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A Fuzzy Analytic Hierarchy Process Based Particle Swarm Optimization for Multi-Objective Domain Optimal Control Problems

Qiyu Liu[1], Longjin Lv[2*]
[1]School of Computing and Data Engineering, NingboTech University, Ningbo, Zhejiang, China
[2]School of Finance and Information, Ningbo University of Finance and Economics, Ningbo, Zhejiang, China
*Corresponding Author.

Abstract:

Aiming to solve the multi-objective domain optimal control problem for a heat conduct system, this study proposes a numerical computation method based on a hybrid particle swarm optimization (PSO) algorithm integrated with fuzzy analytic hierarchy process (FAHP). After being discretized by finite element method (FEM) and control parameterization, this problem is solved by a hybrid PSO in which global optimal particle is selected by AHP based on fuzzy consistent judgment matrices. The numerical simulations are presented, and the result shows the proposed method can provide a feasible and effective solution for the multi-objective domain optimal control problem for the heat conduct system.

***Keywords**: Domain optimal control, Control parameterization method, Multi-objective optimization, Particle swarm optimization, Fuzzy analytic hierarchy process.*

I. INTRODUCTION

There are often chemical reaction, mass and energy exchange coupled with diffusion in industrial processes. Their dynamic characteristics can be typically modeled by partial differential equation (PDE), which can quantitatively describe the multivariable phenomenon of chemical process and the spatio-temporal evolution of process dynamics, and has been widely used in the field of chemical engineering. Examples of such PDE systems include hydrodynamic characteristics and temperature distribution models of heat exchangers, heat and mass transfer models of chemical reactors, etc.

In industrial production, it is often necessary to adjust the temperature or concentration of the reactor to make it close to the optimal operating conditions. Therefore, it is very important to study its effective control strategy. The optimal control problem of PDE aims to better optimize and control the process, so as to improve industrial production efficiency and product quality

A special optimization problem for controlled systems is to obtain the positions of the actuators. In this kind of problem, the action position of the controllers should be chosen to optimize the performance

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objectives. Among these problems, the optimal domain for the solved control is called the optimal supporting position. This kind of problem needs to solve not only the trajectory of the optimal control, but also the optimal location of the control.

The current studies are mainly in the case of point control. When the controller do not act on a point, but are distributed on a small area near the optimal supporting position, and multiple controls act on their respective optimal areas respectively, then the optimal control problem becomes to solve both the optimal control trajectory satisfying the performance index and its respective optimal action area (actually find the optimal support position). This kind of problem is called domain optimal control problem[1]. There are some studies from different aspects in this field[2-7], whereas there are few literatures on the computational aspect for domain optimal control problem governed by PDEs.

Due to the complexity of the process itself, it is difficult to find an analytical solution to this kind of problem. With the rapid development of information technology, many researchers are interested in finding the approximate solution of control problem through numerical calculation. Control parameterization is a very effective method to numerically solve the optimal control problem. It transforms the optimal control problem of infinite dimensional PDE system into a finite dimensional optimal parameter selection problem by discretizing control variables. Lin[8] and Yang[9] proposed the main steps of the control parameterization method and improved calculation processes.

In practice, most engineering problems need to simultaneously optimize multiple conflicting objectives, which restrict each other through decision variables. The optimization of one objective often costs other objectives, and further, the units of each objective are often inconsistent, so it is difficult to objectively evaluate the advantages and disadvantages of the solution of multi-objective problems. Therefore, different from single objective optimization, the final result of multi-objective optimization problems (MOPs) is only a series of compromises.

In MOPs, the main challenge is to find the Pareto solutions set which is closest to the true Pareto front highly diversified within the search space. There is a large amount of literature devoted to the studies of the related problems of the MOPs, e.g. refer to references[10-14] and references therein. Particle swarm optimization (PSO) has also achieved good results in some aspects. For example, the external archive is introduced into the multi-objective PSO (MOPSO). Generally, MOPSO does not need to assign fitness, but it must select an appropriate global best position for each particle from the external archive.

In order to further improve the objectivity of the optimal particle selection in the multi-objective evolution process of PSO, a hybrid particle swarm optimization algorithm (PSO) based on fuzzy analytic hierarchy process (FAHP) is proposed to solve the multi-objective domain optimal control problem in this paper.

