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Hopf Bifurcation Controller Design for Wireless Access Network Congestion Control System

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Abstract:

A novel Hopf bifurcation controller used in wireless access network congestion control system is proposed in this paper. Communication delay is selected as the bifurcation parameter. The conditions of the existence for the Hopf bifurcation are obtained by analyzing the associated characteristic equation. The delay feedback controller is designed to control the Hopf bifurcation for the wireless access network congestion control system. By using the center manifold theorem and the normal form theory, the direction and stability of bifurcating periodic solutions are confirmed. The simulations verify the controller can delay the onset of the Hopf bifurcation and achieve some desirable dynamical behaviors.

Keywords: *Wireless access network, Hopf bifurcation; Bifurcation control, Congestion control, Communication delay*

I. INTRODUCTION

With the rapid development of the communication technology, Internet technology and artificial intelligence technology, wireless access has become the mainstream of the Internet application[1]. The application scenarios of wireless access technologies such as smart factory[2] and smart agriculture[3] have been relatively common. In order to guarantee the communication performance of the network, network congestion control mechanism is necessary[4]. From the perspective of control theory, the Internet can be regarded as a nonlinear dynamic system with feedback and communication delay. The communication delay is an important factor that causes the system control performance to decline or even unstable. Therefore, it is necessary to research the relationship between communication delay and internal dynamic characteristics of the system, so as to improve the actual performance of the congestion control system. In recent years, many scholars have studied the nonlinear behavior of wire Internet congestion

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control system[5-7], but there are few researches on the dynamic analysis and control of wireless access network congestion control system.

Hopf bifurcation analysis is an effective approach to obtain more information around the operating point of the complex dynamical networks[8-10]. Bifurcation control has absorbed a great deal of researcher. In general, bifurcation control is to design a controller for a given nonlinear system to suppress or reduce some existing bifurcation dynamics, so as to obtain some ideal dynamics behavior.

In this paper, the fluid flow model of network congestion control system via wireless access is studied, and the Hopf bifurcation and control problem of the system are studied. The rest of this paper is organized as follows. In Section 2, communication delay is selected as the bifurcation parameter, and it is proved in detail that Hopf bifurcation will occur when communication delay passes through a certain critical value. In Section 3, in order to improve the stability of the system, the delay feedback control strategy is used to control the Hopf bifurcation without changing the original equilibrium point of the system. Based on the normal form theory and the central manifold theorem, the Hopf bifurcation direction and the stability of the bifurcation periodic solution of the controlled system are obtained. Finally, the existence of the system's Hopf bifurcation and the effectiveness of the designed controller on Hopf bifurcation control are verified by simulation.

II. HOPF BIFURCATION ANALYSIS OF THE WIRELESS ACCESS NETWORK CONGESTION CONTROL SYSTEM

The backbone of the wireless access network studied in this paper is a traditional wired network that supports TCP stream, and the source node is accessed through wireless mode. The network topology is shown in Fig.1. Considering the packet loss caused by wireless channel transmission fading in the above-mentioned network, in [11] the fluid flow model describing the congestion control process of wireless access network was proposed.

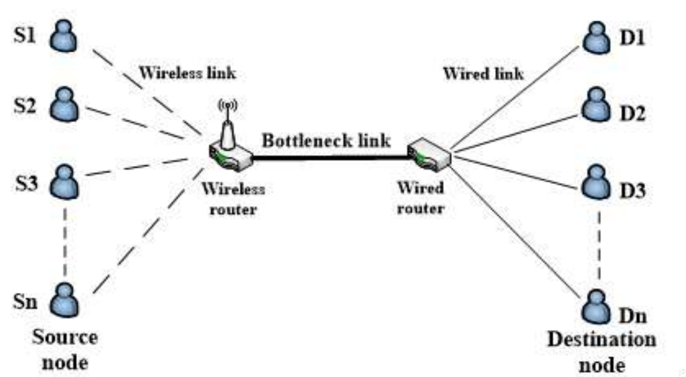

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Fig.1. Wireless access to single-route TCP/AQM network topology

