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Research on Synchronization Capability of Supply Chain Network Based on Master Stability Function

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Abstract:

According to the diversity of enterprises, the supply chain network is often a mixed network of multiple enterprises, and the existing research rarely involves the hierarchical type and coupling strength between enterprises, which is not conducive to the study of the influence of the relationship between enterprises on network synchronization. This paper proposes a research method of supply chain network synchronization capability, which provides a theoretical basis for supply chain network synchronization. Based on the gravitational formula to construct the coupling strength between enterprises, the degree value is defined as the mass, the serial number difference of the enterprise is distance and the industry driving coefficient is the gravitational constant; again, considering the different types of enterprises, the enterprise level difference is defined as the internal connection in the master stability function, The influence component of the matrix. The coupling strength and interconnection matrix in the master stability function are improved through two aspects, so that the research results of the master stability function can better guide the improvement of the synchronization ability of the supply chain network. The study draws the following three conclusions: 1) The comparison of the improved master stability function and the stability domain of the main stability function shows that in order to increase the stability area in the supply chain network, cross-level cooperation between enterprises should be minimized; 2) New enterprises In the development, try to avoid the choice of "the rich get richer". This kind of cooperation among enterprises has the influence of lower availability and higher inequality in costs and terms. It will generally limit the synchronization of supply chain networks. 3) When the supply chain network topology is stable, the government can adjust the industry driving coefficient to accurately remove the synchronous enterprise relationship that requires a high price to achieve. The deficiencies of the research are mainly reflected in the fact that the industry driving coefficient is not set according to industry differences, and the setting of the influence components of the interconnection matrix only considers the impact of stratification.

***Keywords**: Complex network, Master stability function, Synchronization, Supply chain network, Evolution.*

I. INTRODUCTION

Since the 21st century, with the rapid development of the global economy, competition is no longer a competition between enterprises. In the current market environment, it is impossible to achieve a competitive advantage alone. The current commercial competition has long become a competition between the supply chain and the supply chain. Enterprises must add themselves to the corresponding supply chain

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and become an alliance, In order to obtain a competitive advantage. In such an environment, more and more companies choose a supply chain that suits them, and the supply chain network is becoming more and more complex, leading to many problems in the complex supply chain network. Among them, collaborative production has become one of the urgent problems to be solved. The highest form of collaborative production is synchronization. Only synchronization can maximize the response speed of the supply chain, increase customer satisfaction, reduce the bullwhip effect in the supply chain, and optimize the social benefits of the supply chain.

Synchronous research on supply chain networks has always been an important part of complex network research. Research in this area has important theoretical guidance and application value for supply chain companies. The development of supply chain networks has both regularity and randomness. It is difficult to explain its deep laws with the existing theoretical knowledge of network models, while complex network theory can explain the laws of supply chain networks to a certain extent. With the rapid development of science and technology, many research results on complex networks have made it possible for research to explain supply chain networks in a deeper level.

Based on the analysis of the master stable function in the synchronization method of the complex dynamic network, this paper explores the conditions for obtaining synchronization based on the unified chaotic dynamics model, and combines the nature of the supply chain to improve the master stable function, and obtains the type of regional bifurcation in the synchronization of the supply chain network. Further analyzes the changes in the network synchronization capability during the evolution of the supply chain network, and provides realistic reference materials for the management and optimization of enterprises in the supply chain.

II. RESEARCH STATUS AT HOME AND ABROAD

With the prosperity and development of the economy and world trade, the supply chain network is showing more and more complex phenomena. Many scholars at home and abroad have studied the synchronization phenomenon of the supply chain network through different methods. But the master stability equation of the complex network synchronization method is currently, The research applied to supply chain network synchronization is very lacking. The research literature related to this paper mainly includes the following two aspects:

2.1 Related research on the master stable function

Pecora and Carroll[1] proposed the master stable function method in 1998, and gave relevant definitions for its dynamic function, the coupling strength value of the network, and the interconnection matrix, and concluded that all nodes in the network are a necessary condition for complete synchronization; Pan Yonghao[2] and others based on the master stability equation to consider the link prediction of the static network introduces the node dynamic model to form a dynamic network. By analyzing the relationship between the link prediction connection and the dynamic network model synchronization, the

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link prediction connection mechanism Conduct analysis and research. Through experimental and theoretical analysis, it is concluded that the link prediction link has the stability of synchronization ability, and the dynamic mechanism of link prediction link is further discussed, and the difference between the link prediction link mechanism and the real network evolution is revealed; Lu Junan[3] et al. conducted related studies on interconnection matrices and node dynamics based on the master stability function method, and the results showed that: for the same type of node dynamics, different interconnection matrices, and the bifurcation of the synchronization region of the network The modes are different; the same interconnection matrix, different types of node dynamics, there are also many differences in the synchronization area bifurcation of the network; different bifurcation modes, the stability of the network synchronization state is also different, it is bound to affect the network. Tang Longkun[4]et al. put forward the bifurcation problem of the synchronization area of the complex dynamic network with the dynamic parameters of the nodes continuously changing under a given interconnection matrix, and discussed that the synchronization state is equilibrium. Regarding the bifurcation methods of the stable point and the chaotic attractor synchronization domain, it is concluded that the interconnection matrix is different, the synchronization stability structure of the network is different, and the synchronization domain bifurcation points of the node dynamic parameters are also different, and will affect the stability of the synchronization structure. Sha Li[5] et al. used the master stability function as an analysis tool, and for a system composed of two Aihara neurons under electrical synaptic coupling, they gave a necessary condition for the system to achieve complete state synchronization, and used the master stability function value of the system. The fully synchronized area of the 2-dimensional parameter plane is drawn. Numerical simulation shows that too large or too small coupling strength can not make the coupling system achieve complete synchronization of the state. And through numerical examples to show the rationality and validity of the theoretical results; Li Xiaoxia[6] and others based on the master stability function method, through simulation results show that: when the coupling strength of some parts of the network changes, the other coupling strengths follow the opposite trend Changes, under certain changing laws, there may be different trends in network synchronization capabilities in different intervals. In addition, the network synchronization capability may reach extremely large values when the synchronization domain is bounded. The following conclusions are drawn: the change trend of network synchronization capability is always the same as the change trend of smaller coupling strength, and it is easier to achieve network synchronization when the synchronization domain is bounded. Liu Gequn[7] et al. analyzed the Jordan standard form of the nodal variational equation system matrix on the basis of the master stability function, and gave several methods for selecting the internal coupling matrix suitable for different situations to synchronize the speed. For the basis, the selection proposition of the better internal coupling matrix is proposed, and the corresponding selection method is given. Finally, three typical examples are used to verify the effectiveness of the proposed method. Existing studies have proved the applicability of the master stability function to different networks, and have also drawn relevant conclusions about the synchronization of different networks.