The remain of this paper is structured as follows. In Section II, the formulation of the optimal control

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problem is introduced. The original optimal control problem is discretized into an optimal parameter selection problem in space and time, which can be solved using a hybrid PSO algorithm. Section III is devoted to the mechanism of fuzzy analytic hierarchy process. In section IV, the FAHP based PSO algorithm is introduced, and the optimization solution procedure are presented. In section V, the numerical simulations are presented to demonstrate the validity and accuracy of the numerical optimization method. Finally, conclusion and future researches are given in section VI.

II. THE MULTI-OBJECTIVE DOMAIN OPTIMAL CONTROL PROBLEM

2.1 Problem Formulation

The controlled heat conduction systems takes the form:

$$\begin{cases} y_t(x,t) - y_{xx}(x,t) = \sum_{i=1}^{m} \chi_{r_i}(x) g_i(x) u_i(t) + f(x,t), \quad 0 < x < 1, 0 < t \le T, \\ y(0,t) = y(1,t) = 0, \quad 0 < t \le T \\ y(x,0) = y^0(x), \end{cases} \tag{1}$$

where m is the number of controlled support positions, r_i is the i-th controlled support position, $g_i(x)$ is the spatial distribution of the i-th controlled area near the support position, $g_i(x) \in L^2(0,1)$, $f(x, t)$ denotes the external force term, and control is subject to the constraints:

$$0 \le u_i(t) \le 1, \quad i = 1, 2, \cdots, m. \tag{2}$$

Let $0 < \varepsilon < 1$ be a given small positive real constant. Let $r_i \in [\underline{r}, \overline{r}], i = 1, 2, \cdots m$, be m constants, where $\underline{r} = \frac{\varepsilon}{2}$, $\overline{r} = 1 - \frac{\varepsilon}{2}$. Define χ_{r_i} is the indictor function of the interval $[r_i - \frac{\varepsilon}{2}, r_i + \frac{\varepsilon}{2}]$ defined by

$$\chi_{r_i}(x) = \begin{cases} 1, & \text{if } x \in [r_i - \frac{\varepsilon}{2}, r_i + \frac{\varepsilon}{2}], \\ 0, & \text{otherwise.} \end{cases}$$

When the controls act on at a single point $r_i \in [\frac{\varepsilon}{2}, L - \frac{\varepsilon}{2}]$, the function $\chi_{r_i}(x)$ is set to nonzero in a finite spatial interval $[r_i - \frac{\varepsilon}{2}, r_i + \frac{\varepsilon}{2}]$, and zero elsewhere.

Define the following set of functions by

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$$U = \{v = (v_1, v_2, \cdots, v_m)' \mid v_i \in L^2(0,T);\ a_i \le v(t) \le b_i \text{ a.e. } t \in [0,T], i = 1,2,\cdots,m\}$$

Where $a_i, b_i \in R$ are given constants, and define

$$R = \{r = (r_1, r_2, \cdots, r_m)' \mid r_i \in [\frac{\varepsilon}{2}, 1 - \frac{\varepsilon}{2}],\ 1 = 1,2,\cdots,m\},\ V = \{v \in L^2(0,L);\ v(0) = v(L) = 0\},\ \ Q = V \times (0,T).$$

Then consider the following objective functions:

$$J_1(u,r) = \int_0^T \int_0^1 (y - y_d)^2 dxdt \tag{3}$$

$$J_2(u) = \sum_{i=1}^{m} \int_0^T |u_i(t)|^2\ dt \tag{4}$$

$$J_3(r) = \sum_{i=1}^{m} |r_i - 0.5|^2 \tag{5}$$

Where $y_d \in L^2(Q)$ is a given target system state function, the objective. $J_1(u,r)$ .indicates that the desired state is close to the expected value, $J_2(u)$ is to control the energy as small as possible, and $J_3(r)$ seeks that the optimal position is as close to the middle area as possible considering the actual needs. The objective $J_1(u,r)$ has a higher priority than the others.

The optimal control problem is as follows:

$$(P) \qquad \min_{u \in U,\, r \in R} J(u,r) = [J_1, J_2, J_3]$$

Subject to the equation (1) and (2).