When the link capacity is large, $\tau(t)$ is mainly determined by the propagation delay, and $\tau(t)=\tau_{ah}(t)=\tau$ can be set, τ as a constant [12]. Referring to [11], it is assumed that packet dropping/marking probability is proportional to queue length, i.e. $p(t)=Kq(t), (K>0)$, and $P_{ul}(t)$, $P_{dl}(t)$, is set as constant. Ignoring the TCP window delay, the system model can be simplified as

$$\begin{cases} \dot{W}(t)=\dfrac{1}{\tau}-(1-P_{dl})\dfrac{W(t)}{2}\dfrac{W(t)}{\tau}Kq(t-\tau)-P_{dl}(W(t)-1)\dfrac{W(t)}{\tau}Kq(t-\tau), \\ \dot{q}(t)=-C+N(1-P_{ul})\dfrac{W(t)}{\tau}。 \end{cases} \tag{1}$$

In which $W(t)$ denotes TCP window size of the source node in packets, $q(t)$ denotes the instantaneous queue length in the router in packets, $\tau(t)$ denotes the round-trip time (RTT) in second, incluing propagation delay and transmission delay are included. T_p denotes propagation delay in second, $p(t)$ denotes the packet-dropping probability, $p(t)\in(0,1)$. $P_{ul}(t)$ denotes the packet loss rate in the uplink due to the channel attenuation characteristics of the wireless network, $P_{dl}(t)$ denotes the occurring probability of "packet mark probability loss fed back to the source node" in the downlink, $\tau_{ah}(t)$ denotes the time interval between the current time and the last successfully receiving a packet token, $C(t)$ denotes link capacity in packets per second, $N(t)$ denotes the number of TCP sessions. $q(t)$ and $W(t)$ satisfy $q\in[0,\bar{q}]$, $W\in[0,\bar{W}]$, respectively. Here, $\bar{q}$, $\bar{W}$ denote buffer capacity and maximum window size. $P_{ul}(t)$, $P_{dl}(t)$ are both evaluated in (0,1).

For the convenience of analysis, the nominal values of network parameters are assumed to be constants N, C. Choosing (W,q) as state variable, (W_0,q_0,p_0) denotes the non-zero operating point of the system. Setting $\begin{cases}\dot{W}(t)=0\\ \dot{q}(t)=0\end{cases}$, it can be obtained that

$$W_0=\frac{\tau C}{N(1-P_{ul})},$$

$$q_0=\frac{2}{K\left[(1+P_{dl})W_0^2-2P_{dl}W_0\right]}=\frac{2N^2(1-P_{ul})^2}{K\left[(1+P_{dl})\tau^2C^2-2P_{dl}\tau NC(1-P_{ul})\right]}=\frac{1}{K}p_0.$$

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Note 1: The actual network congestion control system via wireless access, the queue length of the wireless access device is always greater than zero, that is to say, $q_0 > 0$. We obtain $W_0 > \frac{2P_{dl}}{1+P_{dl}}$.

Setting $x_1(t) = W(t) - W_0$, $x_2(t) = q(t) - q_0$, it's linearized at the operating point. It can be deduce that

$$\begin{cases} \dot{x}_1(t) = a_1 x_1(t) + a_2 x_2(t-\tau) \\ \dot{x}_2(t) = a_3 x_1(t) \end{cases} \tag{2}$$

where,

$$a_1 = -\frac{1}{\tau}(1+P_{dl})W_0 p_0 + \frac{p_0 P_{dl}}{\tau},\quad a_2 = \left(-\frac{(1+P_{dl})}{2\tau}W_0^2 + \frac{P_{dl}}{\tau}W_0\right)K,\quad a_3 = \frac{N}{\tau}(1-P_{ul}).$$

The operating point changes to $x^* = (0,0)$. Eq. (2) is a set of delay differential equations. Then the local stability of the original differential equation is studied by discussing its characteristic equation, and finally the properties of the whole system are obtained.

The characteristic equation of Eq. (2) is

$$\lambda^2 - b_1\lambda - b_2 e^{-\lambda\tau} = 0. \tag{3}$$

In which,

$$\begin{cases} b_1 = -\frac{1}{\tau}(1+P_{dl})W_0 p_0 + \frac{p_0 P_{dl}}{\tau} < 0, \\ b_2 = K\frac{N}{\tau}(1-P_{ul})\left(-\frac{(1+P_{dl})}{2\tau}W_0^2 + \frac{P_{dl}}{\tau}W_0\right) < 0. \end{cases} \tag{4}$$

Eq. (2) is transcendental equation. When $\tau = 0$, it can be written as

$$\lambda^2 - b_1\lambda - b_2 = 0.$$

Because b_1, b_2 are less than zero, therefore, according to the stability theory of delay differential equations, the system is locally asymptotically stable at the operating point.

