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Research on Social Network Clustering Synchronization Based on Improved Coupling Time-Delay Complex Network Synchronization Model

Xiaodong Qian[1,*], Zhengzhong Chen[2]
[1]Economics and Management School, Lanzhou Jiaotong University, Lanzhou, Gansu, China
[2]Automation and Electrical Engineering School, Lanzhou Jiaotong University, Lanzhou, Gansu, China

Abstract:

In order to study the synchronous evolution of social networks and the process of reaching consensus, the coupled time-delay complex network model is improved. Aiming at the problem of constant coupling delay in the original model, a random coupling delay rule is proposed to adjust the network evolution rate. Aiming at the same importance of nodes in the original model, the multi-attribute decision method is introduced to change the importance of nodes and optimize the efficiency of network synchronization. Based on Lyapunov stability theory and matrix theory, the conditions for social networks to achieve cluster synchronization are deduced. Finally, the evolution experiment is carried out with Twitter social network data set, which verifies the theoretical feasibility and effectiveness.

Keywords*: Complex network theory, Social networks, Cluster synchronization.*

I. INTRODUCTION

With the comprehensive promotion and application of Internet technology, online social media tools such as online forums, Facebook, Twitter, and WeChat have become important communication channels for netizens to express opinions, share emotions, spread and obtain information. In addition, with the rapid growth of netizens, information dissemination on the Internet has an increasing influence on product promotion, brand building, social emergency evolution, and public emotions and attitudes. People will encounter all kinds of information in social networks. Due to the different social experience, education level and social status of users, they often have different understandings and opinions on the same information. In fact, the dissemination of information on the Internet is usually accompanied by the dissemination of online public opinion. It is essentially a synchronous phenomenon of typical social collective behavior, which means that many people have reached a consensus in certain aspects, leading to a certain social event. In other words, synchronization is a metaphor for the phenomenon that many people take the same view or take action at the same time. Social network user opinion dynamics, that is, the process of opinion evolution is a type of problem in the study of complex network dynamics. To study

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consensus issues from the perspective of social network user viewpoint dynamics, that is, how to reach consensus among users in the dynamic process, study the generation of user viewpoints or behaviors in social networks, the mutual influence between users, and the dissemination of viewpoints through the network, And finally reached a consensus on the views from different users. This paper uses a complex network synchronization model to study the consensus process of social networks. The synchronization research of social networks can not only provide a scientific theoretical basis for large-scale Internet incidents, but also provide reasonable preventive measures and strategies before and after the incident. Therefore, this is an important subject worthy of study.

Ⅱ. RESEARCH STATUS

At present, the research on synchronization types of complex networks mainly includes: complete synchronization[2],exponential synchronization[3],cluster synchronization[4] and projection synchronization[5]. In most cases, unless some controllers are added, the network will not synchronize autonomously. In order to realize the synchronization dynamics of complex networks, many control strategies are proposed, such as adaptive control[6], sampled data control[7], pulse control[8], intermittent control[9] and many other effective control methods. The research related to this paper at home and abroad generally includes the following two aspects:

2.1Research on Cluster Synchronization

In [10], the author studied the adaptive cluster synchronization of directed networks and gave the minimum number of pinned nodes. Moreover, when the root nodes in all clusters are pinned, the cluster synchronization with adaptive coupling strength can be realized. In [11], the author studied the clustering synchronization of the network through the methods of local control and local adaptive coupling strength, in which the coupling strength of each node is adaptively adjusted only according to the state information of its neighbor nodes, and the sufficient conditions for realizing clustering synchronization are obtained. In [12] and [13], Su et al. Used a relatively novel decentralized adaptive pinning control method to study the cluster synchronization of linear coupled networks and the cluster synchronization of networks with multiple agents and oscillators. In [14], the author designed a linear feedback controller to eliminate the interaction between clusters to realize the cluster synchronization of complex networks with non identical nodes. In [15], the author discusses the clustering synchronization of a class of directed networks by using intermittent pinning control. In [16], the author discusses the cluster synchronization problem of complex networks with linear coupling by using a pinning control method. The dynamic behavior of each node in the network is the same, and then obtains the sufficient conditions for complex networks to achieve cluster synchronization.