2.2 Related research on supply chain network synchronization

At present, most researches on supply chain synchronization focus on core enterprises or key links, and

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fail to integrate upstream and downstream suppliers, distributors, and retail networks. Judging from the existing research results of supply chain network synchronization, the use of complex network theory to study the dynamic characteristics of supply chain networks is mainly to study the robustness and risk propagation of various supply chain networks. However, there are few studies on the synchronization dynamics of supply chain networks. As early as 1994, David Anderson [8], a world-renowned supply chain management expert, published an article entitled "Collaborative Supply Chain: A New Frontier". It is clearly pointed out that the new generation of supply chain strategy is the collaborative supply chain. Lei [9] et al. (2007) studied the chaotic synchronization of the bullwhip effect in the supply chain based on the Lorenz chaos model; Lin Yu [10] et al. took the system cost as the optimization objective to obtain the optimal security at all levels of the supply chain Inventory factor, set the optimal inventory of the system, and use evaluation indicators to compare and analyze the supply chain model, which confirmed the superiority of the synchronous supply chain model combined with inventory optimization; R.McIvor[11]et al. studied the support of e-commerce for supply chain collaborative management. Li Yongkang [12] considered the synchronization optimization model of the supply chain network, aiming at the optimization problem of the four-level supply chain network of multi-period and multi-product facilities under the situation of studying the design of enterprise production and supply chain network at the same time, it is mainly concentrated on the micro level, such as Manufacturing facility dynamic location selection, capacity planning, balance of cross-cycle production plans etc; Metin Turkay[13] modeled and quantitatively analyzed the synergy between enterprises in the chemical industry; Zhang Haiyan[14] analyzed how to The strategy of realizing supply chain synchronization in the supply chain shows that a corresponding mechanism needs to be established within the enterprise to ensure the realization of supply chain synchronization. Akkermans[15] and others established a theoretical model of supply chain synergy, focusing on the study of the important influence of non-technical factors on the realization of synergy. However, these existing studies lack the analysis of the synchronization dynamics of the supply chain network.

As a typical complex network, the supply chain network is divided into four layers according to the type of enterprise. The first layer is composed of suppliers; the second layer is composed of manufacturers; the third layer is composed of distributors; and the fourth layer is composed of retailers. If there is a cooperative relationship between enterprises, the effect is promotion; if it is a competitive relationship, the effect is consumption. In the supply chain network, it is not only necessary to indicate the direction of the coupling relationship between enterprises, but also to determine the size of this coupling capability; the influence between enterprises does not depend on a specific factor, among which the influencing factors are: supply, storage and sales and many more. These points are equivalent to the coupling function and the interconnection matrix in the master stability function. The coupling function can indicate the competition and cooperation between enterprises, and the interconnection matrix can separately express the influence of different influencing factors between enterprises to obtain more information For accurate inter-enterprise influence. Regarding the master stability function, it is currently one of the most important methods to study network synchronization. It has a relatively complete explanation of the stability of the network synchronization area. In the existing research on network synchronization[5-7,12], the master stability function is very good. Explains the relevant attributes of many networks. As an actual large-scale

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complex network, the master stability function is improved according to the supply chain rules to make it more in line with the research of the supply chain network, so as to guide the realization of the synchronization state between the supply chain network enterprises. Only by achieving synchronization between enterprises can we speed up the response speed of the supply chain, reduce the production cost of the supply chain, improve customer satisfaction, and increase the competitiveness of the supply chain.

III. ANALYSIS AND IMPROVEMENT OF THE MASTER STABILITY FUNCTION

3.1 Introduction to the master stability function

Consider a continuous time complex dynamic network composed of N identical nodes[17]:

$$\dot{x}_i = f(x_i) - c\sum_{j=1}^{N} l_{ij} H(x_j), i = 1, 2, \ldots\ldots, N \qquad (1)$$

Where the nodal dynamics equation is $\dot{x}_i = f(x_i)$, $x_i \in R^{N\times N}$, it is the state variable of the i-th node, The constant c is the coupling strength of the network, the coupling matrix $L = (l_{ij})_{N\times N}$ is Laplacian, (satisfying the dissipative coupling condition $\sum_j l_{ij} = 0$), $H: R^n \to R^n$ is the internal coupling function (referred to as the internal connection matrix) between the state variables of each node. In the research, it is generally assumed that the internal coupling relationship is exactly the same, Let s(t) be the solution of the isolated node dynamic equation $s = \dot{f}(s)$. The master stability function must meet the following four conditions: ①The dynamics of all nodes are exactly the same; ②The coupling function between each node is also exactly the same; ③The synchronous manifold is an invariant manifold; ④Linearization can be done near the synchronous manifold . Under these assumptions, Formula (1) do the variation at s, Let $\varepsilon_j(t) = x_i(t) - s(t)$,then the variational equation of formula (1) on s is obtained

$$\varepsilon_i = Df(s)\varepsilon_i - c\sum_{j=1}^{N} l_{ij} DH(s)\varepsilon_j, i = 1, 2, \ldots\ldots, N \qquad (2)$$

Among them, $Df(s)$ and $DH(s)$ $\dot{f}(x)$ are the Jacobian matrices of $f(x)$ and $H(x)$ at s respectively. Let $\varepsilon = [\varepsilon_1, \varepsilon_2, \ldots., \varepsilon_N]$, then formula (2) can be rewritten for

$$\dot{\varepsilon}_i = Df(s)\varepsilon - cDH(x)\varepsilon L^T \qquad (3)$$

Here we assume that the matrix L can be diagonalized, denoted as $L^T = P\Lambda P^{-1}$, $\Lambda =$ *diag* $(\lambda_1, \lambda_2, \ldots\ldots, \lambda_N)$ where $\lambda_K (K = 1, 2, \ldots., N)$ is the eigenvalue of the coupled Laplacian matrix L, and then let

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$\eta = (\eta_1, \eta_2, ..., \eta_N) = \varepsilon P$, then we have

$$\dot{\eta} = [Df(s) - c\lambda_K DH(s)]\eta_K, K = 1, 2, ..., N \quad (4)$$

A commonly used criterion for judging the stability of a synchronous manifold is to require that the transverse Lyapunov exponent of formula (4) be all negative. When the coupling matrix L is an asymmetric matrix, its eigenvalues may be complex numbers, so the master stable equation can be written as

$$\dot{y} = [Df(s) - (\alpha + \beta i)DH(s)]y \quad (5)$$

Where $i = \sqrt{-1}$. Given the node dynamic function f and the interconnection matrix H, the maximum Lyapunov exponent L_{max} of the master stability function is a function of the variables α and β, which is called the master stability function of the dynamic network formula (1), written as $L_{\max} = L_{\max}(\alpha + \beta i)$ [17]. We call the area where the master stability function is negative as the main stable area, denoted as $SR = \{\alpha + \beta_i | L_{\max}(\alpha + \beta_i) < 0\}$. Therefore, if the product of the coupling strength and all the eigenvalues of the coupling matrix L falls into the main stable region SR, it becomes a necessary condition for the network to achieve partial complete synchronization.

However, there are still many shortcomings in the study of the master stability function applied to the supply chain network. First, among the four conditions that the primary stability function hypothesis satisfies, the second is that the coupling function between nodes is the same, which is too ideal for the supply chain network, and there are many influencing factors in the development and evolution of the enterprise, and the coupling function must be Will be different. This article will determine its value through the distance between enterprises and the number of cooperative competitors, and distinguish the positive and negative coupling values through competition and cooperation between enterprises. The inter-firm coupling capability does not blindly reduce or increase the development of the state difference between enterprises. Secondly, the fixed value of the influence component of the interconnection matrix in the master stability function cannot indicate the difference in influence ability between different levels of enterprises in the supply chain network. The setting of the interconnection matrix elements must consider the hierarchical properties of the supply chain network. The relationship between enterprise layers is determined, taking into account the different impacts of different layers of enterprises on the development of the enterprise, eliminating the assumption that the same level and different levels of enterprises have the same influence ability. The improved master stability function in terms of coupling strength and interconnection matrix will be able to more reasonably represent the relationship between enterprises in the supply chain network, so that the research results can better guide the supply chain network to improve its own synchronization capabilities.