2.2 Time and Space Discretization of the Problem

In this section, the Galerkin finite element method[15-17] is employed to discretize the original infinite-dimensional problem (P) into a semi-discrete approximation problem governed by a finite-dimensional system. Then, we will use the control parameterization method[8, 18] to reduce the original problem to an optimal parameter selection problem.

Firstly, we standard triangulation to construct a finite-dimensional subspace V^h of the space V. Select $\{\phi_i; i = 1,2,\cdots,N\}$ as the continuous piecewise linear basis functions of V^h, which are defined by

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$$\phi_i(x_j) = \begin{cases} 1, & \text{if } i = j; \\ 0, & \text{if } i \neq j. \end{cases}$$

Then, By FEM for spatial discretization, equation (1) is transformed into an approximately equivalent system of ordinary differential equations in matrix form

$$\begin{cases} M\dot{Y}(t) + KY(t) = B_r u(t) + F(t), \quad 0 < t \leq T, \\ Y(0) = Y^0, \end{cases} \tag{6}$$

Where

$$Y(t) = [Y_1(t), Y_2(t), \cdots, Y_N(t)]', \qquad Y^0 = [Y_1^0, Y_2^0, \cdots, Y_N^0]',$$
$$M = [(\phi_i, \phi_j)]_{N\times N}, \qquad K = [(\partial_x \phi_i, \partial_x \phi_j)]_{N\times N},$$
$$u(t) = [u_1(t), u_2(t), \cdots, u_m(t)]',$$
$$F(t) = [(f(t)_i, \phi_j)]_{N\times 1}, \quad B_r = [(\phi_i, \chi_{r_j} g_j)]_{N\times m}.$$

To approximate problem in space, the control parameterization method is used to restrict the admissible controls to a linear combination of basis functions, in which the coefficients are decision variables to be optimized.

Define

$$U^p = \{\sigma = (\sigma^1, \sigma^2, \cdots, \sigma^p) \in R^p \mid \sigma^k \in U\}.$$

Further, the dynamic system (6) is transformed into

$$\begin{cases} \dot{Y}(t) = \sum_{k=1}^{p} M^{-1}[-KY(t) + B_r \boldsymbol{\sigma}^{\mathbf{k}} + F(t)]\chi_{[t_{k-1},t_k)}(t), \quad t \in [0,T], \\ Y(0) = Y^0. \end{cases} \tag{7}$$

After discretization in time and space, we obtain an ordinary differentiate equations (ODEs) (7) which can be solved by Runge Kutta method. Let $Y^p(\cdot \mid \boldsymbol{\sigma})$ denotes the solution of this system, where $\boldsymbol{\sigma} \in U^p$. Now, the objective functions of the problem (*P*) become

$$J_{1h}^p(\boldsymbol{\sigma}, r) = \frac{1}{2}\int_0^T (Y^p(t \mid \boldsymbol{\sigma}) - Y^d, M(Y^p(t \mid \boldsymbol{\sigma}) - Y^d))_{\mathbf{R}^N} \, dt, \tag{8}$$

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$$J_{2h}^{p}(\boldsymbol{\sigma}, r) = \sum_{i=1}^{m}\sum_{k=1}^{p}\int_{t_{k-1}}^{t_k}\left|\sigma_i^k\right|^2 dt. \tag{9}$$

$$J_3(r) = \sum_{i=1}^{m} | r_i - 0.5 |^2 \tag{10}$$

Then, the approximate problem (P_h^p) of the original problem (P) is defined as follows:

$$(P_h^p) \qquad \min_{u \in U,\, r \in R} J_h^p(\boldsymbol{\sigma}, r) = [J_1, J_2, J_3]$$

Subject to the system (7).

III. FUZZY ANALYTIC HIERARCHY PROCESS

Analytic hierarchy process (AHP) is a multi-objective decision analysis method, which was proposed by American operational research scientist T. L. Saaty. It combines qualitative and quantitative analysis, especially quantifies the experience judgment of decision makers[19, 20].

