{\omega t}$ with some constant σ and grown bound ω.

II. PROBLEM FORMULATION

In this paper, we consider an array of forest fire processes which described by spatial diffusion processes with uncertainty. This one-dimensional system represents the forest belt and its model can approximate the actual combustion process.

$$\begin{aligned}\frac{\partial Q(t,\xi)}{\partial t} &= \frac{\partial}{\partial \xi}\left(\alpha(\xi)\frac{\partial Q(t,\xi)}{\partial \xi}\right) - (a + \Delta a(t))Q(t,\xi) \\ &\quad + \sum_{i=1}^{n} f_i\left(\xi;\xi_i^a(t)\right)(b_i + \Delta b_i(t))u_i(t), \\ y(t) &= \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{bmatrix} = \begin{bmatrix} \int_0^l \gamma_1\, g_1\big(\xi;\xi_1^s(t)\big)c_1 Q(t,\xi)d\xi \\ \int_0^l \gamma_2\, g_2\big(\xi;\xi_2^s(t)\big)c_2 Q(t,\xi)d\xi \\ \vdots \\ \int_0^l \gamma_n\, g_n\big(\xi;\xi_n^s(t)\big)c_n Q(t,\xi)d\xi \end{bmatrix}, \#(1)\end{aligned}$$

subject to the Dilichlet boundary condition

$$Q(t,0) = Q(t,l) = 0, \#(2)$$

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and the initial condition

$$Q(0,\xi)=Q_0(\xi), \#(3)$$

where $Q(t,\xi)$ denotes temperature of the diffusion process, $\xi\in\Omega=[0,l]$ is the spatial variable and $t\in\mathbf{R}^+$ is the time variable. The transmission diffusion operator $\alpha(\xi)\geq\alpha_0>0$, a, b_i and c_i are constant parameters. The nonnegative smooth function $f_i(\xi;\xi_i^a(t))$ represents the spatial distribution of ith moving actuator, where $\xi_i^a(t)\in[0,l]$ is the time-varying centroid of ith actuator. Similarly, the spatial distribution of jth moving sensor is represented by the nonnegative smooth function $g_i(\xi;\xi_i^s(t))$, where $\xi_i^s(t)\in[0,l]$ is the time-varying centroid of ith sensor. The spatial distribution of moving actuators and sensors mentioned above shows each different actuator and sensor has different distribution. $u_i(t)$ denotes the ith component of a control signal.

In this paper, moving sensors for the measurement data missing are considered. $\Lambda_\gamma=\mathrm{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_n\}$ with the stochastic variable $\gamma_i\in\mathbf{R}$ is a Bernoulli distributed white sequence taking values of 1 and 0. It has

$$\begin{cases}\mathrm{Prob}\{\gamma_i=1\}=\overline{\gamma_i},\\ \mathrm{Prob}\{\gamma_i=0\}=1-\overline{\gamma_i}.\end{cases}$$

Here, $\overline{\gamma_i}\in[0,1]$ are known constants. We denote $\overline{\gamma_i}=\mathbb{E}\{\gamma_i\}$, while $\overline{\Lambda}_\gamma=\mathrm{diag}\{\overline{\gamma}_1,\overline{\gamma}_2,\cdots,\overline{\gamma}_n\}$.

Then, the system (1) can be rewritten in the following compact form:

$$\begin{cases}\dot{Q}(t)=(\mathcal{A}+\Delta\mathcal{A})Q(t)+\mathcal{F}\big(\xi^a(t)\big)(\mathcal{B}+\Delta\mathcal{B})u(t),\\ y(t)=\Lambda_\gamma\mathcal{C}\mathcal{G}\big(\xi^s(t)\big)Q(t),\end{cases}\#(4)$$

Here, the state space is $\mathcal{H}=L_2(\Omega)$, where $Q(t,\cdot)=\{Q(t,\xi):0\leq\xi\leq l\}$ is the instantaneous state of the system. Let infinitesimal operator $\mathcal{A}=\frac{d}{d\xi}(\alpha(\xi)\frac{d}{d\xi})-a$ and its domain is given by $\mathcal{D}(\mathcal{A})=\{\psi\in L_2(\Omega):\psi,\psi'\text{ are absolutely continuous},\psi''\in L_2(\Omega)\text{ and }\psi(0)=\psi(l)=0\}$. Obviously, since $\alpha(\xi)>0$, then the operator $\mathcal{A}$ is bounded and satisfy $-\mathcal{A}>0$. The infinitesimal operator $\mathcal{A}$ generates a strongly continuous semigroup $T(t), t\geq 0$ and the domain $\mathcal{D}(\mathcal{A})$ of the operator $\mathcal{A}$ is dense in $\mathcal{H}$. $\mathcal{B}$ and $\mathcal{C}$ denote, respectively, input coefficient operator and output coefficient operator.

Uncertain linear operators $\Delta\mathcal{A},\Delta\mathcal{B}\in\mathcal{L}(\mathcal{H})$ are bounded perturbations of the infinitesimal operator $\mathcal{A}$, $\mathcal{B}$ respectively. Such parameter uncertainties satisfy the following admissible condition:

$$\Delta\mathcal{A}^*\Delta\mathcal{A}\leq\hat{\mathcal{A}}, \Delta\mathcal{B}^*\Delta\mathcal{B}\leq\hat{\mathcal{B}}, \#(5)$$

where $\hat{\mathcal{A}}>0,\hat{\mathcal{B}}>0$.

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The input operator $\mathcal{F}(\xi^a(t))$ is linear and bounded, which is formulated as

$$\mathcal{F}(\xi^a(t))u(t) = [f_1(\xi_1^a(t)), f_2(\xi_2^a(t)), \cdots, f_n(\xi_n^a(t))] \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_n(t) \end{bmatrix},$$

with actuator location vector provided by $\xi^a(t) = [\xi_1^a(t), \xi_2^a(t), \cdots, \xi_n^a(t)]$. Similarly, the output operator $\mathcal{G}(\xi^s(t))$ is also linear and bounded, which is written as

$$\mathcal{G}(\xi^s(t))Q(t) = \begin{bmatrix} \int_0^l g_1(\xi; \xi_1^s(t))Q(t,\xi)d\xi \\ \int_0^l g_2(\xi; \xi_2^s(t))Q(t,\xi)d\xi \\ \vdots \\ \int_0^l g_n(\xi; \xi_n^s(t))Q(t,\xi)d\xi \end{bmatrix},$$

where the output measurement operator is the vector of sensor location parameterized by $\xi^s(t) = [\xi_1^s(t), \xi_2^s(t), \cdots, \xi_n^s(t)]$. According to the description in (1), it is easy to get $\mathcal{F}(\xi^a(t)) \geq 0$ and $\mathcal{G}^*(\xi^s(t)) \geq 0$.

In this paper, we consider the following decentralized output feedback control strategy:

$$u_i(t) = -k_i y_i(t) = -k_i \int_0^l \gamma_i c_i g_i\big(\xi; \xi_i^s(t)\big)Q(t,\xi)d\xi, \#(6)$$

for $k_i > 0, i = 1,2,\cdots,n$.

Also, it can be written in matrix form

$$u(t) = -Ky(t), \#(7)$$

with $K = \mathrm{diag}\{k_1, k_2, \cdots, k_n\}$ is the control gain matrix.

In fact, the locally decentralized controller (6) or (7) for actuators and sensors can be utilized effectively owing to the advanced technology of actuation and sensing. The recent advances of micro-electro-mechanical systems, make it easy to implement this class of controllers in a large number of moving actuators and sensors.