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3.2 Improvement of coupling function based on gravitational field

3.2.1 Improvement of the coupling function

The definition of the coupling function in the master stability function is inconsistent with the coupling strength between enterprises in the supply chain network. Competitiveness at the same level and cooperation at different levels have different effects on synchronization between enterprises. To promote cooperation, the state gap between enterprises should be gradually reduced, and the coupling function in the master stability function should take a positive value; while competition is the effect of reducing the other's strengths, the state gap between enterprises will increase, and the coupling function in the master stability function should be negative value. In the calculation of the coupling function, the concept of gravitational field is introduced, and the degree value of the node is considered as the mass in the gravitational formula; the distance can be calculated by the location of the company, but for the sake of simplicity, the company is numbered according to the establishment time, as With the expansion of market size, enterprises will become more and more dispersed (after the initial enterprise area is saturated, the area can only be expanded to seek development). Therefore, the greater the difference in node numbers, the greater the distance between nodes. The difference between the numbers is used as the distance for research; the gravitational coefficient is considered as the inter-industry driving coefficient, and the definition of the industry driving coefficient comes from the industry investment table. Therefore, the value of C_{ij} satisfies the following formula:

$$c_{ij} = \begin{cases} G_{k_i k_j} \dfrac{m_i \times m_j}{r_{ij}^2}, i, j \notin samelayer \\ -G_{k_i k_j} \dfrac{m_i \times m_j}{r_{ij}^2}, i, j \in samelayer \end{cases} \quad (6)$$

$$G_{k_i k_j} = h_{k_j k_i} + d_{k_i k_j}$$

Note: m_i represents the degree value of node i, i and j represent the number of nodes, r_{ij} represents the distance between enterprises, $r_{ij} = |i-j|$, k_i represents the industry of enterprise i, and $G_{k_i k_j}$ represents industry k_i versus industry k_j, $h_{k_j k_i}$ represents the complete consumption coefficient of industry k_i to industry k_j ,and $d_{k_i k_j}$ represents the complete distribution coefficient of industry k_i to industry k_j.

3.2.2 Analysis of the synchronization area after the coupling function is improved

The interconnection matrix is $H = diag(1,0,0)$, and the unified chaotic dynamics model is used as the network node dynamics. The coupling function is determined according to the degree value and distance of the node, and its value will vary with the change of the node degree value According to formula (4), for the situation where the synchronization state is the equilibrium point, the main stability equation can be written as

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$$\dot{y} = [A - \theta H]y \qquad (7)$$

Where $\theta = c_{ij}\lambda_i$, λ_i represents the eigenvalue of the Laplacian matrix.

Chaos Dynamics System of Supply Chain Network:

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x) \\ \dot{y} = (28 - 35\alpha)x - xz - (29\alpha - 1)y \\ \dot{z} = xy - \dfrac{8+\alpha}{3} z \end{cases} \qquad (8)$$

In the formula: $\alpha \in (0,1)$.

Type 1: When the node enterprises i and j belong to the same level enterprises, the enterprises belong to the competitive relationship $c_{ij} = -G_{k_i k_j} \dfrac{m_i \times m_j}{r_{ij}^2}$, the master stability function equation is:

$$\dot{y} = [A + \theta H]y$$

To make a simple proof, take the case of $H = diag(1,0,0)$. The other cases are similar. The coefficient matrix of the master stability equation at (0,0,0) is:

$$Df + \theta DH = \begin{bmatrix} -(25\alpha+10)+\theta & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\dfrac{8+\alpha}{3} \end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha) = (\lambda + \frac{8+\alpha}{3})[\lambda^2 + (11-4\alpha-\theta)\lambda + (29\alpha-1)\theta - (25\alpha+10)(27-6\alpha)] = 0$$

Obviously it can be seen that the first eigenvalue is negative $\lambda_1 = -\dfrac{8+\alpha}{3} < 0$, in order to satisfy that the eigenvalues are all negative, we know that $11-4\alpha-\theta > 0$, and $-(1-29\alpha) - (25\alpha+10)(27-6\alpha) > 0$, that is, $\theta < 11-4\alpha$, and when $\alpha \in (0,1/29), \theta < \dfrac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, $SR = (-\infty, \dfrac{(25\alpha+10)(27-6\alpha)}{1-29\alpha})$ .When these conditions are met, the remaining two eigenvalues of the characteristic equation are all negative.

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Therefore, the two nodes can be fully synchronized. When $\alpha \in (0,1/29)$, $SR = (-\infty, \frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha})$, which means that it can also be obtained in this area Synchronize. Therefore, α=1/29 is the "unbounded-bounded" bifurcation point.

Type 2: When the node enterprises i and j belong to different level enterprises, the enterprises belong to a cooperative relationship $c_{ij} = G_{k_i k_j} \frac{m_i \times m_j}{r_{ij}^2}$, the master stability equation is:

$$\dot{y} = [A - \theta H]y$$

As with the above conditions, the coefficient matrix of the master stability equation at (0, 0, 0) can be obtained as:

$$Df - \theta DH = \begin{bmatrix} -(25\alpha+10)-\theta & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha) = (\lambda + \frac{8+\alpha}{3})[\lambda^2 + (11-4\alpha+\theta)\lambda - (29\alpha-1)\theta - (25\alpha+10)(27-6\alpha)] = 0$$

Obtain $\lambda_1 = -\frac{8+\alpha}{3} < 0$, also in order to ensure that the eigenvalues of the characteristic equation are all negative, it is still necessary to satisfy $11-4\alpha+\theta > 0$ and $(1-29\alpha)\theta - (25\alpha+10)(27-6\alpha) > 0$, that is, when $\alpha \in (0,1/29), \theta > \frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, $SR = (\frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$. When these conditions are met, the remaining two eigenvalues of the characteristic equation are both negative, so the two nodes can be completely synchronized. When $\alpha \in (1/29,1], SR = \Phi$, which means that synchronization cannot be achieved in this area. Therefore, α=1/29 is also a "unbounded-empty set" bifurcation point.

Obviously, it is concluded that the stability domains in which the cooperation and competition between enterprises in the supply chain network are synchronized are mutually exclusive, and what is needed in the synchronous development of the supply chain network is to promote the cooperation between enterprises. Therefore, it is concluded that the bifurcation type of the synchronization area of the supply chain network is "unbounded-empty set" bifurcation point. The synchronization capability of the network can be judged

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according to the smallest non-zero value of the product of the eigenvalue of the external coupling matrix and the coupling capability. The larger the value, the stronger the synchronization capability of the supply chain network.

3.3 Improvement of the influence component of the interconnection matrix based on the supply chain network layering

3.3.1 Improvement of interconnection matrix

Regarding the supply chain network, make assumptions: S is the supplier network; M is the manufacturer network; D is the distributor network; R is the retailer network. In the interconnection matrix of enterprises i and j, the supply of enterprise i has an impact on the supply, storage, and demand of enterprise j (when the supply of enterprise i increases by a%, it will inevitably cause the supply of enterprise j Supply quantity ± b%, storage capacity ± c%, demand quantity ± d%), the impact value needs to be determined according to the number of levels between enterprises, the greater the difference between the levels, the smaller the number of transactions and the quantity in general, so The influence will naturally decrease. Assuming that the influence component of the same-level enterprise a=1, the other are all taking the reciprocal of the absolute value of the difference between the enterprise series as the influence component. For the convenience of calculation, only the coupling value of the first influence component supply is considered. As shown in the following formula:

$$H_{ij} = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{9}$$

$$a = \begin{cases} 1, when k_i = k_j \\ \dfrac{1}{|k_i - k_j|}, when k_i \neq k_j \end{cases}$$

Note: When $i \in S, k_i = 1$, and so on, S=1; M=2; D=3; R=4.