For complex multi-objective decision-making, AHP first arranges and classifies the influencing factors involved, and constructs a hierarchical structure model on this basis. In this structure, they are connected with each other according to their mutual influence relationship, and they are arranged to different layers from top to bottom to form a hierarchical structure diagram. On this basis, the weight coefficient of the lower element dominated by the factor x_i of each layer with respect to the relative importance of this x_i is determined until the relative weight of each scheme is calculated, and finally the order of advantages and disadvantages of each scheme with respect to the target layer is obtained.

The key of AHP is the construction of consistency judgment matrix, which is established by comparing the elements in each level. However, due to the fuzziness and ambiguity of problem evaluation, using fuzzy analytic hierarchy process (FAHP) to establish fuzzy consistent judgment matrix is more conducive to fuzzy expression of relative comparison of attributes. Here, three scales are used to construct fuzzy consistent judgment matrix.

Define the scheme set as follows

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{pmatrix}$$

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Where m is the number of schemes and n is the number of attribute elements in each scheme. For a scheme c_l, the importance of any two attributes c_{il} and c_{jl} is compared respectively, and the importance index is scaled.

Let e_{ij}^l be the importance scale, and its value principle is as follows:

$$e_{ij}^l = \begin{cases} 0, & c_{jl} \text{ is more important than } c_{il}; \\ 0.5, & c_{jl} \text{ is as important as } c_{il}; \\ 1, & c_{il} \text{ is more important than } c_{jl}. \end{cases} \tag{11}$$

Where $i, j \in \{1, 2, \cdots, n\}$, $l \in \{1, 2, \cdots, m\}$. Let $E_i^l = \sum_{i=1}^{n} e_{ij}^l$, then the fuzzy consistent judgment matrix can be constructed as follows:

$$A^l = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \tag{12}$$

Where each element of the matrix is defined as

$$a_{ij} = \frac{E_i^l - E_j^l}{2^{\alpha} n} + 1. \tag{13}$$

Where α is called sensitivity factor, and its value can be 0, 1, 2 and 3, which can be determined according to the actual situation. In this way, the constructed matrix (12) can be used as a fuzzy consistency matrix in application with some excellent properties.

The calculation of eigenvector of matrix does not need high accuracy, so the square root method is used here. Calculate the weight of each group, that is, the average of each row of the matrix:

$$\bar{w}_{il} = \left(\prod_{j=1}^{n} a_{ij}^l \right)^{1/n}.$$

Normalize $\bar{w}_{il}$ as

$$w_{il} = \frac{\bar{w}_{il}}{\sum_{i=1}^{n} \bar{w}_{il}},$$

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We obtain $w_l = \{w_{1l}, w_{2l}, \cdots, w_{nl}\}$, which is the eigenvector, and is also a weight relative to c_l. If the weight of scheme c_l is $\bar{v}_l$, $l = 1, 2, \cdots, m$, the comprehensive evaluation (i.e. weight) of scheme i relative to the objective is

$$v_i = \sum_{j=1}^{m} w_{ij} \bar{v}_j \tag{14}$$

IV. OPTIMIZATION SOLUTION PROCEDURE

4.1 Adaptive Parameter Adjustment

In PSO algorithm, when the particle position is updated, the search ability of the algorithm is closely related to the value of parameters. The inertia weight w represents the inertia of maintaining the original motion direction, and c_1 and c_2 represent the evolution speed of particles to the individual optimal and global optimal particle direction respectively.

In order to make the particle adaptively adjust various parameters according to the current evolution state, we use the evolution factor as the measurement.

Let N be the population size, and D the dimension of particles. The average distance h_i between the particle i and other particles is calculated.

$$h_i = \frac{1}{N-1} \sum_{j=1, j \neq i}^{N} \left(\sum_{k=1}^{D} (x_i^i - x_j^k)^2 \right)^{1/2} \tag{15}$$

Denote h_g as the average distance of the globally optimal particle, and z as the evolution factor, which is defined as follows:

$$z = \frac{h_g - h_{min}}{h_{max} - h_{min}}. \tag{16}$$

We choose inertia weight $w \in [0.4, 0.9]$, acceleration factors $c_1, c_2 \in [0.5, 2.5]$. Then, their dynamic adjustment formula according to the evolution factor is as follows:

$$w = \frac{1}{1 + 1.5 e^{-2.6z}} \tag{17}$$

If $c_1 + c_2 \leq 4$,

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$$c_1 = \frac{1}{0.2 + 0.2e^{2.2z}},\quad c_2 = \frac{1}{0.1 + 0.2e^{-1.85z}}\ . \tag{18}$$