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For presentation convenience, we denote $\tilde{\mathcal{A}} = \mathcal{A} + \Delta\mathcal{A}$ and $\tilde{\mathcal{B}} = \mathcal{B} + \Delta\mathcal{B}$. With the decentralized output feedback control policy (6), we can study the system in the following form

$$\begin{aligned} \dot{Q}(t) = & \left(\tilde{\mathcal{A}} - \tilde{\mathcal{B}}\left(\xi^a(t)\right)K\tilde{\mathcal{C}}\left(\xi(t)\right)\right)Q(t) \\ = & \mathcal{A}_c\left(\xi(t)\right)Q(t) \end{aligned} \#(8)$$

where $\tilde{\mathcal{B}}(\xi^a(t)) = \mathcal{F}(\xi^a(t))\tilde{\mathcal{B}}, \tilde{\mathcal{C}}(\xi^s(t)) = \Lambda_\gamma \mathcal{C}\mathcal{G}(\xi^s(t))$ and denotes $\mathbb{E}\{\tilde{\mathcal{C}}(\xi^s(t))\} = \mathcal{C}(\xi^s(t))$. From distributed parameter system (8) with parameter uncertainties, $\Delta\mathcal{A}$ and $\Delta\mathcal{B}$ are bounded linear operators. Then the operator $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ are dissipative operators follows from an application of Kato-Rellich Theorem. In this case, one may easily assume that $\mathcal{F}(\xi^a(t))$ and $\mathcal{G}(\xi^s(t))$ are commutative, then the product of $\mathcal{F}(\xi^a(t))$ and $\mathcal{G}(\xi^s(t))$ is nonnegative definite. Consequently, $\tilde{\mathcal{B}}(\xi^a(t))K\tilde{\mathcal{C}}(\xi^s(t))$ is bounded and nonnegative definite. Also, $-\mathcal{A}_c(\xi(t))$ is bounded and nonnegative definite.

Definition 1 *The spatial diffusion processes is said to be globally asymptotically stable in the mean square if*

$$\lim_{t\to+\infty} \mathbb{E}|Q(t)|^2 = 0, \qquad (9)$$

holds.

Definition 2 *The uncertain spatial diffusion processes (4) is said to be robustly globally asymptotically stable in the mean square if the system (4) is globally asymptotically stable in the mean square for all admissible parameter uncertainties.*

III. MAIN RESULTS AND PROOFS

To achieve the major results, the following lemmas are introduced..

Lemma 1 *Given linear operators* $\mathcal{P}$ *and* $\mathcal{Q}$ *and a scalar* $\beta > 0$. *Then for any* $x, y \in \mathcal{H}$,

$$2\langle x, \mathcal{P}\mathcal{Q}y\rangle \leq \beta^{-1}\langle x, \mathcal{P}\mathcal{P}^*x\rangle + \beta\langle y, \mathcal{Q}^*\mathcal{Q}y\rangle.$$

Proof: It is readily seen that

$$\begin{aligned} 0 \leq & \langle \mathcal{P}^*x - \beta\mathcal{Q}y, \mathcal{P}^*x - \beta\mathcal{Q}y\rangle \\ = & \langle x, \mathcal{P}\mathcal{P}^*x\rangle - 2\beta\langle x, \mathcal{P}\mathcal{Q}y\rangle + \beta^2\langle y, \mathcal{Q}^*\mathcal{Q}y\rangle \end{aligned}$$

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Hence, we have

$$2\langle x,\mathcal{P}\mathcal{Q}y\rangle \le \beta^{-1}\langle x,\mathcal{P}\mathcal{P}^{*}x\rangle + \beta\langle y,\mathcal{Q}^{*}\mathcal{Q}y\rangle.$$

Especially,

$$2\langle x,\mathcal{P}\mathcal{Q}y\rangle \le \beta^{-1}\langle x,\mathcal{P}\mathcal{P}^{*}x\rangle + \beta\langle y,\hat{\mathcal{Q}}\,y\rangle.$$

if $\mathcal{Q}$ satisfying $\mathcal{Q}^{*}\mathcal{Q} \le \hat{\mathcal{Q}}$.

Lemma 2 *(Barbalat's Lemma) A function* $f(t)$ *satisfies* $\lim_{t\to+\infty} f(t) = 0$, *if* $f(t) \ge 0$ *is Lebesgue integrable and uniformly continuous on* $[0,+\infty)$.

The main results of this paper are given in the following theorem.

Theorem 1 *The uncertain spatial diffusion processes with multiple packet losses (4) is robustly globally asymptotically stable in the mean square if there exist constants* $\beta > 0$, *a linear operator* $\mathcal{P}(\xi(t)) > 0$ *subject to* $c\langle Q(t),Q(t)\rangle \le \langle Q(t),\mathcal{P}(\xi(t))Q(t)\rangle \le b\,[\langle Q(t),Q(t)\rangle + \langle\mathcal{A}Q(t),\mathcal{A}Q(t)\rangle]$, *for two positive constants* b, c *such that the following operator-dependent LOIs hold in the Hilbert space* $\mathcal{D}(\mathcal{A})$*:*

$$\Psi(\xi(t)) = \begin{bmatrix} \Upsilon(\xi(t)) & \mathcal{P}(\xi(t)) & \mathcal{P}(\xi(t))\mathcal{F}(\xi^{a}(t)) & \beta_2\mathcal{G}^{*}(\xi^{s}(t))\mathcal{C}^{*}\overline{\Lambda}_{\gamma}K\,\hat{\mathcal{B}} \\ \star & -\beta_1 I & 0 & 0 \\ \star & \star & -\beta_2 I & 0 \\ \star & \star & \star & -\beta_2\,\hat{\mathcal{B}} \end{bmatrix} < 0, \#(11)$$

Where

$$\begin{aligned}\Upsilon(\xi(t)) = \; & \mathcal{P}(\xi(t))\left(\mathcal{A} - \mathcal{F}(\xi^{a}(t))\mathcal{B}K\overline{\Lambda}_{\gamma}\mathcal{C}\mathcal{G}(\xi^{s}(t))\right) \\ & +(\mathcal{A} - \mathcal{F}(\xi^{a}(t))\mathcal{B}K\overline{\Lambda}_{\gamma}\mathcal{C}\mathcal{G}(\xi^{s}(t)))^{*}\mathcal{P}(\xi(t)) + \beta_1\,\hat{\mathcal{A}}\,,\end{aligned} \#(12)$$

and each moving agent's velocity law is provided by

$$\dot{\xi}\,(t) = -\rho^{d}\left\langle Q(t), \frac{\partial\mathcal{P}(\xi(t))}{\partial t}Q(t)\right\rangle \#(13)$$

with $\rho^{d} > 0$.