3.3.2 Analysis of the synchronization area after the improvement of the internal connection matrix

According to the four-tier supply chain network described in 3.3.1, four different interconnection matrices will be derived, the difference being the value of the coupling component of the supply. The level difference between enterprise i and j may be: 0, 1, 2 and 3, 0 represents the node level difference is 0, and a is defined as 1;1 represents the enterprise level difference is 1, a is 1/1; and so on, In this section, two cases where the stratification difference is 1 and 3 are selected for description. For other specific analysis, please refer to section 4.2.

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1. When the difference between the number of layers of enterprise i and j is 1, a=1

The internal connection matrix is written as:

$$H_{ij}=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

The master stability equation is: $\dot{y}=[A-\theta H]y$

$$Df-\theta DH=\begin{bmatrix}-(25\alpha+10)-\theta & 25\alpha+10 & 0\\ 28-35\alpha & 29\alpha-1 & 0\\ 0 & 0 & -\dfrac{8+\alpha}{3}\end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha)=(\lambda+\frac{8+\alpha}{3})[\lambda^2+(11-4\alpha+\theta)\lambda+(1-29\alpha)\theta-(25\alpha+10)(27-6\alpha)]=0$$

Obtain $\lambda_1=-\frac{8+\alpha}{3}<0$, also in order to ensure that the eigenvalues of the characteristic equation are all negative values, it is still necessary to satisfy $11-4\alpha+\theta>0$ and $(1-29\alpha)\theta-(25\alpha+10)(27-6\alpha)>0$, that is, when $\alpha\in(0,1/29),\theta>\frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, $SR=(\frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha},\infty)$ .When these conditions are met, the remaining two eigenvalues of the characteristic equation are both negative, so the two nodes can be completely synchronized. When $\alpha\in[1/29,1), SR=\Phi$. which means that synchronization cannot be achieved in this area. Therefore, α=1/29 is the "unbounded-empty set" bifurcation point.

2. When the difference between the number of layers of enterprise i and j is 3, a=1/3

The internal connection matrix is written as:

The master stability equation is: $\dot{y}=[A-1/3\theta H]y$

$$H_{ij}=\begin{pmatrix}1/3&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

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$$Df-\frac{1}{3}\theta DH=\begin{bmatrix}-(25\alpha+10)-\frac{1}{3}\theta & 25\alpha+10 & 0\\ 28-35\alpha & 29\alpha-1 & 0\\ 0 & 0 & -\frac{8+\alpha}{3}\end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha)=(\lambda+\frac{8+\alpha}{3})[\lambda^2+(11-4\alpha+\frac{1}{3}\theta)\lambda+(1-29\alpha)\frac{1}{3}\theta-(25\alpha+10)(27-6\alpha)]=0$$

Obtain $\lambda_1=-\frac{8+\alpha}{3}<0$, also in order to ensure that the eigenvalues of the characteristic equation are all negative values, it is still necessary to satisfy $11-4\alpha+\frac{1}{3}\theta>0$ and $(1-29\alpha)\frac{1}{3}\theta-(25\alpha+10)(27-6\alpha)>0$, that is, when $\alpha\in(0,1/29),\theta>\frac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, SR= $(\frac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha},\infty)$ .When these conditions are met, the remaining two eigenvalues of the characteristic equation are both negative, so the two nodes can be completely synchronized. When $\alpha\in[1/29,1), SR=\Phi$. which means that synchronization cannot be achieved in this area. Therefore, α=1/29 is the "unbounded-empty set" bifurcation point.

From the above calculations, the cross-level cooperation of enterprises in the supply chain network causes the reduction of the synchronization stability area. Under the same unified dynamic parameter α, the supply chain network needs a higher coupling ability to reach the synchronization stability area.

IV. INTRODUCTION TO THE SUPPLY CHAIN NETWORK MODEL BASED ON THE MASTER STABILITY FUNCTION

4.1 Introduction to Supply Chain Network Model

According to the master stability function method proposed by Pecora and Carroll[1], the master stability equation is obtained as $\dot{y}=[A-\theta H]y$, where $\theta=c_{ij}\lambda_i$. The existing research on the coupling function and the interconnection matrix has only been considered in the homogeneous network, and the consideration of the heterogeneous supply chain network is not sufficient, the results obtained have deviations in explaining the supply chain network. This paper uses the concept of gravitational field to determine the value of coupling strength, and further considers the competition and cooperation between enterprises to determine the positive and negative coupling strength; on the other hand, considering the hierarchical properties of the supply chain, the influence capability between nodes is expressed by the reciprocal of the inter-layer difference. The research results can better guide the supply chain network to

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achieve synchronization.

Step 1: Determine the local world of m_1 enterprise at time t,

$$\Omega_{m_1} = \{s_1, s_2, s_3, m_2, m_3, d_2, d_3, r_2, r_5\};$$

Step 2: Determine the value of the coupling function c_{ij} according to the number and distance between the company and m_2 companies' competitors and collaborators in the local world, and consider the nature of the node relationship to determine the level of influence of the contacting company in the interconnection matrix;

Step 3: Write the coupling strength function and interconnection matrix of the supply chain network according to the above rules:

Coupling function:

$$c_{ij} = \begin{cases} G_{k_i k_j} \dfrac{m_i \times m_j}{r_{ij}^2}, i, j \notin samelayer \\ -G_{k_i k_j} \dfrac{m_i \times m_j}{r_{ij}^2}, i, j \in samelayer \end{cases}$$

Interconnection matrix:

$$H_{ij} = \begin{cases} \begin{pmatrix} 1/|k_i - k_j| & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, i, j \notin samelayer \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, i, j \in samelayer \end{cases}$$

Step 4: Combine the changed coupling function and interconnection matrix according to the level of the enterprise and the nature of the relationship. Draw the supply chain network model:

$$\dot{x}_i = f(x_i) - \sum_{g \in G} c_{ig} l_{ig} H_{|k_i - 1|}(x_g) - \sum_{m \in M} c_{im} l_{im} H_{|k_i - 2|}(x_m) - \sum_{d \in D} c_{id} l_{id} H_{|k_i - 3|}(x_d) - \sum_{r \in R} c_{ir} l_{ir} H_{|k_i - 4|}(x_r) \quad (10)$$

Note: $G = [g_1, g_2, g_3], M = [m_1, m_2, m_3], D = [d_2, d_3], R = [r_2, r_5]$, where G, M, D and R represent the supply chain respectively Suppliers, manufacturers, distributors, and retailers in the network.

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$$ki = \begin{cases} 1, i \in G \\ 2, i \in M \\ 3, i \in D \\ 4, i \in R \end{cases} \tag{11}$$

When enterprises i and j belong to the same set, the coupling strength is: $c_{ij} = -G_{k_i k_j} \frac{m_i \times m_j}{r_{ij}^2}$, otherwise the coupling strength is: $c_{ij} = G_{k_i k_j} \frac{m_i \times m_j}{r_{ij}^2}$. It is assumed that the driving coefficient of all industries is G.