Then, when $\tau \neq 0$, assuming that the characteristic Eq. (3) has a pair of pure virtual roots $\lambda = \pm i\omega_p$, ($\omega_p > 0$), and substituted into the characteristic equation, we can get

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$$\begin{cases} \omega_p^2 + b_2 \cos(\tau\omega_p) = 0, \\ -b_1\omega_p + b_2 \sin(\tau\omega_p) = 0. \end{cases} \tag{5}$$

Solve Eq. (5), and get

$$\omega_p^4 + b_1^2\omega_p^2 - b_2^2 = 0., \tag{6}$$

Here, $\omega_p = \sqrt{\frac{-b_1^2 + \sqrt{b_1^4 + 4b_2^2}}{2}}$. Accordingly, if the characteristic equation has a pure imaginary root, then $b_2^2 > 0$ in Eq. (6). It is obviously satisfied. Further get

$$\cos(\tau\omega_p) = \frac{-\omega_p^2}{b_2} > 0, \quad \sin(\tau\omega_p) = \frac{b_1\omega_p}{b_2} > 0.$$

Assuming $\theta(\tau) \in (0, \pi/2)$, then

$$\cos\theta(\tau) = \frac{-\omega_p^2}{b_2}, \quad \sin\theta(\tau) = \frac{b_1\omega_p}{b_2}.$$

Combined with the Eq (5), set

$$S_n(\tau) = \tau - \frac{\theta(\tau) + 2n\pi}{\omega_p}, \quad n \in N.$$

Lemma 1: If the characteristic equation (3) has a pair of conjugate pure imaginary roots $\lambda = \pm i\omega_p$, then it must be at τ_0, and τ_0 satisfies $S_0(\tau_0) = 0$.

According to Lemma 1, we can know $0 < \tau_0\omega_p < \pi/2$,

$$\tau_0 = \frac{1}{\omega_p} \arctan \frac{-b_1}{\omega_p}. \tag{7}$$

Next, further analysis is made according to the conditions of Hopf bifurcation generation in the time-delay system mentioned in [3]:

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(1) When $\tau=\tau_0$, $\lambda_{\tau_0}=\pm i\sqrt{\frac{-b_1^2+\sqrt{b_1^4+4b_2^2}}{2}}$ is a simple root of Eq. (3).

Proof.

Assuming

$$\Delta(\lambda,\tau)=\lambda^2-b_1\lambda-b_2e^{-\lambda\tau},$$

take the derivative with respect to λ. We can get

$$\frac{d\Delta(\lambda,\tau)}{d\lambda}=2\lambda-b_1+b_2\tau e^{-\lambda\tau}.$$

The value can be get under the condition of $(\lambda=i\omega_p,\tau=\tau_0)$.

$$\left.\frac{d\Delta(\lambda,\tau)}{d\lambda}\right|_{\substack{\lambda=i\omega_p\\ \tau=\tau_0}}=(-b_1-\tau_0\omega_p^2)+(2-b_1\tau_0)\omega_p i. \tag{8}$$

If Eq. (8) is zero, then $-b_1^2=2\omega_p^2$. Obviously this cannot be true, therefore $\left.\frac{d\Delta(\lambda,\tau)}{d\lambda}\right|_{\substack{\lambda=i\omega_p\\ \tau=\tau_0}}\neq 0$.

Similarly, $\left.\frac{d\Delta(\lambda,\tau)}{d\lambda}\right|_{\substack{\lambda=-i\omega_p\\ \tau=\tau_0}}\neq 0$ can be get.

The proof is completed.

(2)Assuming that Eq. (3) has a pair of conjugate complex eigenroots $\lambda_\tau=\alpha(\tau)\pm i\omega(\tau)$, and when $\tau=\tau_0$, $\begin{cases}\alpha(\tau_0)=0\\ \omega(\tau_0)=\omega_p\end{cases}$, then $\mathrm{Re}\left[\left.\frac{d\lambda(\tau)}{d\tau}\right|_{\tau=\tau_0}\right]>0$.