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Proof: The following operator-dependent Lyapunov functional is used to establish the stability conditions.

$$V(t) = \langle Q(t), \mathcal{P}\big(\xi(t)\big)Q(t)\rangle. \#(14)$$

The infinitesimal operator $\mathcal{L}$ of $V(t)$ is determined as

$$\mathcal{L}V(Q(t),t) = \lim_{\Delta\to 0^+}\frac{1}{\Delta}\{\mathbb{E}\{V(Q(t+\Delta),t+\Delta)|Q(t)\} - V(Q(t),t)\}. \#(15)$$

Along the trajectories of (8), the derivative of the Lyapunov functional

$$\mathcal{L}V(t) = \mathbb{E}\langle \dot{x}(t), \mathcal{P}\big(\xi(t)\big)Q(t)\rangle + \mathbb{E}\langle Q(t), \mathcal{P}\big(\xi(t)\big)\dot{x}(t)\rangle + \left\langle Q(t), \frac{d\mathcal{P}\big(\xi(t)\big)}{dt}Q(t)\right\rangle. \#(16)$$

Firstly, substituting (8) into the first and second term of (16), and considering $\mathcal{A}_c(\xi(t))$ can be decompose as $\mathcal{A}_c(\xi(t)) = \mathcal{A}(\xi(t)) + \Delta\mathcal{A}(\xi(t))$, where $\mathcal{A}(\xi(t)) = \mathcal{A} - \mathcal{F}(\xi^a(t))\mathcal{B}K\overline{\Lambda}_\gamma\mathcal{C}\mathcal{G}(\xi^s(t))$ and $\Delta\mathcal{A}(\xi(t)) = \Delta\mathcal{A} - \mathcal{F}(\xi^a(t))\Delta\mathcal{B}K\overline{\Lambda}_\gamma\mathcal{C}\mathcal{G}(\xi^s(t))$. Utilizing the fact that the linear operator $\mathcal{P}(\xi(t))$ is self-adjoint, and combining the admissible condition of parameter uncertainties and Lemma 1, we can deduce

$$\begin{aligned}
& 2\mathbb{E}\langle Q(t), \mathcal{P}(\xi(t))\,\dot{Q}(t)\rangle \\
= \; & 2\mathbb{E}\langle Q(t), \mathcal{P}(\xi(t))\mathcal{A}_c(\xi(t))Q(t)\rangle \\
= \; & 2\langle Q(t), \mathcal{P}(\xi(t))(\mathcal{A}(\xi(t)) + \Delta\mathcal{A}(\xi(t)))Q(t)\rangle \\
\leq \; & 2\langle Q(t), \mathcal{P}(\xi(t))\mathcal{A}(\xi(t))Q(t)\rangle + \beta_1^{-1}\langle Q(t), \mathcal{P}(\xi(t))\mathcal{P}^*(\xi(t))Q(t)\rangle \\
& +\beta_1\langle Q(t), \hat{\mathcal{A}}\,Q(t)\rangle + \beta_2^{-1}\langle Q(t), (\mathcal{P}(\xi(t))\mathcal{F}(\xi^a(t)))(\mathcal{P}(\xi(t))\mathcal{F}(\xi^a(t)))^*Q(t)\rangle \\
& +\beta_2\langle Q(t), (K\overline{\Lambda}_\gamma\mathcal{C}\mathcal{G}(\xi^s(t)))^*\,\hat{\mathcal{B}}\,(K\overline{\Lambda}_\gamma\mathcal{C}\mathcal{G}(\xi^s(t)))Q(t)\rangle \\
= \; & \langle Q(t), \Psi(\xi(t))Q(t)\rangle.
\end{aligned}$$

where $\Psi(\xi(t))$ are defined as in (11).

Next, the third term of (16) is

$$\langle Q(t), \frac{d\mathcal{P}(\xi(t))}{dt}Q(t)\rangle = \langle Q(t), \dot{\xi}(t)\frac{\partial\mathcal{P}(\xi(t))}{\partial\xi}Q(t)\rangle,$$

and, for ρ^d is any positive gain, which can be made negative by selecting

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$$\dot{\xi}(t) = -\rho^d \langle Q(t), \frac{\partial \mathcal{P}(\xi(t))}{\partial \xi} Q(t) \rangle.$$

It follows from (11) -(13) that

$$\mathcal{L}V(t) \leq -c\ \mathbb{E}|Q(t)|^2, t \geq 0, \quad (17)$$

Therefore, we have

$$\mathbb{E}V(t) - V(0) \leq -c \int_0^t \mathbb{E}\,|Q(s)|^2 ds$$

whereas the linear positive definite operator $\mathcal{P}(\xi(t))\colon \mathcal{D}(\mathcal{A}) \to \mathcal{H}$ satisfy the inequality (10). Hence, it implies that

$$\int_0^t \mathbb{E}\,|Q(s)|^2 ds \leq \alpha \ \parallel Q_0 \parallel^2,$$

where $\alpha = \frac{b}{c}$. Namely,

$$\int_0^t \mathbb{E}\,|Q(s)|^2 ds \leq \alpha(|Q_0|^2 + |\mathcal{A}Q_0|^2) < +\infty.$$

Moreover, it is simple to confirm that $\mathbb{E}|Q(t)|^2$ is uniformly continuous on $[0, +\infty)$. From Lemma 2, we can obtain

$$\lim_{t \to +\infty} \mathbb{E}|Q(t)|^2 = 0.$$

Accordingly, the system (4) is globally asymptotically stable.

Remark 1: It is worth mentioning that the system (4) also can be proved globally exponentially stable in the mean square. In this case, we choose the Lyapunov functional as $V(t) = e^{rt}\langle Q(t), \mathcal{P}(\xi(t))Q(t)\rangle$, the similar results can be obtained easily, and the proof is omitted here.

Theorem 1 presents an abstract stability framework on uncertain distributed parameter systems with multiple missing measurements. In the following, the general spatial distribution of moving sensors and actuators is considered under the proposed framework. Simultaneously, the velocity law of each moving actuator and sensor will be proposed for the stability conditions of such system and enhance the controller performance via choose the proper $\mathcal{P}(\xi(t))$.

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The spatial distribution of each mobile actuator which centroid at ξ_i^a, given by

$$f_i(\xi;\xi_i^a)=\begin{cases} f_i(\xi) & \text{if} \quad \xi\in[\xi_i^a-\varepsilon_i^-,\xi_i^a+\varepsilon_i^+] \\ 0 & \text{otherwise}\end{cases}, \#(18)$$

or depict

$$f_i(\xi;\xi_i^a)=f_i(\xi)\left[H\left(\xi-(\xi_i^a-\varepsilon_i^-)\right)-H\left(\xi-(\xi_i^a+\varepsilon_i^+)\right)\right], \#(19)$$

what employs 2 different Heaviside functions. The spatial distribution is followed by each mobile sensor with its centroid at ξ_i^s.

$$g_i(\xi;\xi_i^s)=\begin{cases} g_i(\xi) & \text{if} \quad \xi\in[\xi_i^s-\vartheta_i^-,\xi_i^s+\vartheta_i^+] \\ 0 & \text{otherwise}\end{cases}, \#(20)$$

or depict

$$g_i(\xi;\xi_i^s)=g_i(\xi)\left[H\left(\xi-(\xi_i^s-\vartheta_i^-)\right)-H\left(\xi-(\xi_i^s+\vartheta_i^+)\right)\right]. \#(21)$$

Remark 2: Here, two notes are given in follows. For one hand, two edges of each moving agent's range is not symmetry, and more general than symmetrical one. For the other hand, each actuator and sensor may have different spatial distribution each other in a homogeneous network. Indeed, the distribution of each agent also can be piecewise smooth in local. For example, one actuator's distribution in symmetric interval $[\xi_i^a-\varepsilon,\xi_i^a+\varepsilon]$ may be shown as

$$f_i(\xi)=\begin{cases} \frac{1}{\varepsilon} & \text{if} \quad \xi\in[\xi_i^a-\frac{\varepsilon}{3},\xi_i^a+\frac{\varepsilon}{3}] \\ \frac{1}{2\varepsilon} & \text{if} \quad \xi\in[\xi_i^a-\varepsilon,\xi_i^a-\frac{\varepsilon}{3}]\cup[\xi_i^a+\frac{\varepsilon}{3},\xi_i^a+\varepsilon]\end{cases}.$$

Therefore, the more general distribution of each actuation-sensing device can given by

$$\begin{aligned} f_i(\xi;\xi_i^a) &= \sum_{j=1}^{m} f_{ij}(\xi)[H(\xi-(\xi_{i0}^a+(j-1)\Delta h))-H(\xi-(\xi_{i0}^a+j\Delta h))], \\ g_i(\xi;\xi_i^s) &= \sum_{j=1}^{m} g_{ij}(\xi)[H(\xi-(\xi_{i0}^s+(j-1)\Delta h))-H(\xi-(\xi_{i0}^s+j\Delta h))], \end{aligned}$$

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where $\Delta h = \frac{\varepsilon^+ + \varepsilon^-}{m}$, $\xi_{i0}^a = \xi_i^a - \varepsilon^-$ and $\xi_{i0}^s = \xi_i^s - \vartheta^-, j = 1,2,\cdots,m$. And f_{ij} and g_{ij} can have the same expression, also they can have the different one each other.