Interconnection matrix:

$$H_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} H_2 = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} H_3 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Still expressing s(t) as the solution of the isolated nodal dynamics equation $\dot{s} = f(s)$, the master stability function is given below.

Let $\varepsilon_i(t) = x_i(t) - s(t)$, and get the variational equation of equation 10 on S

$$\dot{\varepsilon}_i = Df(s)\varepsilon_i - \sum_{g \in G} c_{ig} l_{ig} DH_{|k_i-1|}(s)\varepsilon_g - \sum_{m \in M} c_{im} l_{im} DH_{|k_i-2|}(s)\varepsilon_m - \sum_{d \in D} c_{id} l_{id} DH_{|k_i-3|}(s)\varepsilon_d - \sum_{r \in R} c_{ir} l_{ir} DH_{|k_i-4|}(s)\varepsilon_r \quad \text{i=1,2,.......,N} \tag{12}$$

Here we still remember $C^T = P\Lambda P^{-1}, \Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_N)$, where $\lambda_k (k = 1, 2, ..., N)$ is the coupling Laplacian matrix L The characteristic value, and then let $\eta = [\eta_1, \eta_2, ..., \eta_N] = \varepsilon P$, then we have

$$\dot{\varepsilon}_i P = Df(s)\varepsilon_i P - \lambda_i DH_{|k_i-1|}(s)\varepsilon_g P_G - \lambda_i DH_{|k_i-2|}(s)\varepsilon_m P_M - \lambda_i DH_{|k_i-3|}(s)\varepsilon_d P_D - \lambda_i DH_{|k_i-4|}(s)\varepsilon_r P_R \tag{13}$$

Note: c_g, c_m, c_d, c_r respectively represent the minimum coupling strength values between node k and the connected nodes of each layer.

According to formula 4, the criterion of synchronous epidemic stability is that the cross-Lyapunov exponent of equation (14) is required to be all negative. The synchronization area of the supply chain

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network can be obtained, and its type and synchronization ability can be judged.

Step 5: Steps 1 to 4 are the network at time t. Assuming that the supply chain network joins an enterprise in each unit time, calculate the change of network synchronization capability. Take a random generation of $k_i \in [1,2,3,4]$, representing G, M, D, and R respectively. After generation, the total sequence number of a given node is i. When joining, two new connections are generated, one of which is randomly connected to less than The set of k_i, and the other randomly connected to the set greater than k_i. In order to ensure the rationality of the supply chain network connection, each enterprise has the source of the goods and the sales channels of the goods.

Step 6: When the total sales of all retailers equals the total market demand, the network stops evolving. Assuming that the sales volume of each retailer is q_i, and the total market demand Q, when $Q = \sum_{i=1}^{i\in R} q_i$, the network evolution is terminated, otherwise proceed to step 5.

4.2 Analysis of Bifurcation Types of Supply Chain Model

In summary, regarding the improved master stability function, four types of master stability equations will appear in the four-tier supply chain network, which are:

1. $H_{|k_i-1|} = H_0, c_{ij} = -G_{k_ik_j}\frac{m_i \times m_j}{r_{ij}^2}$,, The master stability equation is： $\dot{y} = [A + \theta H_0]y$

$$DF + \theta DH_0 = \begin{bmatrix} -(25\alpha+10)+\theta & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha) = (\lambda + \frac{8+\alpha}{3})[\lambda^2 + (11-4\alpha-\theta)\lambda - (1-29\alpha)\theta - (25\alpha+10)(27-6\alpha)] = 0$$

Obtain $\lambda_1 = -\frac{8+\alpha}{3} < 0$, also in order to ensure that the eigenvalues of the characteristic equation are all negative values, it is still necessary to satisfy $11-4\alpha+\theta > 0$ and $-(1-29\alpha)\theta \ +(25\alpha+10)(27-6\alpha) > 0$, that is, when $\alpha \in (0,1/29), \theta < \frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, SR= $(-\infty, \frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha})$ .When these

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conditions are met, the remaining two eigenvalues of the characteristic equation are both negative, so the two nodes can be completely synchronized. When $\alpha \in [1/29,1), SR = \Phi$. which means that synchronization cannot be achieved in this area. Therefore, α=1/29 is the "unbounded-empty set" bifurcation point.

2. $H_{|k_i-1|} = H_1, c_{ij} = G_{k_i k_j}\dfrac{m_i \times m_j}{r_{ij}^2}$, The master stability equation is: $\dot{y} = [A - \theta H_1]y$

$$DF - \theta DH_1 = \begin{bmatrix} -(25\alpha+10)-\theta & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\dfrac{8+\alpha}{3} \end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha) = (\lambda + \frac{8+\alpha}{3})[\lambda^2 + (11-4\alpha+\theta)\lambda + (1-29\alpha)\theta - (25\alpha+10)(27-6\alpha)] = 0$$

Obtain $\lambda_1 = -\dfrac{8+\alpha}{3} < 0$, also in order to ensure that the eigenvalues of the characteristic equation are all negative values, it is still necessary to satisfy $11-4\alpha+\theta > 0$ and $(1-29\alpha)\theta - (25\alpha+10)(27-6\alpha) > 0$, that is, when $\alpha \in (0,1/29), \theta > \dfrac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, SR=$(\dfrac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$ .When these conditions are met, the remaining two eigenvalues of the characteristic equation are both negative, so the two nodes can be completely synchronized. When $\alpha \in [1/29,1), SR = \Phi.$, which means that synchronization cannot be achieved in this area. Therefore, α=1/29 is the "unbounded-empty set" bifurcation point.

3. $H_{|k_i-1|} = H_2, c_{ij} = G_{k_i k_j}\dfrac{m_i \times m_j}{r_{ij}^2}$, The master stability equation is: $\dot{y} = [A - \theta H_2]y$

$$DF - \frac{1}{2}\theta DH_2 = \begin{bmatrix} -(25\alpha+10)-\dfrac{1}{2}\theta & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\dfrac{8+\alpha}{3} \end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha) = (\lambda + \frac{8+\alpha}{3})[\lambda^2 + (11-4\alpha+\frac{1}{2}\theta)\lambda + (1-29\alpha)\frac{1}{2}\theta - (25\alpha+10)(27-6\alpha)] = 0$$

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Obtain $\lambda_1 = -\frac{8+\alpha}{3} < 0$, also in order to ensure that the eigenvalues of the characteristic equation are all negative values, it is still necessary to satisfy $11-4\alpha+\frac{1}{2}\theta > 0$, and $(1-29\alpha)\frac{1}{2}\theta-(25\alpha+10)(27-6\alpha) > 0$, that is, when $\alpha \in (0,1/29), \theta > \frac{2(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, $SR = (\frac{2(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$ .When these conditions are met, the remaining two eigenvalues of the characteristic equation are both negative, so the two nodes can be completely synchronized. When $\alpha \in [1/29,1), SR = \Phi$. which means that synchronization cannot be achieved in this area. Therefore, α=1/29 is the "unbounded-empty set" bifurcation point.