2.2 Research on Network Synchronization with Time Delay

Time delay is a common social phenomenon in nature and human society. It is usually caused by limited signal transmission and memory effect. Time delay also exists in the node characteristics of complex networks and network topology. In recent years, there have been many research results on the time-delay effect in network topology and its dynamic behavior [17-29]. In [17], the author studied the fixed time synchronization problem of complex networks with multiple weights and coupling delays based on aperiodic intermittent control. By constructing a complex network model with multiple weights, and based on the fixed time stability lemma and matrix theory, the sufficient conditions for realizing the fixed time synchronization of complex networks are given. In [18], the author studies the adaptive finite time clustering synchronization problem of a class of complex dynamic networks coupled by non constant and discontinuous Lur'e systems, designs an effective pinning feedback controller, and obtains the synchronization conditions of adaptive finite time clustering synchronization. In [19], the author establishes a fractional order complex network model with coupling time delay and model parameter uncertainty, and gives the delay projection synchronization error model of driving network and response network. Secondly, an effective controller and parameter adaptive law are designed to realize the delay projection synchronization and parameter identification between the two networks. In [20], the author considers the time-varying delay and network uncertainty in the complex network model, and studies the exponential synchronization problem under the condition that the dynamic changes of nodes and the coupling between nodes are nonlinear. In [21], the global synchronization of a class of complex dynamic networks with time-delay and non time-delay coupling terms is studied based on adaptive control technology; In [22] studies the exponential synchronization of time-delay coupled dynamic networks and the influence of time delay. In [23], a suitable state feedback controller is designed for a class of complex networks with mixed time delays, and a sufficient condition to ensure the finite time $H\infty$ synchronization of the system is obtained. In [24] Studies exponential synchronization of complex networks with time-delay in general topology, considering both directed and undirected network models respectively; In [25] discusses the exponential synchronization of dynamic complex networks with time-delay and non time-delay coupling terms; In [26] studies a class of complex network models with directional topology and time delay. In [27] studies the exponential synchronization problem of a class of mixed coupling complex dynamic networks with variable time delay. By designing an appropriate intermittent feedback controller, a new synchronization criterion is given; Based on Lyapunov function method and Razumikhin technology, In [28] studies the exponential synchronization problem of complex networks with variable delay.

At present, the research on synchronization and control of complex time-delay dynamic network has attracted wide attention of many scientists at home and abroad, but most scholars have studied the complete synchronization and exponential synchronization of the network, and the group synchronization of complex time-delay dynamic network Less research.

Ⅲ. SYNCHRONIZATION MODEL AND IMPROVEMENT OF COUPLED TIME-DELAY COMPLEX NETWORKS

3.1 Coupling time-delay complex network synchronization model

The typical synchronization model of coupled time-delay complex networks can be described as: when all nodes in the system have the same dynamics, the network synchronization is realized by mutual time-delay coupling between nodes and the action of external controller. The specific formula is as

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follows:

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}x_j(t-\tau) + \mu_i(t) \quad i = 1,2,\cdots,N \qquad (1)$$

Where$x_i(t) = [x_{i1}, x_{i2}, \cdots x_{iN}]^T \in R^N$represents the state variable of the i-th node in the complex network,; the function $f: R^n \to R^n$ describes the dynamic equation of each node; τ is the coupling delay between nodes; c is the coupling strength; the asymmetric matrix $A = (a_{ij})_{N\times N}$ is used to describe the network Coupling matrix, a_{ij} is defined as follows: if there is a connection between node i and node j,$a_{ij} = -a_{ji} = 1$, otherwise $a_{ij} = a_{ji} = 0$; $u_i(t)$ is the external controller, $u_i(t) = -d_i e_i(t)$. d_i where is the control intensity.

In the network (1), when $t \to \infty$, $x_1(t) = x_2(t) = \cdots = x_N(t) = s(t)$ can also be expressed as $\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0, i, j = 1,2, \ldots N$, which means that the network is fully synchronized. Where s(t) is the solution of a single isolated node.

However, the model believes that the coupling delay and coupling strength between all nodes are the same, which does not conform to the characteristics of real social interaction. In real social networks, due to the different life behaviors of users, the time lag in the process of network communication is different; in social networks, the importance of users is not the same, and the impact on other users is also different. The following will optimize and improve the network model based on the appeal question.

3.2 Coupling delay optimization based on random function

From the coupling time delay setting rules of the coupled time delay complex network model, it can be known that the model considers the coupling time delay between all individuals to be the same constant. However, in the real social network, the coupling time delay of each individual interacting is uncertain, so the typical coupled time delay complex network model has insufficient research on the synchronous evolution process of social networks.

In fact, in clusters of real social networks, individuals often have different time lags in their interactions in social networks according to their own living habits or emergencies. Only when all individuals in the entire social network have the same living habits and no emergencies occur, the most perfect ideal situation occurs, each individual in the social network can be generated according to the rules in the typical coupled time-lag complex network model The same time lag, but this situation is too idealistic and unrealistic, and cannot accurately represent the diversity of individuals in social networks.

In order to explore the time lag in the interaction process of each user in the real social network, a small-scale social survey was conducted. The survey results are shown in Table Ⅰ and Table Ⅱ:

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Table Ⅰ. Causes of time lag and the number of people

Reasons for interaction delay	No interaction delay	Work	study	sports	other
Number of people	15	9	15	9	18

Table Ⅱ. Specific time lag and number of people

Time delay(minutes)	0-5	5-10	10-15	15-20	20 and above
Number of people	18	15	6	12	9

From the results of the survey, everyone in the real network will have some special reasons to cause some delays in social network interactions, and the length of the delays varies due to different reasons. The generation of this delay is often uncertain and cannot be described correctly. Based on the above analysis, this paper proposes a stochastic strategy to represent the coupling time delay between individuals in a complex network. The coupling time delay between individuals in the interaction should be completely randomly generated, so this paper adopts a random coupling time delay generation strategy, in which the coupling time delay generated by each node is different, and the strategy obeys the standard uniform distribution,$\tau_i \sim U(0,1), i \in N$.

In order to study the synchronization process between the complex network model with random coupling delay and the complex network model with constant coupling delay, the following small example analysis is made.