Otherwise, if $c_1 + c_2 > 4$, then

$$c_1 = \frac{4c_1}{c_1 + c_2},\quad c_2 = \frac{4c_2}{c_1 + c_2}. \tag{19}$$

4.2 External Archive Maintenance Strategies

In the process of population evolution, the convergence speed will slow down as the Pareto solutions set expands. Therefore, strategies need to be adopted to eliminate relatively inferior individuals. Here, a mechanism based on the degree of congestion is used to update and manage external files. The crowding distance d_i of individual i is defined as the sum of the horizontal distances of two adjacent points along each objective function, as shown in Fig 1.

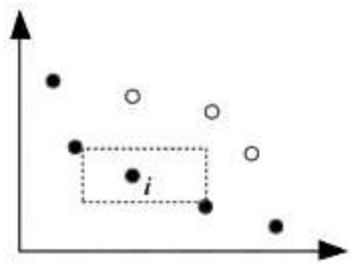

Fig 1: The congestion degree distance of particle i

Let the objective function be $J_s,\ s = 1, 2, \cdots, q$, the calculation steps according to the crowding distance are as follows.

Step 1. Initialization. The individual size of the external archive set to L, the number of objective functions is q, the distance of all members is 0, and the current individual i=1.

Step 2. For the objective $J_s,\ s = 1, 2, \cdots, q$, sort the members in descending order of distance, and set the crowding distance of the members at the boundary position (i.e. i=1 and i=L) to +∞.

Step 3. For each individual, calculate d_i as follows:

$$d_i = d_i + \frac{J_s^{i+1} - J_s^{i-1}}{J_s^{max} - J_s^{min}} \tag{20}$$

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Where J_s^{i+1}, J_s^{i-1} respectively represent the value of the *s*-th objective function of the (*i*+1)-th and (*i*-1)-th individual. J_s^{max}, J_s^{min} are the maximum and minimum of the *s*-th objective function values of all individuals, respectively.

Step 4. Let $s = s+1$, if $s \leq q$, go back to Step 2; otherwise, stop.

When a new non-inferior solution is added to the external archive set, if the number of members does not exceed the given upper limit, it is added directly; if the number exceeds the upper limit, the crowding distance of each member is calculated and the set is sorted, and then, the member with the smallest distance should be eliminated to keep the solution with better distribution.

4.3 FAHP Based PSO Algorithm

In this algorithm, the optimal position of the individual is selected according to the dominance of the solution. If the current solution of the particle dominates the historical optimal solution, the position is the individual optimal; if the current solution and the individual optimal solution have no dominance relationship, the individual optimal solution is randomly determined; otherwise, the individual optimal solution remains unchanged.

In the optimization process, the global optimal particle guides the evolutionary optimization direction of the entire population, and its selection strategy has a great impact on the final result. In the process of multi-objective optimization, the Pareto solutions set composed of non-dominated solutions are continuously updated in the iterative process, gradually approaching the global optimal solution. Since the non-inferior solutions in the archives obtained after each generation of evolution are not dominated, the FAHP can be used to evaluate multi-objectives. According to formula (14), the highest combination weight is selected as the global optimal particle.

Integrated with FAHP, the hybrid multi-objective optimization algorithm FAHP-PSO is described as follows.

Step 1. Initialization. Set the swarm size *M*, the solution interval [X_{max}, X_{min}], and initialize parameters of each particle to random values within the solution space: position X_i, velocity V_i, and individual optimal value P_i=X_i.

Step 2. Calculate the initial objective function value J_s, $s = 1, 2, \cdots, q$. Set the initial individual optimal P_i as X_i; calculate the weight according to the fuzzy consistent judgment matrix, evaluate the particles, select the global optimal value P_g, and save it in the external archive.

Step 3. According to (17), (18) and(19), calculate the parameters to dynamically adjust the evolution

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process. Then update the position and velocity of each particle.

Step 4. Calculate the objective function value $J_s,\ s=1,2,\cdots,q$. Calculate the individual optimal P_i according to the dominance relationship; calculate the weight according to the fuzzy consistent judgment matrix, evaluate the particles, select the global optimal value P_g, store it in the external archives, and maintain it according to the crowded distance.