Theorem 2 *Under the decentralized output feedback control strategy (6), uncertain spatial diffusion processes with multiple packet losses (4) is robustly globally asymptotically stable in the mean square, if the spatial distribution of mobile actuators and sensors are given by (18) and (20) respectively and satisfy* $[\xi_i^a - \varepsilon_i^-, \xi_i^a + \varepsilon_i^+] \cap [\xi_i^s - \vartheta_i^-, \xi_i^s + \vartheta_i^+] \neq \varnothing$, *such that the following velocity law of each moving agent holds,*

$$\dot{\xi}_i^a(t) = -\rho_i^a W_i^a b_i k_i \bar{\gamma}_i c_i \quad (24)$$

$$\dot{\xi}_i^s(t) = -\rho_i^s W_i^s b_i k_i \bar{\gamma}_i c_i \quad (25)$$

with $\rho_i^a > 0$ *and* $\rho_i^s > 0, i = 1,2,\cdots,n,$ *are velocity gain of each actuation-sensing device, where the expression of* W_i^a *and* W_i^s *are depend on the mobile actuators and sensors intersecting part of the spatial distribution of zone, also express as following:*

(i) $[\xi_i^a - \varepsilon_i^-, \xi_i^a + \varepsilon_i^+] \subset [\xi_i^s - \vartheta_i^-, \xi_i^s + \vartheta_i^+]$

$$W_i^a = \int_{\xi_i^a - \varepsilon_i^-}^{\xi_i^a + \varepsilon_i^+} f'_i(\xi) g_i(\xi) x^2(t,\xi) d\xi + f_i(\xi_i^a - \varepsilon_i^- + 0) g_i(\xi_i^a - \varepsilon_i^-) x^2(t, \xi_i^a - \varepsilon_i^-) - f_i(\xi_i^a + \varepsilon_i^+ - 0) g_i(\xi_i^a + \varepsilon_i^+) x^2(t, \xi_i^a + \varepsilon_i^+), \quad (26)$$

$$W_i^s = \int_{\xi_i^a - \varepsilon_i^-}^{\xi_i^a + \varepsilon_i^+} g'_i(\xi) f_i(\xi) x^2(t,\xi) d\xi; \quad (27)$$

(ii) $[\xi_i^s - \vartheta_i^-, \xi_i^s + \vartheta_i^+] \subset [\xi_i^a - \varepsilon_i^-, \xi_i^a + \varepsilon_i^+]$

$$W_i^a = \int_{\xi_i^s - \vartheta_i^-}^{\xi_i^s + \vartheta_i^+} f'_i(\xi) g_i(\xi) x^2(t,\xi) d\xi, \quad (28)$$

$$W_i^s = \int_{\xi_i^s - \vartheta_i^-}^{\xi_i^s + \vartheta_i^+} g'_i(\xi) f_i(\xi) x^2(t,\xi) d\xi + g_i(\xi_i^s - \vartheta_i^- + 0) f_i(\xi_i^s - \vartheta_i^-) x^2(t, \xi_i^s - \vartheta_i^-) - g_i(\xi_i^s + \vartheta_i^+ - 0) f_i(\xi_i^s + \vartheta_i^+) x^2(t, \xi_i^s + \vartheta_i^+); \quad (29)$$

(iii) $[\xi_i^s - \vartheta_i^-, \xi_i^s + \vartheta_i^+] \cap [\xi_i^a - \varepsilon_i^-, \xi_i^a + \varepsilon_i^+] = [\xi_i^a - \varepsilon_i^-, \xi_i^s + \vartheta_i^+]$

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$$W_i^a = \int_{\xi_i^a-\varepsilon_i^-}^{\xi_i^s+\vartheta_i^+} f'_i(\xi)g_i(\xi)x^2(t,\xi)d\xi + f_i(\xi_i^a-\varepsilon_i^-+0)g_i(\xi_i^a-\varepsilon_i^-)x^2(t,\xi_i^a-\varepsilon_i^-), \quad (30)$$

$$W_i^s = \int_{\xi_i^a-\varepsilon_i^-}^{\xi_i^s+\vartheta_i^+} g'_i(\xi)f_i(\xi)x^2(t,\xi)d\xi - g_i(\xi_i^s+\vartheta_i^+-0)f_i(\xi_i^s+\vartheta_i^+)x^2(t,\xi_i^s+\vartheta_i^+); \quad (31)$$

(iv)$[\xi_i^a-\varepsilon_i^-,\xi_i^a+\varepsilon_i^+]\cap[\xi_i^s-\vartheta_i^-,\xi_i^s+\vartheta_i^+] = [\xi_i^s-\vartheta_i^-,\xi_i^a+\varepsilon_i^+]$

$$W_i^a = \int_{\xi_i^s-\vartheta_i^-}^{\xi_i^a+\varepsilon_i^+} f'_i(\xi)g_i(\xi)x^2(t,\xi)d\xi - f_i(\xi_i^a+\varepsilon_i^+-0)g_i(\xi_i^a+\varepsilon_i^+)x^2(t,\xi_i^a+\varepsilon_i^+), \quad (32)$$

$$W_i^s = \int_{\xi_i^s-\vartheta_i^-}^{\xi_i^a+\varepsilon_i^+} g'_i(\xi)f_i(\xi)x^2(t,\xi)d\xi + g_i(\xi_i^s-\vartheta_i^-+0)f_i(\xi_i^s-\vartheta_i^-)x^2(t,\xi_i^s-\vartheta_i^-). \quad (33)$$

Proof: Consider the following Lyapunov functional, which is operator-dependent.

$$V(t) = -\langle Q(t), \mathcal{A}(\xi(t))Q(t)\rangle, \#(34)$$

where the boundedness and nonnegative definite of the operator $-\mathcal{A}(\xi(t))$ easily see from the discussion above.