Taking a directed network model composed of ten nodes as an example, its topology is shown in Figure 1:

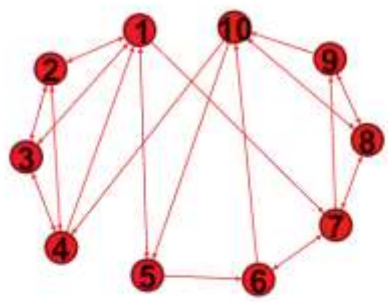

Fig 1: directed network composed of 10 nodes

In this analysis, the complex network is described by equation (1), where C = 1, and the dynamic

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system of the node is described by Lorenz system, $f(x_i(t)) = \begin{cases} 10(x_2 - x_1) \\ 28x_1 - x_2 - x_1x_3 \\ x_1x_2 - {}^{8x_3}/_{3} \end{cases}$.

In order to verify the influence of the fixed coupling delay τ and the randomly generated coupling delay τ_i on the convergence of the complex network synchronization model with the fixed coupling delay $\tau = 1$ and the random coupling delay $\tau_i = \{0, 0.2, 0.8, 0.8, 0.5, 0.2, 0.9, 0.4, 0.1, 0.7\}$, different coupling delay strategies simulation experiment was carried out under. The result of constant delay synchronization is shown in Figure 2, and the result of random delay synchronization is shown in Figure 3:

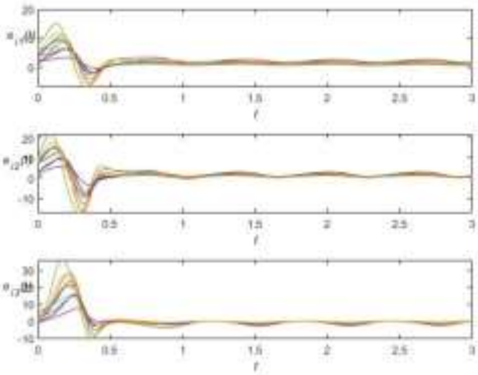

Fig 2: Time change diagram of synchronization error under constant coupling time delay $\tau = 1$

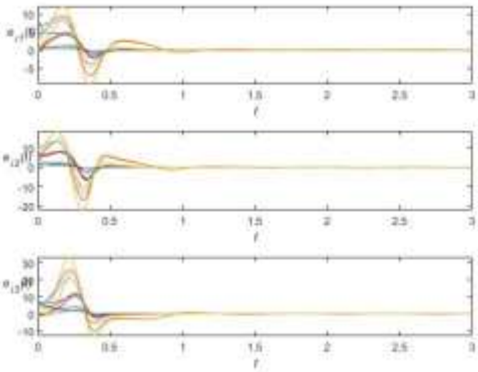

Fig 3: Time variation diagram of synchronization error under random coupling time delay τ_i

Comparing Figure 2 and Figure 3, from the perspective of synchronization time, at t=1.5, the synchronization error components of all nodes in the random coupled time-delay network model are equal to 0, and the nodes reach a fully synchronized state; while the fixed coupled time-delay network In the model, it can be clearly seen that the various stability error components of each node are not equal, and the node has not yet reached the synchronization state. From the perspective of synchronization accuracy, from t=1.5 to t=3, all synchronization error components of all nodes in the random coupling time-delay network model are stable at 0; The synchronization error of the node in the fixed coupled time-delay model is still in a fluctuating state during the time period, and its state change is not stable.

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The experimental results show that the convergence time of the stochastically coupled time-delay synchronization model is lower than that of the constant-coupled time-delay synchronization model, that is, under the same conditions, compared with the constant coupling time-delay synchronization model, the convergence is higher, which means that the optimized synchronization model has Conducive to promote system synchronization.

3.3 Coupling strength optimization based on node importance

From the setting rules of the coupling strength of the coupled time-delay complex network model, it can be known that the model believes that the coupling strength between all individuals is the same during the evolution of the system. In fact, the non homogeneity of social network topology determines the importance of nodes in complex networks. Individuals with high importance in social networks will have greater influence on other individuals, so the coupling strength of each node should be unequal. Therefore, this paper considers the influence of node importance on the coupling strength between individuals.

There are many angles to evaluate the importance of nodes in complex networks, such as degree centrality, intermediate centrality, proximity centrality, eigenvector and so on. There are some limitations in using single methods to evaluate the importance of nodes in the network. In the complex network of the real world, it is difficult to use a single index to measure the importance of nodes in the network. The importance of nodes in the network is related to the overall structure of the network. It is necessary to use multiple importance indexes of nodes for comprehensive evaluation. Therefore, this paper uses multi-attribute decision-making method to evaluate the importance of nodes in complex networks. This article intends to evaluate the importance of network nodes from the following three aspects, First, in terms of the degree value (D) of the node, the more user interactions in social networks and the higher the activity, the greater the importance of the node, $D_i = \sum_{j=1}^{N} a_{ij}$; second, the clustering coefficient (C) of the node, in The aggregation coefficient in social networks is the degree of interconnection between a user and nearby users. The higher the degree of connection, the greater the importance of the node. Assuming that node i is directly connected to v nodes, the aggregation coefficient of node i is $C_i = M_i/[v(v-1)]$; Third, in terms of node betweenness (B), in social networks, some nodes may have a relatively small degree, but it may be an intermediate contact between the two groups. If the node is deleted, it will result in two The connection of the group is interrupted, so the node plays an important role in the network. Therefore, the index is measured by betweenness. It is defined as:$B_i = \sum_{i \le j < l \le N, j \ne i \ne l} [n_{jl}(i)/n_{jl}]$, where n_{jl} is the number of shortest paths between nodes j and l; $n_{jl}(i)$ is the number of shortest paths between nodes j and l through node i, and N is the total number of nodes in the network. The higher the value of the node betweenness, the greater the influence of the node, and the more important the corresponding status. However, the greater the maximum betweenness of the node in network synchronization, the weaker the synchronization ability of the network.