Step 5. Determine whether the termination condition (maximum number of iterations) is met, and if so, output the result; otherwise, go to Step 3.

V. NUMERICAL SIMULATION

In this section, the numerical simulation is performed in Matlab environment.

In the experiments, we take $T=1,\ y^0(x)=0,\ \varepsilon=0.2,\ r\in[0.1,0.9],\ g(x)=1$, $f(x,t)=2t^2x$, y_d is discrete data.

Set the number of finite element discrete section partitions N=20, and the interval number of control parameterization

p=20.

Let $h=\dfrac{1}{N+1}$, then $F(t)=[(f(t),\phi_i)]_{N\times 1}=2t^2[h^2\ \ 2h^2\ \ \cdots\ \ Nh^2]'$.

Now calculate the explicit formula of the matrix $B_r=[(\phi_i,\chi_{r_j})]_{N\times m}$.

For $i\in[1,N],\ j\in[1,m]$, the expression of B_r element can be obtained by calculation.

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$$(\phi_i,\chi_{r_j})=\int_{r_j-\frac{\varepsilon}{2}}^{r_j+\frac{\varepsilon}{2}}\phi_i(x)dx=\begin{cases}0, & x_{i+1}+\frac{\varepsilon}{2}\le r_j;\\ 0, & r_j<x_{i-1}-\frac{\varepsilon}{2};\\ h, & x_{i+1}-\frac{\varepsilon}{2}\le r_j<x_{i-1}+\frac{\varepsilon}{2};\\ \frac{1}{2h}(r_j+\frac{\varepsilon}{2}-x_{i-1})^2, & x_{i-1}-\frac{\varepsilon}{2}\le r_j<x_i-\frac{\varepsilon}{2};\\ \frac{h}{2}-\frac{1}{2h}[(r_j+\frac{\varepsilon}{2}-x_{i+1})^2-h^2], & x_i-\frac{\varepsilon}{2}\le r_j<x_{i+1}-\frac{\varepsilon}{2};\\ \frac{h}{2}+\frac{1}{2h}[h^2-(r_j-\frac{\varepsilon}{2}-x_{i-1})^2], & x_{i-1}+\frac{\varepsilon}{2}\le r_j<x_i+\frac{\varepsilon}{2};\\ \frac{1}{2h}(r_j-\frac{\varepsilon}{2}-x_{i+1})^2, & x_i+\frac{\varepsilon}{2}\le r_j<x_{i+1}+\frac{\varepsilon}{2}.\end{cases}$$

Other coefficient matrices can be calculated in the same way.

In this experiment, we study the case of only one control position.

In order to compare the optimization results of single objective and multi-objective case, we calculate the optimal control and position obtained for the problem (P) with only objective J_1, as well as the multi-objective case, respectively. In the first case, r = 0.6984; in the second case, r = 0.4735. The optimal control trajectory in the two cases is shown in Fig 2 and Fig 3.