The infinitesimal operator $\mathcal{L}V$ along (34) is given by

$$\mathcal{L}V = -\mathbb{E}\langle \dot{Q}(t), \mathcal{A}(\xi(t))Q(t)\rangle - \mathbb{E}\langle Q(t), \mathcal{A}(\xi(t))\dot{Q}(t)\rangle - \left\langle Q(t), \frac{d\mathcal{A}(\xi(t))}{dt}Q(t)\right\rangle. \#(35)$$

By using the properties of the operators $-\mathcal{A}(\xi(t))$ and $-\mathcal{A}_c(\xi(t))$, the following results deduced easily.

$$-\mathbb{E}\langle \dot{Q}(t), \mathcal{A}(\xi(t))Q(t)\rangle - \mathbb{E}\langle Q(t), \mathcal{A}(\xi(t))\dot{Q}(t)\rangle \le 0. \#(36)$$

Then, the third one of (35) has

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$$
\begin{aligned}
&-\left\langle Q(t),\frac{d\mathcal{A}(\xi(t))}{dt}Q(t)\right\rangle\\
=&\left\langle Q(t),\frac{d}{dt}\left(\mathcal{F}(\xi^a(t))\mathcal{B}K\bar{\Lambda}_\gamma\mathcal{C}\mathcal{G}(\xi^s(t))\right)Q(t)\right\rangle\\
=&\left\langle Q(t),\dot{\xi}^a(t)\frac{\partial\mathcal{F}(\xi^a(t))}{\partial\xi}\mathcal{B}K\bar{\Lambda}_\gamma\mathcal{C}\mathcal{G}(\xi^s(t))Q(t)\right\rangle\\
&+\left\langle Q(t),\mathcal{F}(\xi^a(t))\mathcal{B}K\bar{\Lambda}_\gamma\mathcal{C}\dot{\xi}^s(t)\frac{\partial\mathcal{G}(\xi^s(t))}{\partial\xi}Q(t)\right\rangle
\end{aligned}
\#(37)
$$

There are two parts to the formula (37) proposed here: one is used to determine the velocity of ith moving actuating device; and the other is employed to determine the velocity of ith moving sensing device.

The first term in (37) can be written in terms of the integral representation, that is,

$$
\begin{aligned}
&\langle Q(t),\dot{\xi}^a(t)\frac{\partial\mathcal{F}(\xi^a(t))}{\partial\xi}\mathcal{B}K\bar{\Lambda}_\gamma\mathcal{C}\mathcal{G}(\xi^s(t))Q(t)\rangle\\
=&\int_0^l\dot{\xi}^a(t)\frac{\partial f(\xi;\xi^a)}{\partial\xi}Q^2(t,\xi)\mathcal{B}K\bar{\Lambda}_\gamma\mathcal{C}g(\xi;\xi^s)d\xi\\
=&\sum_{i=1}^n\dot{\xi}_i^a(t)\int_0^l\frac{\partial}{\partial\xi}[f_i(\xi)(H(\xi-(\xi_i^a-\varepsilon_i^-))-H(\xi-(\xi_i^a+\varepsilon_i^+)))]Q^2(t,\xi)\\
&\quad\times[g_i(\xi)(H(\xi-(\xi_i^s-\vartheta_i^-))-H(\xi-(\xi_i^s+\vartheta_i^+)))]d\xi b_ik_i\bar{\gamma}_ic_i\\
=&\sum_{i=1}^n\dot{\xi}_i^a(t)\int_{\xi_i^s-\vartheta_i^-}^{\xi_i^s+\vartheta_i^+}[f'_i(\xi)(H(\xi-(\xi_i^a-\varepsilon_i^-))-H(\xi-(\xi_i^a+\varepsilon_i^+)))\\
&\quad+f_i(\xi_i^a-\varepsilon_i^-+0)\delta(\xi-(\xi_i^a-\varepsilon_i^-))\\
&-f_i(\xi_i^a+\varepsilon_i^+-0)\delta(\xi-(\xi_i^a+\varepsilon_i^+))]Q^2(t,\xi)g_i(\xi)d\xi b_ik_i\bar{\gamma}_ic_i
\end{aligned}
$$

For presentation convenience, we denote

$$
\begin{aligned}
W_i^a=&\int_{\xi_i^s-\vartheta_i^-}^{\xi_i^s+\vartheta_i^+}[f'_i(\xi)(H(\xi-(\xi_i^a-\varepsilon_i^-))-H(\xi-(\xi_i^a+\varepsilon_i^+)))+f_i(\xi_i^a-\varepsilon_i^-+0)\delta(\xi-(\xi_i^a-\varepsilon_i^-))\\
&-f_i(\xi_i^a+\varepsilon_i^+-0)\delta(\xi-(\xi_i^a+\varepsilon_i^+))]Q^2(t,\xi)g_i(\xi)d\xi.
\end{aligned}
$$

And, for ρ_i^a is any positive gain, it can be made negative by the selecting

$$\dot{\xi}_i^a(t)=-\rho_i^aW_i^ab_ik_i\bar{\gamma}_ic_i,$$

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where the expression of W_i^a can be calculated as the form of (26) in $[\xi_i^a - \varepsilon_i^-, \xi_i^a + \varepsilon_i^+]$, (28) in $[\xi_i^s - \vartheta_i^-, \xi_i^s + \vartheta_i^+]$, (30) in $[\xi_i^a - \varepsilon_i^-, \xi_i^s + \vartheta_i^+]$ and (32) in $[\xi_i^s - \vartheta_i^-, \xi_i^a + \varepsilon_i^+]$.

The second term in (37) is calculated as follows

$$\begin{aligned}
&\langle Q(t), \mathcal{F}(\xi^a(t))\mathcal{B}K\overline{\Lambda}_\gamma \mathcal{C}\dot{\xi}^s(t)\frac{\partial \mathcal{G}(\xi^s(t))}{\partial \xi}Q(t)\rangle \\
=\ &\int_0^l f(\xi;\xi^a)\mathcal{B}K\overline{\Lambda}_\gamma \mathcal{C}\dot{\xi}^s(t)\frac{\partial g(\xi;\xi^s)}{\partial \xi}Q^2(t,\xi)d\xi \\
=\ &\sum_{i=1}^n \dot{\xi}_i^s(t)\int_0^l \frac{\partial}{\partial \xi}[g_i(\xi)(H(\xi-(\xi_i^s-\vartheta_i^-))-H(\xi-(\xi_i^s+\vartheta_i^+)))]Q^2(t,\xi) \\
&\quad \times [f_i(\xi)(H(\xi-(\xi_i^a-\varepsilon_i^-))-H(\xi-(\xi_i^a+\varepsilon_i^+)))]d\xi b_i k_i \overline{\gamma}_i c_i \\
=\ &\sum_{i=1}^n \dot{\xi}_i^s(t)\int_{\xi_i^a-\varepsilon_i^-}^{\xi_i^a+\varepsilon_i^+}[g'_i(\xi)(H(\xi-(\xi_i^s-\vartheta_i^-))-H(\xi-(\xi_i^s+\vartheta_i^+))) \\
&\quad + g_i(\xi_i^s-\vartheta_i^-+0)\delta(\xi-(\xi_i^s-\vartheta_i^-)) \\
&- g_i(\xi_i^s+\vartheta_i^+-0)\delta(\xi-(\xi_i^s+\vartheta_i^+))]Q^2(t,\xi)f_i(\xi)d\xi b_i k_i \overline{\gamma}_i c_i
\end{aligned}$$

We denote

$$\begin{aligned}
W_i^s = &\int_{\xi_i^a-\varepsilon_i^-}^{\xi_i^a+\varepsilon_i^+}[g'_i(\xi)\left(H\left(\xi-(\xi_i^s-\vartheta_i^-)\right)-H\left(\xi-(\xi_i^s+\vartheta_i^+)\right)\right)+g_i(\xi_i^s-\vartheta_i^-+0)\delta\left(\xi-(\xi_i^s-\vartheta_i^-)\right) \\
&- g_i(\xi_i^s+\vartheta_i^+-0)\delta(\xi-(\xi_i^s+\vartheta_i^+))]Q^2(t,\xi)f_i(\xi)d\xi.
\end{aligned} \#(39)$$

Then, choose

$$\dot{\xi}_i^s(t) = -\rho_i^s W_i^s b_i k_i \overline{\gamma}_i c_i,$$

for ρ_i^s is any positive gain. W_i^s can be calculated as the expression of (27) in $[\xi_i^a - \varepsilon_i^-, \xi_i^a + \varepsilon_i^+]$, (29) in $[\xi_i^s - \vartheta_i^-, \xi_i^s + \vartheta_i^+]$, (31) in $[\xi_i^a - \varepsilon_i^-, \xi_i^s + \vartheta_i^+]$ and (33) in $[\xi_i^s - \vartheta_i^-, \xi_i^a + \varepsilon_i^+]$.