4. $H_{|k_i-1|} = H_3, c_{ij} = G_{k_ik_j}\frac{m_i \times m_j}{r_{ij}^2}$, The master stability equation is: $\dot{y} = [A-\theta H_3]y$

$$DF-\theta DH_3 = \begin{bmatrix} -(25\alpha+10)-\frac{1}{3}\theta & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}$$

The corresponding characteristic equation is:

$$f(\lambda,\theta,\alpha) = (\lambda+\frac{8+\alpha}{3})[\lambda^2+(11-4\alpha+\frac{1}{3}\theta)\lambda+(1-29\alpha)\frac{1}{3}\theta-(25\alpha+10)(27-6\alpha)] = 0$$

Obtain $\lambda_1 = -\frac{8+\alpha}{3} < 0$, also in order to ensure that the eigenvalues of the characteristic equation are all negative values, it is still necessary to satisfy $11-4\alpha+\frac{1}{3}\theta > 0$ and $(1-29\alpha)\ \frac{1}{3}\theta-(25\alpha+10)(27-6\alpha) > 0$, that is, when $\alpha \in (0,1/29), \theta > \frac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha}$, SR= $(\frac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$ .When these conditions are met, the remaining two eigenvalues of the characteristic equation are both negative, so the two nodes can be completely synchronized. When $\alpha \in [1/29,1), SR = \Phi$. , which means that synchronization cannot be achieved in this area. Therefore, α =1/29 is the "unbounded-empty set" bifurcation point.

Through the calculation of the above four types of nodes, it can be concluded that the stability domains for nodes in the network to achieve synchronization are four intervals. $SR_1 = (-\infty, \frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha})$;

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$SR_2 = (\frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$; $SR_3 = (\frac{2(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$; $SR_4 = (\frac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$. It is concluded that in the interval of $\alpha \in (0, 1/29)$, the synchronization and stability area of nodes on the same layer is $(-\infty, \frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha})$; The synchronization stability area of heterogeneous nodes is $SR_4 = (\frac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha}, \infty)$.

It can be concluded that the synchronization area of the supply chain network is the second type of the four types of network synchronization areas [17], namely SR=(a, ∞) type, that is, when $a_1 < c\lambda_2 < c\lambda_3 < .... < c\lambda_N$, The largest lyapunov exponent is less than 0. For the case where the synchronization area is unbounded, when the coupling strength is greater than a certain value, the synchronization manifold is stable. Therefore, in this case, the minimum non-zero eigenvalue λ_2 of the coupling matrix can be used as an indicator to measure the network synchronization capability. If λ_2 is larger, the supply chain network is easier to synchronize, that is, the supply chain network's synchronization capability is stronger. Since the coupling strength of the improved main stability function is not a fixed value, the synchronization area is $a < c_{ik}\lambda_2 < c_{kj}\lambda_3 < .... < c_{in}\lambda_N .i, j, n \in [1, N]$. the same reason shows that when the minimum non-zero value of $c_{ij}\lambda_i$ is larger, the supply chain network is easier to synchronize, that is, the synchronization capability of the supply chain network is stronger.

V. SIMULATION RESEARCH ON THE EVOLUTION OF SUPPLY CHAIN NETWORK

5.1 Supply chain network simulation

From the analysis in Section 4.2, it can be seen that in the supply chain network model, the synchronization conditions of the same-tier enterprises are contrary to the synchronization conditions of the non-peer-tier companies. What is needed in the economic market is the synchronization of cooperation between non-peer-tier companies, due to the competitive relationship between the same-tier companies. So define $H_0 = zeros(3)$. The following describes the simulated supply chain network shown in Fig 1 and Fig 2. Assuming that the market demand is Q and the sales volume of each retailer is Q/6, when the retailer's total supply meets the market demand, the supply chain network is stable.

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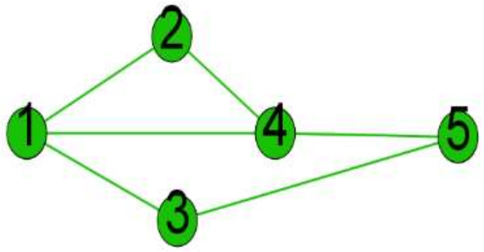

Fig.1 t=0 Initial network

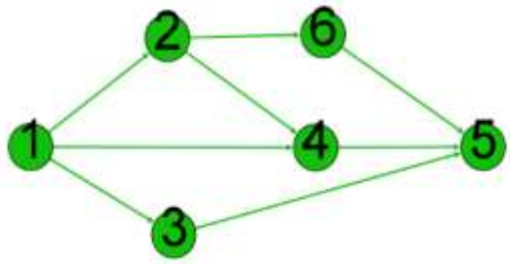

Fig.2 t=1 time network

5.2 Analysis of Supply Chain Network Topology

In the calculation, the Laplacian matrix is replaced by the coupling strength matrix of the network. Due to the contradiction between the synchronization conditions of the same-level and non-same-level enterprises, the competitive relationship of the same-level enterprises is not considered in the simulation network. From Fig. 1 and Fig. 2, the Laplacian matrix L_0 and L_1 can be obtained. According to the rule of formula 6, the coupling strength matrix C_0 and C_1 of the corresponding network at the corresponding time can be obtained. $G_0=[1_g], M_0=[2_m,3_m]$, $D_0=[4_d], R_0=[5_r]$; $G_1=[1_g], M_1=[2_m,3_m], D_1=[4_d,6_d], R_1=[5_r]$.

$$L_0=\begin{pmatrix}-3&1&1&1&0\\1&-2&0&1&0\\1&0&-2&0&1\\1&1&0&-3&1\\0&0&1&1&-2\end{pmatrix} L_1=\begin{pmatrix}-3&1&1&1&0&0\\1&-3&0&1&0&1\\1&0&-2&0&1&0\\1&1&0&-3&1&0\\0&0&1&1&-3&1\\0&1&0&0&1&-2\end{pmatrix}$$

$$C_0=\begin{pmatrix}0&40.02&10.00&6.67&0\\40.02&0&0&10.00&0\\10.00&0&0&0&6.67\\6.67&10.00&0&0&40.02\\0&0&6.67&40.02&0\end{pmatrix} C_1=\begin{pmatrix}0&60.03&10.00&6.67&0&0\\60.03&0&0&15.00&0&2.50\\10.00&0&0&0&10.00&0\\6.67&15.00&0&0&60.03&0\\0&0&10.00&60.03&0&40.02\\0&2.50&0&0&40.02&0\end{pmatrix}$$

$$P_0=\begin{pmatrix}-0.6015&0.5117&0.3717&-0.1954&0.4472\\0.0000&-0.6325&0.0000&-0.6325&0.4472\\0.3717&-0.1954&0.6015&0.5117&0.4472\\0.6015&0.5117&-0.3717&-0.1954&0.4472\\-0.3717&-0.1954&-0.6015&0.5117&0.4472\end{pmatrix} P_1=\begin{pmatrix}0.2887&0.6533&0.2887&0.4082&-0.2706&0.4082\\0.2887&-0.6533&0.2887&0.4082&0.2706&0.4082\\-0.2887&-0.2706&0.2887&-0.4082&-0.6533&0.4082\\-0.5774&0.0000&-0.5774&0.4082&0.0000&0.4082\\0.5774&0.0000&-0.5774&-0.4082&0.0000&0.4082\\-0.2887&0.2706&0.2887&-0.4082&0.6533&0.4082\end{pmatrix}$$