Constructing a decision matrix: If there are L nodes in each subgroup of the social network, the corresponding decision plan set can be expressed as $G = \{G_1, G_2, \cdots G_L\}$, because there are 3 indicators for evaluating the importance of each node, the corresponding plan attribute set is $I = \{I_1, I_2, I_3\}$,The j-th index of the i-th node is $G_i(I_j), (i = 1,2,\cdots,L; j = 1,2,3)$to form the decision matrix J. Where I_1, I_2, I_3 is the degree of nodes, the agglomeration coefficient of nodes and the intermediate number of nodes respectively.

Decision matrix standardization: Because this method has more indicators, all indicators are divided into positive indicators (the higher the indicator, the stronger the ability) and the reverse indicator (the higher the indicator, the weaker the ability).

$$\begin{cases} Positive\ indicators: r_{ij} = G_i(I_j) \Big/ G_i(I_j)^{max} \\ Inverse\ indicators: r_{ij} = G_i(I_j)^{min} \Big/ G_i(I_j) \end{cases} \quad (2)$$

$$\begin{cases} G_i(I_j)^{max} = max\{G_i(I_j)|1 \le i \le L\} \\ G_i(I_j)^{min} = min\{G_i(I_j)|1 \le i \le L\} \end{cases} \quad (3)$$

The standardized decision matrix is recorded as: $R(r_{ij})_{L\times 3}$

Determine the positive ideal plan $A^+$and the negative ideal plan A^-according to the matrix R, where

$$\begin{cases} A^+ = \left\{\max_{i\in L}(r_{i1}, r_{i2}, r_{i3})\right\} = \left\{r_{i1}^{\max}, r_{i2}^{\max}, r_{i3}^{\max}\right\} \\ A^- = \left\{\min_{i\in L}(r_{i1}, r_{i2}, r_{i3})\right\} = \left\{r_{i1}^{\min}, r_{i2}^{\min}, r_{i3}^{\min}\right\} \end{cases} \quad (4)$$

Calculate the distance from each scheme to the positive ideal scheme and the negative ideal scheme according to formula (5).

$$\begin{cases} D_i^+ = \left[\sum_{j=1}^{3}(r_{ij} - r_j^{max})^2\right]^{\frac{1}{2}} \\ D_i^- = \left[\sum_{j=1}^{3}(r_{ij} - r_j^{min})^2\right]^{\frac{1}{2}} \end{cases} \quad (5)$$

Finally, the importance is sorted according to the paste progress Z of the ideal scheme, where $Z_i = D_i^- \Big/ D_i^+ + D_i^-$.

Finally, through the importance of individuals, the following formula is proposed to express the

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coupling strength of interaction between individuals: $c_i = \sqrt{Z_i}/N$.

Take kite network as an example:

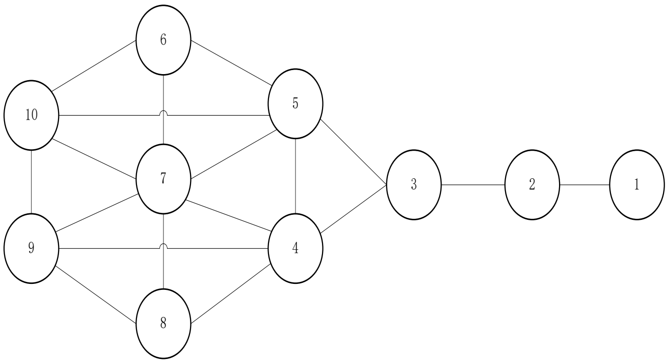

Figure 4: Kite network topology

The importance of nodes in the network is comprehensively calculated based on the three indexes of degree value, clustering coefficient and betweenness. Among them, degree value and clustering coefficient are positive indicators, and betweenness is a reverse indicator. The calculated results of each indicator are shown in Table Ⅲ. Secondly, compare the synchronicity of the coupled time-delay complex network model with the constant coupling strength and the optimized coupling strength:

Table Ⅲ. The calculation results of each index of the nodes in the kite network

node	Degree value	Agglomeration coefficient	betweenness
1	1	0	0
2	2	0	16
3	3	1/3	28
4	5	1/2	16.67
5	5	1/2	16.67
6	3	1	0
7	6	8/15	7.33
8	3	1	0
9	4	2/3	1.67
10	4	2/3	1.67

From Table 3, it can be obtained that $A^+ = \{6,1,28\}$, $A^- = \{1,0,0\}$. The multi-attribute decision-making evaluation results of the kite network nodes are calculated as shown in Table Ⅳ.