From the above discussion, we have $\mathcal{L}V(t) \le -c_1\ \mathbb{E}|Q(t)|^2, t \ge 0$, whereas the linear positive definite and bounded operator $-\mathcal{A}(\xi(t))$ on $\mathcal{D}(\mathcal{A})$ satisfy the following inequality:

$$c_1\langle Q(t), Q(t)\rangle \le \langle Q(t), -\mathcal{A}(\xi(t))Q(t)\rangle \le b_1[\langle Q(t), Q(t)\rangle + \langle \mathcal{A}Q(t), \mathcal{A}Q(t)\rangle], \#(40)$$

for $b_1 > 0$ and $c_1 > 0$. We can deduce that there exists $\alpha_1 > 0$ such that

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$$\int_0^t \mathbb{E}\,|Q(s)|^2 ds \leq \alpha_1(|Q_0|^2 + |\mathcal{A}Q_0|^2),$$

in the similar way that Theorem 1 was proved.

From Lemma 2, it implies that

$$\lim_{t\to+\infty} \mathbb{E}|Q(t)|^2 = 0.$$

Therefore, the system (4) is globally asymptotically stable if the velocity law of each moving actuation-sensing device is given in (24) and (25), when their spatial distribution is (18) and (20) respectively.

IV. DISCUSSION

The system (4) with spatial distribution of each moving agent is rather general. Specially, consider the scenario when there are no uncertainties, in which case system (4) may be altered to the following model.

$$\frac{\partial Q(t,\xi)}{\partial t} = \frac{\partial}{\partial \xi}(\alpha(\xi)\frac{\partial Q(t,\xi)}{\partial \xi}) - aQ(t,\xi) + \sum_{i=1}^{n} f\,(\xi;\xi_i^a(t))b_i u_i(t),$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{bmatrix} = \begin{bmatrix} \int_0^l \gamma_1\, g\big(\xi;\xi_1^s(t)\big)c_1 Q(t,\xi)d\xi \\ \int_0^l \gamma_2\, g\big(\xi;\xi_2^s(t)\big)c_2 Q(t,\xi)d\xi \\ \vdots \\ \int_0^l \gamma_n\, g\big(\xi;\xi_n^s(t)\big)c_n Q(t,\xi)d\xi \end{bmatrix}. \#(41)$$

where the spatial distribution of each moving actuator which centroid at ξ_i^a, given by

$$f(\xi;\xi_i^a) = \begin{cases} f(\xi) & \text{if} \quad \xi \in [\xi_i^a - \varepsilon^-, \xi_i^a + \varepsilon^+] \\ 0 & \text{otherwise} \end{cases} \#(42)$$

Each moving sensor which centroid at ξ_i^s follow the spatial distribution as

$$g(\xi;\xi_i^s) = \begin{cases} g(\xi) & \text{if} \quad \xi \in [\xi_i^s - \varepsilon^-, \xi_i^s + \varepsilon^+] \\ 0 & \text{otherwise} \end{cases} \#(43)$$

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Remark 3: The spatial distribution of each mobile agent discussed in (42) and (43), implies that the mobile actuator and sensor networks is homogeneous. In such network, every actuator and sensor is identical to each other, only different at the location of their centroids ξ_i^a and ξ_i^s. That is $f_i(\xi) = f(\xi;\xi_i^a)$ and $g_i(\xi) = g(\xi;\xi_i^s), i = 1,2,\cdots,n$.

Corollary 1 *Under the decentralized output feedback control strategy (6), spatial diffusion processes with multiple packet losses (4) is globally asymptotically stable in the mean square, if the spatial distribution of mobile actuators and sensors are given by (42) and (43) respectively and satisfy* $[\xi_i^a - \varepsilon^-, \xi_i^a + \varepsilon^+] = [\xi_i^s - \vartheta^-, \xi_i^s + \vartheta^+]$, *such that the following velocity law of each moving agent holds,*

$$\begin{aligned}\dot{\xi}_i^a(t) = & -\rho_i^a b_i k_i \bar{\gamma}_i c_i [\int_{\xi_i^s-\vartheta^-}^{\xi_i^s+\vartheta^+} f'(\xi) g(\xi) Q^2(t,\xi) d\xi \\ & + f(\xi_i^a - \varepsilon^- + 0) g(\xi_i^a - \varepsilon^-) Q^2(t, \xi_i^a - \varepsilon^-) \\ & - f(\xi_i^a + \varepsilon^+ - 0) g(\xi_i^a + \varepsilon^+) Q^2(t, \xi_i^a + \varepsilon^+)], \quad (44) \\ \dot{\xi}_i^s(t) = & -\rho_i^s b_i k_i \bar{\gamma}_i c_i [\int_{\xi_i^a-\varepsilon^-}^{\xi_i^a+\varepsilon^+} g'(\xi) f(\xi) Q^2(t,\xi) d\xi \\ & + g(\xi_i^s - \vartheta^- + 0) f(\xi_i^s - \vartheta^-) Q^2(t, \xi_i^s - \vartheta^-) \\ & - g(\xi_i^s + \vartheta^+ - 0) f(\xi_i^s + \vartheta^+) Q^2(t, \xi_i^s + \vartheta^+)], \quad (45)\end{aligned}$$

with $\rho_i^a > 0$ *and* $\rho_i^s > 0, i = 1,2,\cdots,n,$ *are velocity gain of each actuation-sensing device.*

Moreover, the system (41) can be further refined to

$$\frac{\partial Q(t,\xi)}{\partial t} = \frac{\partial}{\partial \xi}(\alpha(\xi)\frac{\partial Q(t,\xi)}{\partial \xi}) + \sum_{i=1}^{n} f(\xi;\xi_i^a(t)) u_i(t),$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{bmatrix} = \begin{bmatrix} \int_0^l g(\xi;\xi_1^s(t)) Q(t,\xi) d\xi \\ \int_0^l g(\xi;\xi_2^s(t)) Q(t,\xi) d\xi \\ \vdots \\ \int_0^l g(\xi;\xi_n^s(t)) Q(t,\xi) d\xi \end{bmatrix}. \#(46)$$

Here, we can assume that the spatial distribution of moving actuators and sensors in the network are collocated, which means that $\xi_i^a(t) = \xi_i^s(t)$ with $f(\xi;\xi_i^a) = g(\xi;\xi_i^s), i = 1,2,\cdots,n$. Then, the spatial distribution of each agent is given by

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$$f(\xi;\xi_i^a)=\begin{cases} f(\xi) & \text{if} \quad \xi\in[\xi_i^a-\varepsilon,\xi_i^a+\varepsilon] \\ 0 & \text{otherwise} \end{cases} \#(47)$$

and we have the following corollary directly.