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$$\Lambda_0 = \begin{pmatrix} -4.6180 & 0 & 0 & 0 & 0 \\ 0 & -3.6180 & 0 & 0 & 0 \\ 0 & 0 & -2.3820 & 0 & 0 \\ 0 & 0 & 0 & -1.3820 & 0 \\ 0 & 0 & 0 & 0 & 0.000o \end{pmatrix} \Lambda_1 = \begin{pmatrix} -5.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4.4142 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.5858 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0000 \end{pmatrix}$$

5.3 Analysis of Supply Chain Network Synchronization Capability

In Fig 1, node 2 is used for illustration, when $k_2 = 2$, the master stability equation is obtained as:

$$\dot{\eta}_2 = [Df(s) - \lambda_2 c_g DH_{|k_i-1|}(s)]\eta_0^G + [Df(s) - \lambda_2 c_m DH_{|k_i-2|}(s)]\eta_0^M + [Df(s) - \lambda_2 c_d DH_{|k_i-3|}(s)]\eta_0^D + [Df(s) - \lambda_2 c_r DH_{|k_i-4|}(s)]\eta_0^R$$

From the eigenvalue matrix Λ_0, we can see that the minimum non-zero eigenvalue $\lambda_2 = 1.382$ at t=0; manufacturer 2 has cooperation with supplier 1 and distributor 4, $c_g = 40.02$; $c_d = 10.005$. If node 2 is not connected to the retailer-level enterprise, then $c_r = 0$.

$$\dot{\eta}_2 = [Df(s) - \lambda 2cgDH1(s)]\eta_0^G + [Df(s) - \lambda 2cdDH1(s)]\eta_0^D$$

Nodal dynamics formula chooses unified hybrid dynamics:

$$DF(s) - c\lambda_2 DH_3 = \begin{bmatrix} -(25\alpha+10) - c\lambda_2 & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}$$

$$f(\lambda, c\lambda_2^{\Lambda_0}, \alpha) = (\lambda + \frac{8+\alpha}{3})[\lambda^2 + (11-4\alpha+c\lambda_2^{\Lambda_0})\lambda + (1-29\alpha)c\lambda_2^{\Lambda_0} - (25\alpha+10)(27-6\alpha)] = 0$$

$\lambda_1 = -\frac{8+\alpha}{3} < 0,$ in order to ensure that the eigenvalues of the characteristic equation are all negative, it needs to satisfy $11-4\alpha+c\lambda_2^{\Lambda_0} > 0$ and $(1-29\alpha)c\lambda_2^{\Lambda_0} - (25\alpha+10)(27-6\alpha) > 0$, i.e. $c\lambda_2^{\Lambda_0} > \frac{(25\alpha+10)(27-6\alpha)}{1-29\alpha}$ when $\alpha \in (0, 1/29)$, when the value of c is $c_g = 40.02$; $c_d = 10.00$, the value condition of α cannot be satisfied. Therefore, in the initial network, when the government does not interfere, manufacturer 2 cannot synchronize with supplier 1 and distributor enterprise 4.

$$\dot{\eta}_3 = [Df(s) - \lambda_3 c_g DH_1(s)]\eta_0^G + [Df(s) - \lambda_3 c_r DH_2(s)]\eta_0^R$$

$$\dot{\eta}_4 = [Df(s) - \lambda_4 c_g DH_2(s)]\eta_0^G + [Df(s) - \lambda_4 c_m DH_1(s)]\eta_0^M + [Df(s) - \lambda_4 c_r DH_1(s)]\eta_0^R$$

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$$\dot{\eta}_5 = [Df(s) - \lambda_5 c_m DH_2(s)]\eta_0^M + [Df(s) - \lambda_5 c_d DH_1(s)]\eta_0^D$$

According to the above conditions, $f(\alpha) = \dfrac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha}$ is an increasing function in $\alpha \in (0,1/29)$, only when $f(0) < c\lambda_k^{\Lambda} H_{k_i}(1,1) < f(\frac{1}{29})$, the supply chain network can be synchronized, $f(0) = 810$, $\lim_{\alpha \to 1/29} f(\alpha) = \infty$ it is concluded that the product of the coupling strength in the supply chain network, the corresponding eigenvalue and the influence component of the internal coupling matrix is only greater than 810, the enterprises in the supply chain network can be synchronized. Obviously, by adjusting the industry driving coefficient, the supply chain network can meet the synchronization conditions. See matrix θ0 for details. When the industry driving coefficient is 733, the supply chain network will be synchronized. It is the minimum value of θ that restricts the supply chain network from achieving synchronization. The topological structure of the supply chain network evolving to t=1 is shown in Figure 2. The distributor node 6_d is added to the initial network. In the same way, the master stability equations between node enterprises are as follows:

$$\dot{\eta}_2 = [Df(s) - \lambda_2 c_g DH_1(s)]\eta_1^G + [Df(s) - \lambda_2 c_d DH_1(s)]\eta_1^D \quad \dot{\eta}_3 = [Df(s) - \lambda_3 c_g DH_1(s)]\eta_1^G + [Df(s) - \lambda_3 c_r DH_2(s)]\eta_1^R$$

$$\dot{\eta}_4 = [Df(s) - \lambda_4 c_g DH_2(s)]\eta_1^G + [Df(s) - \lambda_4 c_m DH_1(s)]\eta_1^M + [Df(s) - \lambda_4 c_r DH_1(s)]\eta_1^R$$

$$\dot{\eta}_5 = [Df(s) - \lambda_5 c_m DH_2(s)]\eta_1^M + [Df(s) - \lambda_5 c_d DH_1(s)]\eta_1^D \quad \dot{\eta}_6 = [Df(s) - \lambda_6 c_m DH_1(s)]\eta_1^M + [Df(s) - \lambda_6 c_r DH_1(s)]\eta_1^R$$

$$\theta_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 6084 & 0 & 0 & 1521 & 0 \\ 2621 & 0 & 0 & 0 & 874 \\ 1327 & 3982 & 0 & 0 & 15927 \\ 0 & 0 & 1694 & 20330 & 0 \end{pmatrix} \theta_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 19991 & 0 & 0 & 4998 & 0 & 833 \\ 4202 & 0 & 0 & 0 & 2101 & 0 \\ 2101 & 9455 & 0 & 0 & 37819 & 0 \\ 0 & 0 & 4637 & 55647 & 0 & 37098 \\ 0 & 2626 & 0 & 0 & 42021 & 0 \end{pmatrix}$$

From the evolution of the matrix θ, it can be seen that the position of enterprises in the supply chain network is fixed, and the hierarchical relationship is also stable. The coupling strength between enterprises increases with the increase in the number of partners. The synchronization restriction relationship in the analog network is manufacturer 3 and retailer 5. it is not difficult to find from Figure 2 that node 4 and node 5 are the two largest connected enterprises in the figure. There is cooperation between the two enterprises. The strong alliance between enterprises in the market will inevitably lead to each other. Increased competitiveness. In the matrix θ_1, it is not difficult to find that the lower limit that restricts the global synchronization of the supply chain network is the coupling ability between new entrants and existing enterprises. This also represents the actual market. The integration of new enterprises into the environment is a development process.