Proof.

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Definig $\lambda_{\tau_0} = i\omega_p$, solve $\dfrac{d\lambda(\tau)}{d\tau}$, then

$$\left.\frac{d\lambda(\tau)}{d\tau}\right|_{\tau=\tau_0} = \frac{i\omega_p b_2 e^{-i\omega_p\tau_0}}{b_1 - 2i\omega_p - b_2\tau_0 e^{-i\omega_p\tau_0}} = \frac{\omega_p b_2 \sin(\omega_p\tau_0) + i\omega_p b_2 \cos(\omega_p\tau_0)}{\left[b_1 - b_2\tau_0\cos(\omega_p\tau_0)\right] + i\left[b_2\tau_0\sin(\omega_p\tau_0) - 2\omega_p\right]},$$

$$\operatorname{Re}\left[\left.\frac{d\lambda(\tau)}{d\tau}\right|_{\substack{\tau=\tau_0\\ \lambda_{\tau_0}=i\omega_p}}\right] = \frac{\left[\omega_p b_2\sin(\omega_p\tau_0)\right]\left[b_1 - b_2\tau_0\cos(\omega_p\tau_0)\right]}{\left[b_1 - b_2\tau_0\cos(\omega_p\tau_0)\right]^2 + \left[b_2\tau_0\sin(\omega_p\tau_0) - 2\omega_p\right]^2}$$
$$+ \frac{\omega_p b_2\cos(\omega_p\tau_0)\left[b_2\tau_0\sin(\omega_p\tau_0) - 2\omega_p\right]}{\left[b_1 - b_2\tau_0\cos(\omega_p\tau_0)\right]^2 + \left[b_2\tau_0\sin(\omega_p\tau_0) - 2\omega_p\right]^2}$$
$$= \frac{b_1 b_2\omega_p\sin(\omega_p\tau_0) - 2\omega_p^2 b_2\cos(\omega_p\tau_0)}{\left[b_1 - b_2\tau_0\cos(\omega_p\tau_0)\right]^2 + \left[b_2\tau_0\sin(\omega_p\tau_0) - 2\omega_p\right]^2}.$$

Following, investigate molecular polynomials $b_1 b_2\omega_p\sin(\omega_p\tau_0) - 2\omega_p^2 b_2\cos(\omega_p\tau_0)$.

Assuming

$$b_1 b_2\omega_p\sin(\omega_p\tau_0) - 2\omega_p^2 b_2\cos(\omega_p\tau_0) \le 0,$$

then $\dfrac{b_1}{2\omega_p} \ge \dfrac{\cos(\omega_p\tau_0)}{\sin(\omega_p\tau_0)}$. According to the Eq. (5), $\dfrac{\cos(\omega_p\tau_0)}{\sin(\omega_p\tau_0)} = -\dfrac{\omega_p}{b_1}$, we can get

$$\frac{b_1}{2\omega_p} \ge -\frac{\omega_p}{b_1}. \tag{9}$$

Because of $b_1 < 0$, $\omega_p > 0$, obviously, Eq. (9) is not valid, so the hypothesis is not valid. Therefore,

$$\operatorname{Re}\left[\left.\frac{d\lambda(\tau)}{d\tau}\right|_{\substack{\tau=\tau_0\\ \lambda_{\tau_0}=i\omega_p}}\right] > 0.$$

Similarly, $\operatorname{Re}\left[\left.\dfrac{d\lambda(\tau)}{d\tau}\right|_{\substack{\tau=\tau_0\\ \lambda_{\tau_0}=-i\omega_p}}\right] > 0$ can be get.

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The proof is completed.

Lemma 2: Analyse the characteristic Eq. (3), and define $N(\tau)=\{\tau:\mathrm{Re}(\lambda(\tau))\geq 0\}$ which represents the number of non-negative real part characteristic roots, here τ denotes communicantion delay. If $\tau\in[\tau_1,\tau_2],(0<\tau_1<\tau_2)$, and the characteristic equation has no characteristic roots on the imaginary axis, then $N(\tau_1)=N(\tau_2)$.

(3) When $\tau<\tau_0$, the roots of the characteristic equation (3) all have negative real parts.

Proof.