Corollary 2 *Under the decentralized output feedback control strategy (6), spatial diffusion processes (46) is globally asymptotically stable, if mobile actuators and sensors are collocated, which spatial distribution is given by (47), such that the following velocity law of each moving agent holds,*

$$\begin{aligned}\dot{\xi}_i^a(t) &= \dot{\xi}_i^s(t)\\ &= -\rho_i k_i\Big[\int_{\xi_i^a-\varepsilon}^{\xi_i^a+\varepsilon} f'(\xi)f(\xi)Q^2(t,\xi)d\xi + f^2(\xi_i^a-\varepsilon)Q^2(t,\xi_i^a-\varepsilon)\\ &\quad -f^2(\xi_i^a+\varepsilon)Q^2(t,\xi_i^a+\varepsilon)\Big]\end{aligned} \#(48)$$

with $\rho_i>0, i=1,2,\cdots,n,$ *is velocity gain of each actuation-sensing device.*

Remark 4: Each sensing device is believed to be able to obtain state measurement information from two edges of the sensor's range in this study. In that case, average value of measurement from each of moving sensor can be indicated in the following:

$$\begin{aligned} y_i(t) &= \int_0^l f\left(\xi;\xi_i^s(t)\right)Q(t,\xi)d\xi\\ &= \int_{\xi_i^a-\varepsilon}^{\xi_i^a+\varepsilon} f(\xi)Q(t,\xi)d\xi \\ &\approx \varepsilon[f(\xi_i^a-\varepsilon)Q(t,\xi_i^a-\varepsilon)+f(\xi_i^a+\varepsilon)Q(t,\xi_i^a+\varepsilon)].\end{aligned} \#(49)$$

Then (48) also can be indicated as following

$$\begin{aligned}\dot{\xi}_i^a(t) &= \dot{\xi}_i^s(t)\\ &= -\rho_i k_i\Big[\int_{\xi_i^a-\varepsilon}^{\xi_i^a+\varepsilon} f'(\xi)f(\xi)Q^2(t,\xi)d\xi + f(\xi_i^a-\varepsilon)Q(t,\xi_i^a-\varepsilon)\\ &\quad -f(\xi_i^a+\varepsilon)Q(t,\xi_i^a+\varepsilon)\Big]y_i(t).\end{aligned} \quad (50)$$

Further, if $f(\xi)=\mu$ in (47), the spatial distribution of mobile agent can be simplified as

$$f(\xi;\xi_i^a)=\begin{cases} \mu & \text{if} \quad \xi\in[\xi_i^a-\varepsilon,\xi_i^a+\varepsilon] \\ 0 & \text{otherwise} \end{cases}. \#(51)$$

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Then we can easily deduce the following corollary.

Corollary 3 *Under the decentralized output feedback control strategy (6), spatial diffusion processes (46) is globally asymptotically stable, if mobile actuators and sensors are collocated, which spatial distribution is given by (51), such that the following velocity law of each moving agent holds,*

$$\dot{\xi}_i^a(t) = \dot{\xi}_i^s(t) = -\rho_i k_i \mu^2\left(Q^2(t,\xi_i^a - \varepsilon) - Q^2(t,\xi_i^a + \varepsilon)\right) \#(52)$$

Or

$$\dot{\xi}_i^a(t) = \dot{\xi}_i^s(t) = -\rho_i k_i \mu\left[\left(Q(t,\xi_i^a - \varepsilon) - Q(t,\xi_i^a + \varepsilon)\right)\right] y_i(t) \#(53)$$

with $\rho_i > 0, i = 1,2,\cdots,n,$ *is velocity gain of each actuation-sensing device.*

V. NUMERICAL RESULTS

In this section, we'll use a simulation to demonstrate the utility of our main conclusions. Consider a forest belt with three mobile actuation-sensing devices in $\Omega = [0,1]$.

$$\begin{aligned} \frac{\partial Q(t,\xi)}{\partial t} &= \alpha_0 \frac{\partial^2 Q(t,\xi)}{\partial \xi^2} - (0.6 + \Delta a)Q(t,\xi) \\ &\quad + [1.3 + \Delta b_1 \quad 1.25 + \Delta b_2 \quad 1.35 + \Delta b_3] \begin{bmatrix} f\left(\xi;\xi_1^a(t)\right)u_1(t) \\ f\left(\xi;\xi_2^a(t)\right)u_2(t) \\ f\left(\xi;\xi_3^a(t)\right)u_3(t) \end{bmatrix}, \\ y(t) &= \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} \int_0^1 1.3\,\overline{\gamma}_1 g\left(\xi;\xi_1^s(t)\right)Q(t,\xi)d\xi \\ \int_0^1 1.15\,\overline{\gamma}_2 g\left(\xi;\xi_2^s(t)\right)Q(t,\xi)d\xi \\ \int_0^1 1.2\,\overline{\gamma}_3 g\left(\xi;\xi_3^s(t)\right)Q(t,\xi)d\xi \end{bmatrix}, \end{aligned} \#(54)$$

where the initial condition $x(0,\xi) = \sin(\pi\xi)e^{-8\xi^2}$ and initial boundary condition $Q(t,0) = Q(t,1) = 0$. The diffusion operator is $\alpha_0 = 0.006$. The probabilities are taken as $\overline{\gamma}_1 = 0.9$, $\overline{\gamma}_2 = 0.8$ and $\overline{\gamma}_3 = 0.85$. The spatial distribution of each moving actuator which centroid at ξ_i^a, given by

$$f(\xi;\xi_i^a) = \begin{cases} 1 & \text{if} \quad \xi \in [\xi_i^a - \varepsilon^-, \xi_i^a + \varepsilon^+] \\ 0 & \text{otherwise} \end{cases}. \quad (55)$$

Each moving sensor which centroid at ξ_i^s follow the spatial distribution as

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$$g(\xi;\xi_i^s)=\begin{cases}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(\xi-\xi_i^s)^2}{2\sigma^2}} & \text{if} \quad \xi\in[\xi_i^s-\vartheta^-,\xi_i^s+\vartheta^+]. \\ 0 & \text{otherwise}\end{cases} \quad (56)$$

Utilizing the following decentralized static output feedback control strategy

$$u(t)=\begin{bmatrix}u_1(t)\\u_2(t)\\u_3(t)\end{bmatrix}=-\begin{bmatrix}k_1 & 0 & 0\\0 & k_2 & 0\\0 & 0 & k_3\end{bmatrix}\begin{bmatrix}y_1(t)\\y_2(t)\\y_3(t)\end{bmatrix},\#(57)$$

where $k_1=5,k_2=8$ and $k_3=10$. In the time $[0,20]$, the closed-loop system is simulated.

From Theorem 2, it is easy to verify under the decentralized static output feedback control scheme (6), the diffusion process (54) is globally asymptotically stable in the mean square, if the spatial distribution of moving actuators and sensors are presented by (55) and (56) respectively and satisfy $[\xi_i^a-\varepsilon,\xi_i^a+\varepsilon]=[\xi_i^s-\vartheta,\xi_i^s+\vartheta]$, such that the following velocity law of each moving agent holds,

$$\begin{aligned}\dot{\xi}_i^a(t)= & -\frac{1}{\sqrt{2\pi}\sigma}\rho_i^a b_i k_i \bar{\gamma}_i c_i[e^{-\frac{(\xi_i^a-\varepsilon^--\xi_i^s)^2}{2\sigma^2}}Q^2(t,\xi_i^a-\varepsilon^-) \\ & -e^{-\frac{(\xi_i^a+\varepsilon^+-\xi_i^s)^2}{2\sigma^2}}Q^2(t,\xi_i^a+\varepsilon^+)], \qquad (58) \\ \dot{\xi}_i^s(t)= & \frac{1}{\sqrt{2\pi}\sigma}\rho_i^s b_i k_i \bar{\gamma}_i c_i[\int_{\xi_i^a-\varepsilon^-}^{\xi_i^a+\varepsilon^+}\frac{\xi-\xi_i^s}{\sigma^2}e^{-\frac{(\xi-\xi_i^s)^2}{2\sigma^2}}x^2(t,\xi)d\xi \\ & -e^{-\frac{(\xi_i^s-\vartheta^--\xi_i^s)^2}{2\sigma^2}}Q^2(t,\xi_i^s-\vartheta^-) \\ & +e^{-\frac{(\vartheta^+)^2}{2\sigma^2}}Q^2(t,\xi_i^s+\vartheta^+)], \qquad (59)\end{aligned}$$

with $\rho_i^a>0$ and $\rho_i^s>0,i=1,2,3$ are velocity gain of each actuation-sensing device.