Table Ⅳ. Multi-attribute decision-making evaluation results of kite network

node	D_i^+	D_i^-	Z_i
1	28.3	0	0

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2	12.4	16.03	13.4
3	2.1	28.07	3.1
4	11.34	17.15	12.3
5	11.34	17.15	12.3
6	28.07	2.23	29
7	20.7	8.89	30.1
8	28.07	2.23	29
9	26.35	3.5	27.4
10	26.35	3.5	27.4

The node importance Z_i of the kite network is obtained from Table 4, and the coupling strength of each node in the network is calculated by the node importance $c_i = \{0, 0.36, 0.17, 0.35, 0.35, 0.53, 0.55, 0.53, 0.52, 0.52\}$. After the constant coupling strength c=1 and the multi-attribute decision-making method are optimized, the coupling strength of the node is analyzed. The synchronization capability of the complex network model under different coupling strengths is analyzed. The coupling delay $\tau = 1$, the dynamic equation of the node is $f(x_i(t)) = \begin{cases} 36x_2 - 36x_1 + 144 \\ 20x_1 - x_1x_3 + 2x_1 + 8x_3 - 64 \\ x_1x_2 - 3x_3 - 4x_1 - 8x_2 - 26 \end{cases}$. The results are shown in Figure 5 and Figure 6.

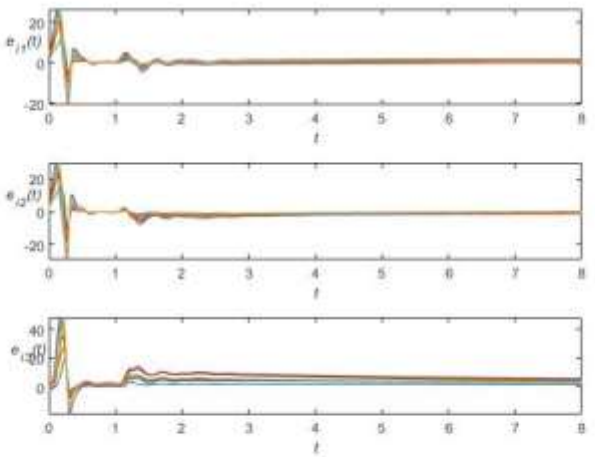

Fig. 5: Time variation diagram of the synchronization error of the kite network under constant coupling strength

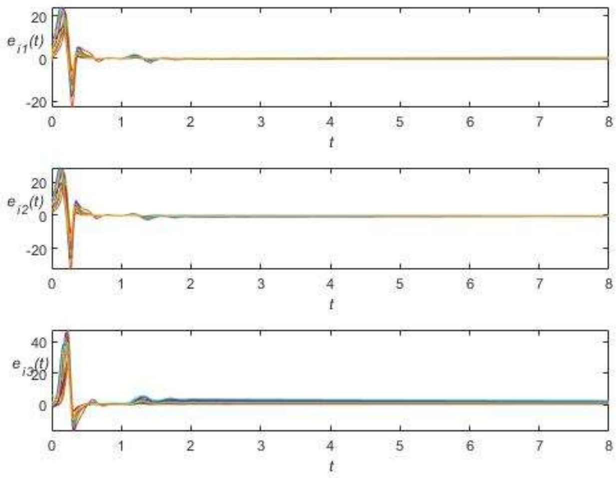

Fig. 6 :Time variation diagram of the synchronization error of the kite network under variable coupling strength

From Figure 5 and Figure 6, we can conclude that at the synchronization rate, at t=3, the synchronization error components e1 and e2 of the nodes in the fixed coupling strength network model tend to stabilize. The synchronization error component e3 of the node is still undergoing synchronization convergence at t=8, and has not reached a stable state; while the node synchronization error components e1 and e2 in the network model under variable coupling strength reach a stable state at t=2. The synchronization error component e3 of the node has reached an asymptotically stable state at t=3. In terms of synchronization accuracy, although the synchronization error components e1 and e2 of the nodes in Figure 5 tend to be stable, they are not completely stable at 0. The synchronization error components e1 and e2 of the nodes in Figure 6 not only tend to stabilize, but also stabilize at At the value of 0; the synchronization error component e3 of the nodes in Figure 5 and Figure 6 at t=8, the node convergence strength of the variable coupling strength network model is significantly higher than that of the constant coupling strength network model, so its synchronization accuracy is also higher than that of the constant coupling strength model.

The analysis results show that the coupling strength of the network nodes is optimized by the multi-attribute decision-making method. The optimized network synchronization rate and synchronization accuracy are significantly improved in a certain range compared with the network model under the fixed coupling strength. Therefore, based on the node importance The optimized coupling strength can better promote the synchronization of the network.

Ⅳ. CLUSTER SYNCHRONIZATION ANALYSIS OF COUPLED TIME-DELAY NETWORKS BASED ON FEEDBACK CONTROL

There are three main analysis methods for the synchronization capability of complex networks, namely, the main stable function method, the connection graph method and the Lyapunov function method. In this paper, the Lyapunov function method is used to analyze the group synchronization of

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social networks, and the Lyapunov function method is used to analyze the local stability or global stability of the system from the energy point of view. This method first constructs an energy function similar to measuring the evolution of the system, and checks whether it decays monotonously over time to judge the stability of the system. This method has universal applicability, can be used in any system, and is widely used in stability analysis and synchronization control analysis of complex networks.