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Further solve the change of synchronization ability during the evolution of supply chain network. When the synchronization state is the equilibrium point (0, 0, 0), the synchronization stability area of the supply chain network is $(\frac{3(25\alpha+10)(27-6\alpha)}{1-29\alpha},\infty)$. When the synchronization state is a chaotic attractor, because the Lyapunov exponent of the chaotic system has no analytical expression, it is difficult to theoretically calculate the bifurcation point of the synchronization area of the supply chain network. Therefore, Matlab can be used for simulation. Fig 3 is a comparison of the master stability function method [1] and the supply chain network model after improving the master stability function. Due to the reduction of the influence component of the interconnection matrix between heterogeneous enterprises, the stable area for synchronization of the supply chain network is reduced. Under the same dynamic parameter α, the larger the stratification difference, the smaller the stable domain for synchronization, indicating that the synchronization domain of the multi-layer network depends on the inter-layer relationship between enterprises. The smaller the enterprise level difference, the larger the stable area. Therefore, the level difference between enterprises should be minimized, and the stability domain for synchronization of the supply chain network should be increased, so that the supply chain network has a stronger synchronization capability. When the

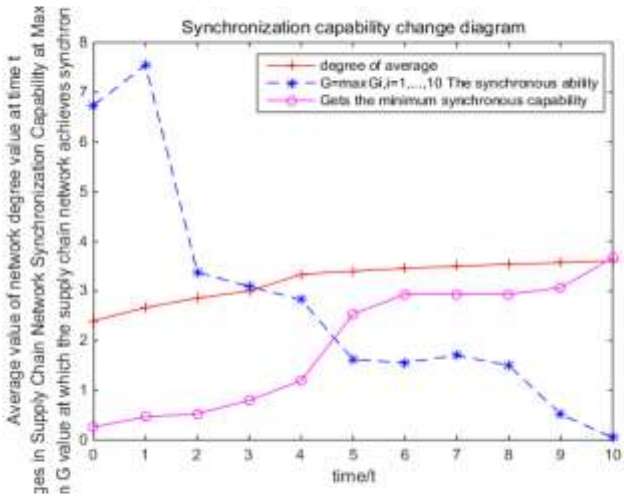

Fig 3. Bifurcation diagram

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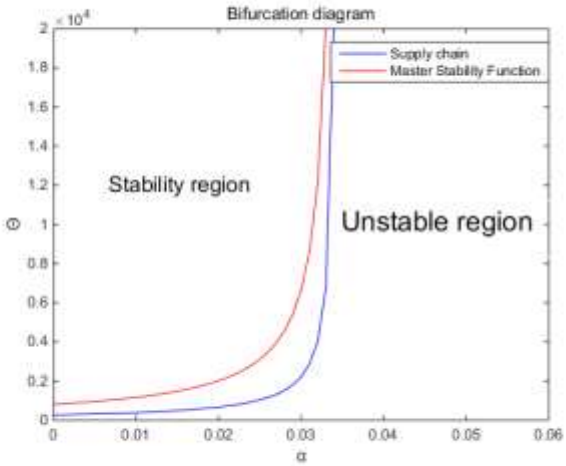

Fig 4. Synchronization capability change diagram

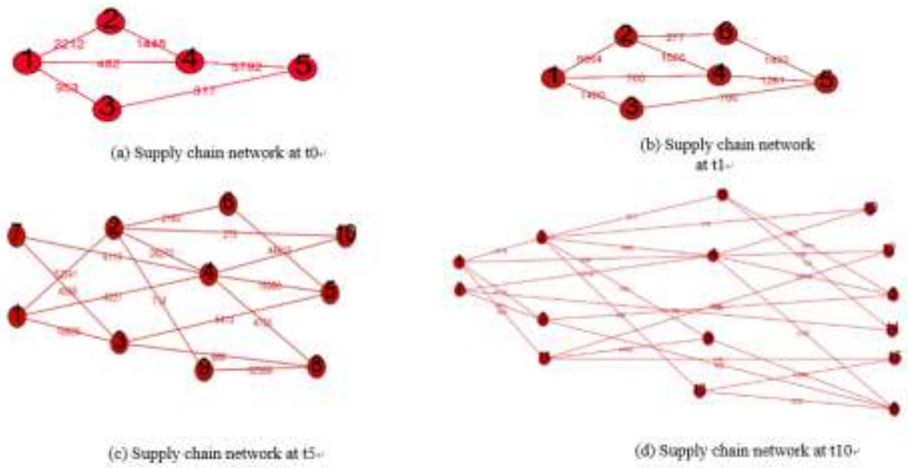

Fig 5. Supply chain network model evolution

Supply chain topology is stable, if you want to synchronize the supply chain network, sometimes you need to increase the G value at a very high cost for a specific relationship. Fig 4 shows that the G value is getting more and more as the supply chain network evolves, so by adjusting the total consumption and distribution between industries, the high-cost synchronization relationship can be eliminated. For example, at t10, the relationship that restricts the synchronization of the supply chain network is enterprise 2 and enterprise 10. When the relationship between the two is not considered, the industry driving coefficient is 3000; and when the relationship between the two is considered, the industry driving coefficient is 3668, at this time To achieve synchronization, the industry driving coefficient is increased by up to 20%, while the synchronization capability of the synchronization restriction relationship is only increased by 2%. At the same time, in the dynamic open supply chain network shown in Fig 5, the synchronization limitation generally comes from the connection of new nodes. For example, the synchronization limitation relationship in Fig 5 (b) and (c) are: 2-6, 2-10, and when a new node Connecting with the magnanimous value node when entering, often becomes a continuous factor restricting synchronization, and Will increase

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with the different development abilities of both parties. For example, in the network of Fig 5(c) and (d), when the enterprise 10 joins the network, then cooperated with large enterprises 2, and it continues to become a relationship that restricts the synchronization of the supply chain network in the subsequent development.

VI. CONCLUSION

This paper studies the push supply chain network based on the master stability function, and derives the measures that can be taken to improve the synchronization capability. The model considers the hierarchical nature of the supply chain network, distinguishes the coupling strength of competition and cooperative enterprises, and explores the influence components of the interconnection matrix. The master stability function is improved through two aspects to make it more suitable for the research of supply chain network synchronization. The following three conclusions are drawn through the research: First, in order to increase the synchronization area of the supply chain network and improve synchronization capabilities, minimize cross-level cooperation between enterprises, and the number of transactions between cross-level cooperative enterprises is small, and the coupling strength is small, The higher requirements for the synchronization environment will become a condition that restricts the synchronization of the supply chain network; second, when the supply chain network topology is stable, the government can control the industry driving coefficient between industries through policy support and restrictions, and eliminate high costs Synchronize corporate relationships; third, advocate for new companies to avoid the option of “the rich get richer” as much as possible during the development of new companies. New companies will cooperate with large companies at the beginning of the industry. This kind of cooperation between asymmetric companies is less available. The impact of higher inequality in costs and terms will generally become conditions that restrict the synchronization of supply chain networks, and the different development capabilities of the two parties will also exacerbate the impact of restrictions.

Regarding the research in this paper, the coupling strength only involves the distance and degree value between enterprises, and the interconnection matrix is only improved between layers. However, in the real supply chain network, the coupling capability is not only affected by the number and distance of the company's competitors, but also by other factors, such as the company's innovation capabilities, national policies, and social opportunities. To determine the influence components of the interconnection matrix, it is far from insufficient to only consider the level difference of the nodes. A single supplier can provide one or more raw materials, but it is often difficult for a single supplier to independently meet the company’s demand for certain raw materials. Need to pay more attention to the interests of those important suppliers. Therefore, the influence of the same-level partners of different levels on the enterprise is also different. This difference is also based on the inter-level relationship. Follow-up research needs to further improve the determination of the coupling strength and the influence component of the interconnection matrix, so that the research results can be Better guide the improvement of supply chain network synchronization capabilities.

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