When $\tau=0$, then $\Delta(\lambda,0)=\lambda^2-b_1\lambda-b_2$. Because b_1, b_2 are less than zero, therefore, all characteristic roots of $\Delta(\lambda,0)=0$ have negative real parts. According to **Lemma 1**, when $\tau<\tau_0$, the characteristic equations do not have roots on imaginary axes. According to **Lemma 2**, so when $\tau<\tau_0$, then $N(\tau)=N(0)=0$, the roots of he characteristic equations all have negative real parts.

The proof is completed.

The root distribution of the characteristic Eq. (3) is analyzed in detail. According to the conditions of Hopf bifurcation generation in [13], the following theorem can be obtained.

Theorem 1: With regard to the congestion control system via wireless access (Eq. (1)), when the communication delay $\tau<\tau_0$, the system is locally asymptotically stable near the operating point. When τ increases and passes through τ_0, Hopf bifurcation will be generated near the operating point.

Ⅲ. HOPF BIFURCATION CONTROLLER DESIGN

According to Theorem 1, when the communication delay exceeds the critical value, Hopf bifurcation will occur in the system. The bifurcation will cause the periodic movement of the system with large amplitude, which will lead to network congestion and even congestion collapse. In order to improve the stability of the system and facilitate its implementation in the actual network, this paper extends the work in the [12,13], adding the delay feedback controller to the source node of the wireless access network system to control the Hopf bifurcation, so as to ensure that the original operating point of the system is not changed and the Hopf bifurcation of the system is delayed. Finally, the stability range of the system is expanded. After adding the controller, the model of the controlled system has the following form.

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$$\begin{cases} \dot{W}(t) = \dfrac{1}{\tau} - (1-P_{dl})\dfrac{W(t)}{2}\dfrac{W(t)}{\tau}Kq(t-\tau) - P_{dl}(W(t)-1)\dfrac{W(t)}{\tau}Kq(t-\tau) \\ \qquad\quad + h[W(t)-W(t-\tau)], \\ \dot{q}(t) = -C + N(1-P_{ul})\dfrac{W(t)}{\tau} 。\end{cases} \tag{10}$$

Where, h Is the gain of the delay feedback controller, $h<0$ (that is negative feedback). Refer to assumptions about network parameters in Section 1, the operating point of the controlled system is obtained.

$$\begin{cases} W_* = \dfrac{\tau C}{N(1-P_{ul})}, \\ q_* = \dfrac{2}{K\left[(1+P_{dl})W_*^2 - 2P_{dl}W_*\right]} = \dfrac{2N^2(1-P_{ul})^2}{K\left[(1+P_{dl})\tau^2C^2 - 2P_{dl}\tau CN(1-P_{ul})\right]} = \dfrac{1}{K}p_*. \end{cases}$$

By comparison with Section 1, it can be seen that the addition of delay feedback controller does not change the operating point of the original system.

Setting $x_1(t) = W(t) - W_*$, $x_2(t) = q(t) - q_*$, It's linearized at the operating point. We obtain

$$\begin{cases} \dot{x}_1(t) = \left(h - \dfrac{1}{\tau}(1+P_{dl})W_*p_* + \dfrac{p_*P_{dl}}{\tau}\right)x_1(t) - hx_1(t-\tau) \\ \qquad\qquad + K\left(-\dfrac{(1+P_{dl})}{2\tau}W_*^2 + \dfrac{P_{dl}}{\tau}W_*\right)x_2(t-\tau), \\ \dot{x}_2(t) = \dfrac{N}{\tau}(1-P_{ul})x_1(t). \end{cases} \tag{11}$$

Further setting

$$\begin{cases} a_1 = -\dfrac{1}{\tau}(1+P_{dl})W_*p_* + \dfrac{p_*P_{dl}}{\tau}, \\ a_2 = K\left(-\dfrac{(1+P_{dl})}{2\tau}W_*^2 + \dfrac{P_{dl}}{\tau}W_*\right), \\ a_3 = \dfrac{N}{\tau}(1-P_{ul}). \end{cases} \tag{12}$$

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Then the Eq. (11) can be written

$$\begin{cases} \dot{x}_1(t) = (a_1 + h)x_1(t) - hx_1(t-\tau) + a_2 x_2(t-\tau), \\ \dot{x}_2(t) = a_3 x_1(t). \end{cases}$$