Due to the massive nature of social network users, countless groups gathered for different reasons are formed in the network, and users in these different groups have different dynamic behaviors due to their self-similarity. Different groups have different opinions on the same event. This feature is consistent with cluster synchronization in the network synchronization model.

The definition of cluster synchronization is as follows. If node i belongs to the k-th cluster, then define $\omega_i = C_k$. Then when the complex network realizes group synchronization, for any node i and j:

$$\begin{cases} \lim\limits_{t\to\infty} \left\| x_i(t) - x_j(t) \right\| = 0, \omega_i = \omega_j \\ \lim\limits_{t\to\infty} \left\| x_i(t) - x_j(t) \right\| \neq 0, \omega_i \neq \omega_j \end{cases} \quad (6)$$

$e_i(t) = x_i(t) - s_k(t), i = 1,2,\cdots,N. k = 1,2,\cdots,m$ is the error vector between the state variable of the i-th node and the target state variable. Among them, the target state $s_k(t)$ satisfies $\dot{s}_k(t) = f(s_k(t))$. If the complex network satisfies the following conditions, the complex network realizes group synchronization:

$$\begin{cases} \lim\limits_{t\to\infty} \|e_i(t)\| = 0, \omega_i = \omega_j \\ \lim\limits_{t\to\infty} \|e_i(t)\| \neq 0, \omega_i \neq \omega_j \end{cases} \quad (7)$$

Therefore, the error system of the network system (1) is:

$$\dot{e}_i(t) = f(x_i(t)) - f\big(s(t)\big) + c\sum_{j=1}^{N} a_{ij} H x_j(t-\tau) - d_i e_i(t) \qquad i = 1,2,\cdots,N \quad (8)$$

In order to prove that the optimized network model can achieve cluster synchronization and cluster synchronization, the following assumptions and lemmas are proposed:

Assumption 1: For any $x, y \in R^n$, there is a constant $\theta > 0$, so that the function $f(\bullet)$ satisfies: $(x - y)^T\big(f(x) - f(y)\big) \leq \theta(x - y)^T(x - y) \quad (9)$

Lemma 1: Linear Matrix Inequality (LMI): $\begin{pmatrix} s_{11} & s_{12} \\ s_{12}^T & s_{22} \end{pmatrix} < 0$, Where $s_{11} = s_{11}^T, s_{22} = s_{22}^T$ is equivalent to $s_{22} < 0, s_{11} - s_{12}s_{22}^{-1}s_{12}^T < 0$.

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Condition 1: Under the condition that hypothesis 1 and the following formula are satisfied, the network model realizes cluster synchronization.

$$\Theta - D + I_N + \frac{C^2}{4} A A^T < 0 \qquad (10)$$

Where $\Theta = diag\{\theta_i; i \in N\}, D = diag\{d_i; i \in N\}$.

Proof: Construct the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i{}^T(t) e_i(t) + \sum_{i=1}^{N} \int_{t-\tau}^{t} e_i{}^T(s) e_i(s) ds$$

Derivative for it:

$$\dot{V}(t) = \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N} e_i^T(t) e_i(t) - \sum_{i=1}^{N} e_i^T(t-\tau) e_i(t-\tau)$$
$$= \sum_{i=1}^{N} e_i^T(t)\left[(f_k(x_i(t)) - f_k(s(t)) + c_{ij}\sum_{j=1, j\neq i}^{N} a_{ij} e_j(t-\tau) - d_i e_i(t)\right]$$
$$+ \sum_{i=1}^{N} e_i^T(t) e_i(t) - \sum_{i=1}^{N} e_i^T(t-\tau) e_i(t-\tau)$$

According to Hypothesis 1:

$$\dot{V}_i(t) \le \sum_{i=1}^{N} e_i^T(t)\left[\theta e_i(t) + c_{ij}\sum_{j=1, j\neq i}^{N} a_{ij} e_j(t-\tau) - d_i\big(s_i(t) - x_i(t)\big) + e_i(t)\right] - \sum_{i=1}^{N} e_i^T(t-\tau) e_i(t-\tau)$$

Where $e(t) = (e_1^T(t), e_2^T(t), \cdots, e_N^T(t))^T$,Through the Kronecker product, $\dot{V}(t)$can be written as

$$\dot{V}(t) \le e^T(t)(\Theta - D + I_N) e(t) + e^T(t) C A e(t-\tau) - e^T(t-\tau) I_N e(t-\tau)$$
$$\le (e(t), e(t-\tau))^T \times \begin{bmatrix} \Theta - D + I_N & \frac{C}{2}A \\ \left(\frac{C}{2}A\right)^T & -I_N \end{bmatrix} \times (e(t), e(t-\tau))$$

Obviously $\Theta - D + I_N$ and $-I_N$ are symmetrical, $-I_N < 0$, according to condition 1, we can get

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$$(\Theta - D + I_N) - \left(\frac{C}{2}A\right) \times (-I_N)^{-1} \left(\frac{C}{2}A\right)^T$$
$$= (\Theta - D + I_N) + \frac{C^2}{4} A I_N A^T$$
$$= \Theta - D + I_N + \frac{C^2}{4} A A^T < 0$$

The inequality $\begin{bmatrix} \Theta - D + I_N & \frac{C}{2}A \\ \left(\frac{C}{2}A\right)^T & -I_N \end{bmatrix} < 0$ satisfies lemma 2, so $\dot{V}(t) < 0$, so $e_1(t), e_2(t), \cdots e_N(t)$ satisfies $\lim_{t\to\infty} e_i(t) = 0, i = 1,2,\cdots,N$,so the clustering synchronization of the optimized network model is realized, and the proof is given.