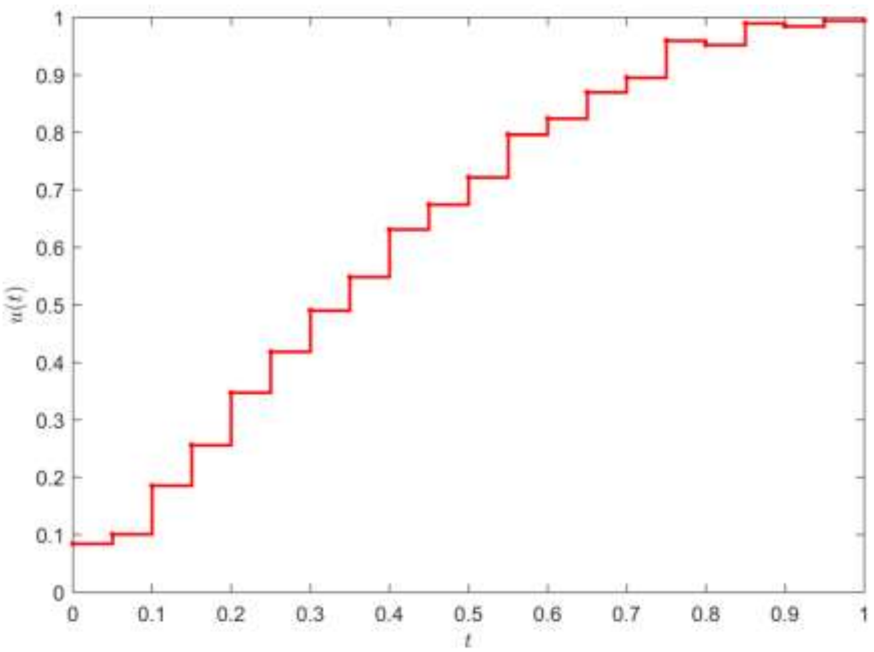

Fig 2: The optimal control trajectory for single objective J_1

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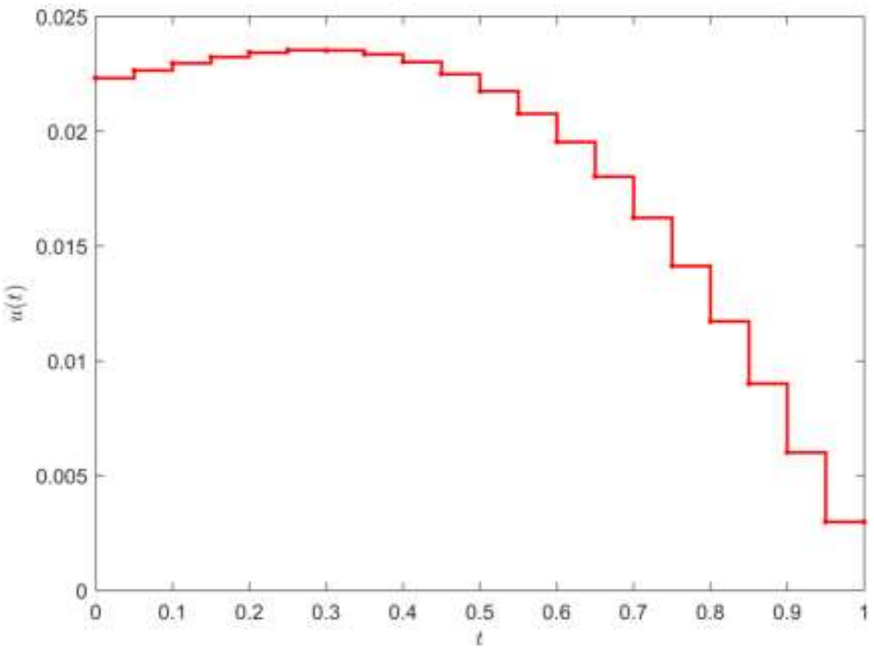

Fig 3: The optimal control trajectory for multi-objective

As can be seen from the figure, the optimal control solutions are quite different in two cases. In the case of single target, the control effect increases gradually to ensure that the state meets the target state trajectory, and the support position deviates more from the middle point; In the case of multi-objective optimization, when the state is made as close as possible to the desired trajectory, in order to ensure the minimum control energy at the same time, the selection of support position is sacrificed. The Pareto optimal solution obtained through optimization provides the decision-making range for the decision-maker. Using the fuzzy consistent judgment matrix, we define the fuzzy subset and fuzzy membership function according to the satisfaction of each objective, and then select three solutions from the Pareto non-inferior solutions for optimization decision-making, thereby obtain the optimal control trajectory as shown in Fig 3.

VI. CONCLUSION

In this paper, we study the multi-objective area optimal control problem of a heat conduction system, and propose a numerical approximation method based on the FAHP-based particle swarm algorithm to solve the multi-objective area optimal control problem. First, finite element and control parameterization methods are used to discretize the original problem into the optimal parameter selection problem of a lumped parameter system; then, the PSO algorithm based on fuzzy analytic hierarchy process is used to optimize the solution. In the multi-objective particle swarm optimization calculation, a fuzzy consistent judgment matrix is established, and the global optimal particle is selected through the analytic hierarchy process. Simulation shows that the algorithm is effective and feasible, and it provides a reference calculation method for solving, optimizing and decision-making of this kind of multi-objective regional optimal control problem.

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