As a comparison, we take into account three fixed-in-space sensors which fixed at $\xi_1^s=0.25,\xi_2^s=0.55$ and $\xi_3^s=0.85$ and three actuators which fixed at $\xi_1^a=0.26,\xi_2^a=0.56$ and $\xi_3^a=0.86$. Figure 1 depicts the state L_2 norm for a closed loop system and the case of mobile networks in the simulation. Figure 2 describes the state distribution of static and mobile networks at four different time instants. The trajectory of three actuators and sensors for the fixed and mobile cases is depicted in Figure 3.

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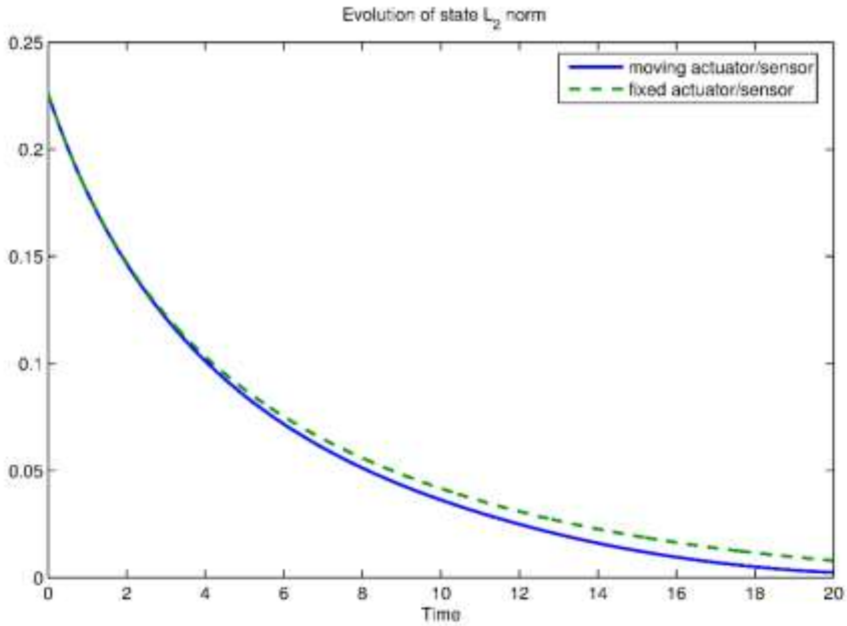

Fig 1: Evolution of spatial L_2 norm

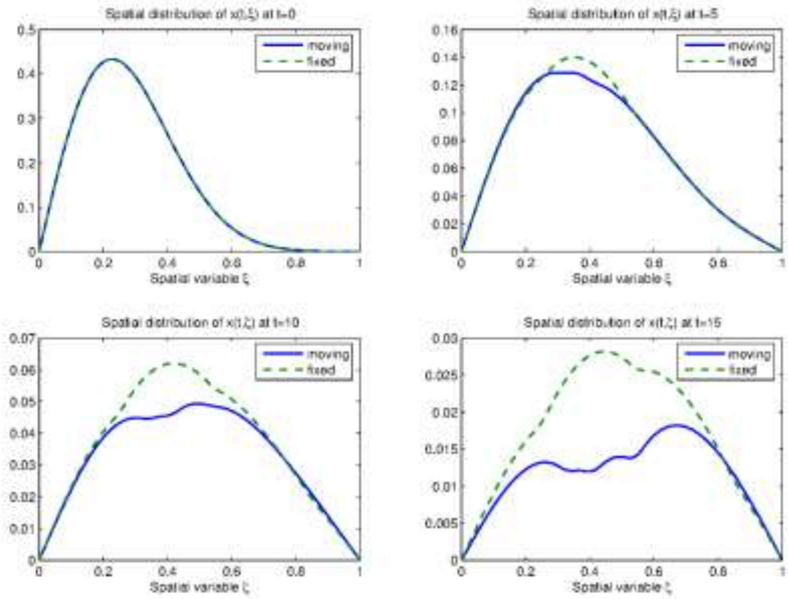

Fig 2: Comparison of closed loop state and spatial variable at different times

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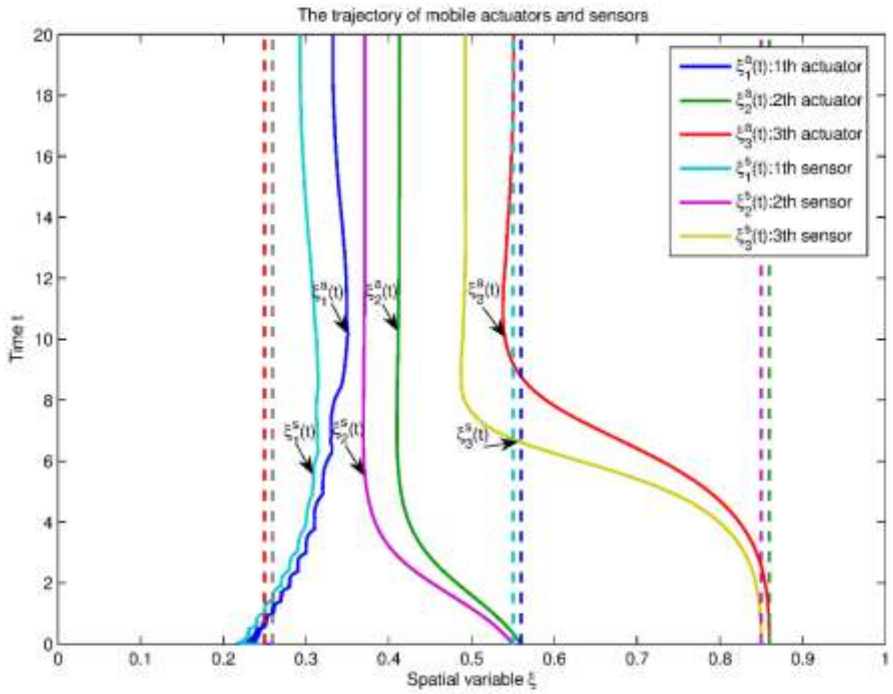

Fig 3: The trajectory of actuation-sensing devices

III. CONCLUSION

This present work has addressed a mobile sensing approach to the optimal monitoring and robust control problem in arrays of forest fire processes in uncertain environments. An optimized framework is given for measurement and control of forest fires using mobile actuator-sensor networks with multiple packet losses. By referring to Lyapunov stability argument, the decentralized output feedback controller is designed to achieve robust stability in the mean square of the addressed systems. Also, the LOIs-based conditions and velocity law of mobile actuation-sensing devices are given. The obtained results improve and extend the earlier works. To exemplify the effectiveness of the presented theoretical conclusions, a numerical example has been provided.

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