Ⅴ. EMPIRICAL ANALYSIS

In this section, an empirical simulation of real social networks will be used to prove the effectiveness of the clustering synchronization theorem obtained in Section 4.

5.1 Data set introduction and data processing

The empirical data of this paper adopts the real network data set of Stanford University - twitter social network data set. Because the social data of twitter social network is too large, this paper intercepts the communication data between 50 users for simulation experiments, and discards 7 isolated nodes. The network consists of 43 points and 233 edges. Without losing generality, the 43 users are divided into two groups, in which different groups express their dynamic equations with different functions, in which the number of group 1 nodes is 22 and the number of group 2 nodes is 21.The network topology is shown in Figure 7:

Figure 7: Topological structure diagram of social network

5.2 Analysis of experimental results

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In this simulation, this complex network is described by the following formula:

$$\dot{x}_i(t) = f(x_i(t)) + c_i \sum_{j=1}^{N} a_{ij} H x_j(t-\tau_i) + \mu_i(t) \qquad i = 1,2,\cdots,N \qquad (11)$$

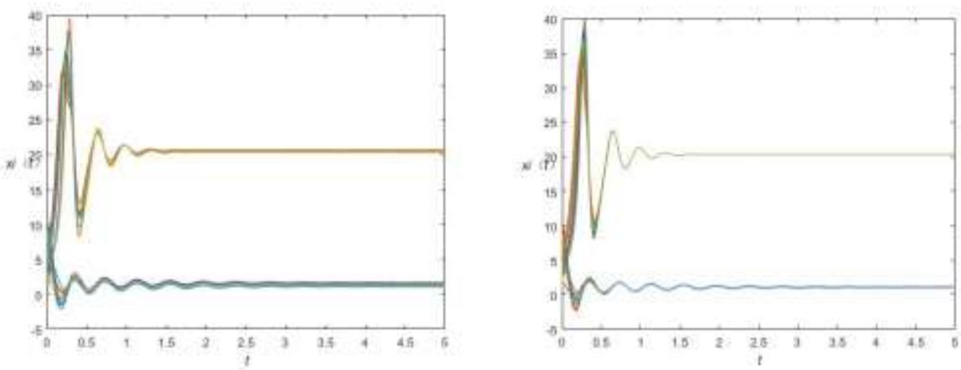
The nodal dynamic equations in different groups are as follows:

$$f_1(x_i(t)) = \begin{cases} 36(x_2 - x_1) + 144 \\ 20x_1 - x_1x_3 + 2x_1 + 8x_3 - 64 \\ x_1x_2 - 3x_3 - 4x_1 - 8x_2 - 26 \end{cases} \qquad f_2(x_i(t)) = \begin{cases} 10x_2 - 10x_1 \\ 28x_1 - x_1x_3 - x_2 \\ x_1x_2 - 8/3x_3 \end{cases}$$

If $\theta_1 = 5, \theta_2 = 9$, $f_1(x_i(t))$ and $f_2(x_i(t))$satisfy formula 9.

$$(x_i - s_1)^T\big(f_1(x_i) - f_1(s_1)\big) \le 5\big(e_{1_1}^2 + e_{1_2}^2 + e_{1_3}^2\big)$$
$$(x_i - s_2)^T\big(f_2(x_i) - f_2(s_2)\big) \le 9\big(e_{2_1}^2 + e_{2_2}^2 + e_{2_3}^2\big)$$

Therefore, $\Theta = diag\left\{\theta_i = 5, \theta_j = 9 | i \in \{1,\cdots 22\}, j \in \{23,\cdots,43\}\right\}$, $d_i = 9, i \in N$, the network satisfies the group synchronization condition 1, and the initial value of the state vector of $\Theta - D + I_N + \frac{c^2}{4}AA^T < 0$.The network node is randomly selected from (0~10), and the synchronization ability of the original network model and the optimized and improved network model is performed Analysis, the simulation results are shown in Figure 8.



(a)Original network model (b)Improved network model

Figure 8 :The trajectory diagram of the state vector $x_{i_1}(t)$ of the social network

Figure 8 shows the trajectories of nodes XI between different clusters. The results show that regardless of the initial value, the trajectories of nodes in the same cluster will be close to the same, achieving synchronization between clusters. However, the trajectories of nodes in different clusters are very different, so different clusters The nodes between the nodes are not synchronized, so the network achieves group synchronization. It can be clearly seen from the above figure that the synchronization accuracy of nodes in the improved network model is higher than that of the original network model; comparing (a) and (b), it is not difficult to see that the nodes in the original network model group 1 are in Synchronization is

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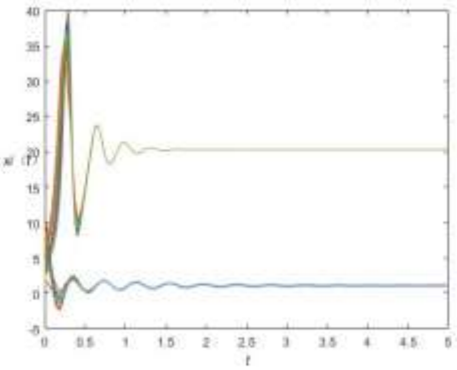
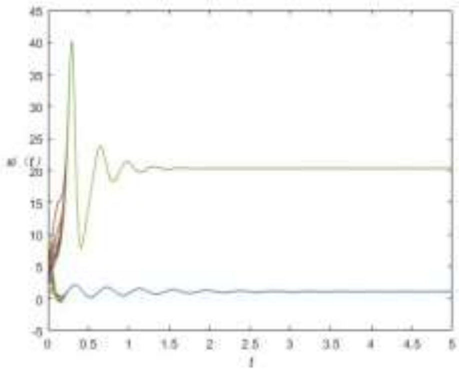
achieved at t=1.5. Compared with the improved network model, the state of the group 1 node reaches the synchronization state at time 0.5, and it stabilizes at time t=1.5. Therefore, the improved network model has better synchronization capabilities than the original network model.

From the perspective of social network, at first, each individual user in the social network had different views on an event, influenced each other in the process of interaction with other users, and adjusted the user's own views on the event under this influence, resulting in a collective behavior, which gathered the user groups with similar social characteristics in the whole social network, And the group reached an agreement on an event, realizing the consensus among network groups. Because of the mass of social network users, under the influence of practical factors such as users' social experience, educational level and social status, the groups formed among users will also have different views on an event. Therefore, the consensus phenomenon in social networks conforms to cluster synchronization.

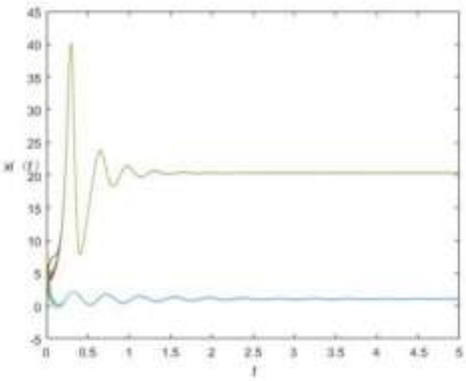
In order to verify the influence of the controller on the group synchronization, the following is the synchronization evolution diagram of the social network when $d_i = 10$、20、30, and the result is shown in Figure 9.



$(a) d_i = 10$



$(b) d_i = 20$



$(c) d_i = 30$

Fig. 9 trajectory diagram of state vector $x_{i_1}(t)$ of social network under different control intensities

As shown in the figure, under different control intensities, the fluctuation range between the state vectors of the social network decreases with the continuous increase of control intensity; while the state of the nodes reaches the time for inter-group synchronization under different control intensities Each is different. When the control strength $d_i = 10$, the network node state reaches the synchronization state at t=0.5; when the control strength $d_i = 20$, the network node state reaches the synchronization state at t=0.3; when the control strength $d_i = 30$, the state of the network node reaches the synchronous state at t=0.1. Therefore, it is concluded that under certain conditions, the time and stability of the synchronization of the network can be adjusted by adjusting the control intensity. In social networks, some network policies or network propaganda methods can be used to control the intensity of control; from the perspective of user consensus, if the viewpoint is a positive viewpoint, social network users can be encouraged or promoted to reach such a consensus. If the opinion is a negative opinion, the above-mentioned theories should be used to suppress or prevent the generation of this consensus.

Ⅵ. CONCLUSION

Based on the complex network synchronization theory, this paper optimizes the coupling delay complex network synchronization model. Considering the characteristics of real social networks, this paper improves the coupling delay complex network synchronization model from two aspects: user coupling delay and individual importance. Firstly, aiming at the problem of constant coupling time delay in the original synchronization model, according to the characteristics of uncontrollable time delay in real social networks, the concept of random time delay is proposed, that is, the time delay caused by the coupling of all nodes in the model is random; Secondly, aiming at the problem that the coupling strength between all nodes in the original model is the same, according to the concept of node importance in complex networks, the importance of user nodes in social networks is evaluated by using multi-attribute decision-making method, and the evaluation index is used as the weight to optimize the node coupling strength. By improving these two aspects, the optimized network model can better describe the law of synchronous evolution in social networks; Then, according to the Lyapunov stability theory, the conditions required to realize the clustering synchronization of social networks are derived; Finally, through the real social network data, the synchronous evolution experiment of social network is carried out by using the improved network synchronization model. The experimental results show that under the condition of meeting the network synchronization criterion, social networks can achieve cluster synchronization. According to this result, we can not only promote the network consensus conducive to the development of social networks through the synchronization criterion, but also suppress or prevent some harmful network consensus. Therefore, the model can not only accurately describe the synchronization phenomenon in social networks, but also help to understand the consensus evolution mechanism of social networks and regulate the synchronization risk.

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