The characteristic equation of the controlled system can be obtained as:

$$\lambda^2 - (a_1 + h)\lambda + h\lambda e^{-\lambda\tau} - a_2 a_3 e^{-\lambda\tau} = 0. \tag{13}$$

Eq. (13) is a transcendental equation. Assuming that there is a pair of pure virtual roots $\lambda = \pm i\omega_{\tilde{p}}$,($\omega_{\tilde{p}} > 0$), and then substituted into the characteristic equation, we can get:

$$\begin{cases} \omega_{\tilde{p}}^2 - h\omega_{\tilde{p}} \sin(\tau_0^{'}\omega_{\tilde{p}}) + a_2 a_3 \cos(\tau_0^{'}\omega_{\tilde{p}}) = 0, \\ -(a_1 + h)\omega_{\tilde{p}} + h\omega_{\tilde{p}} \cos(\tau_0^{'}\omega_{\tilde{p}}) + a_2 a_3 \sin(\tau_0^{'}\omega_{\tilde{p}}) = 0. \end{cases}$$

Solve the above equation, which can be obtained

$$\omega_{\tilde{p}}^4 + (a_1^2 + 2a_1 h)\omega_{\tilde{p}}^2 - a_2^2 a_3^2 = 0. \tag{14}$$

Where, $\omega_{\tilde{p}} = \sqrt{\dfrac{-(a_1^2 + 2a_1 h) + \sqrt{(a_1^2 + 2a_1 h)^2 + 4a_2^2 a_3^2}}{2}}$. Therefore, if the characteristic equation has pure imaginary roots, then $4a_2^2 a_3^2 > 0$ in Eq. (14). It is obvious that the condition satisfies.

It can be further obtained that

$$\cos(\tau_0^{'}\omega_{\tilde{p}}) = \frac{\left[(a_1 + h)h - a_2 a_3\right]\omega_{\tilde{p}}^2}{a_2^2 a_3^2 + h^2 \omega_{\tilde{p}}^2} > 0, \quad \sin(\tau_0^{'}\omega_{\tilde{p}}) = \frac{h\omega_{\tilde{p}}^3 + (a_1 + h)\omega_{\tilde{p}} a_2 a_3}{a_2^2 a_3^2 + h^2 \omega_{\tilde{p}}^2} > 0.$$

According to **Lemma 1**, we can get that $0 < \tau_0^{'}\omega_{\tilde{p}} < \pi/2$,

$$\tau_0^{'} = \frac{1}{\omega_{\tilde{p}}} \arctan \frac{h\omega_{\tilde{p}}^2 + (a_1 + h)a_2 a_3}{\left[(a_1 + h)h - a_2 a_3\right]\omega_{\tilde{p}}}. \tag{15}$$

According to the Hopf bifurcation generation condition [3] and the derivation process in Section 1, and the following theorem is obtained. Theorem 2 proof is omitted here

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Theorem 2: With regard to the congestion control system equation (10) of the controlled wireless access network, when the communication delay $\tau < \tau_0^{'}$, the system is locally asymptotically stable near the operating point. When the communication delay increases, Hopf bifurcation is generated near the equilibrium point.

Note 2: According to Eq. (14), the critical value of bifurcation $\tau_0^{'}$ in system (Eq. 10) is controlled by the gain of the delay feedback controller (i.e. h), and the control of Hopf bifurcation in system (Eq. 10) can be realized by changing h.

Ⅳ. BIFURCATION DIRECTION AND PERIODIC SOLUTION STABILITY ANALYSIS

In Section 2, it is proved that the controlled system will produce Hopf bifurcation in the operating point when the communication delay τ pass through the bifurcation critical value $\tau_0^{'}$. In this section, we will refer to the method proposed in [14]. The directions of Hopf bifurcations and the stability of bifurcations periodic solutions are studied by using the normal form theory and the central manifold theorem, which is very important to reveal the influence law of communication delay on bifurcations of wireless access network TCP/AQM system.

Firstly, write the right-hand side of the Eq. (10) in the following form, thereinto q_τ, W_τ denote $q(t-\tau)$ and $W(t-\tau)$ respectively.

$$\begin{cases} f(W,W_\tau,q_\tau) = \dfrac{1}{\tau} - (1-P_{dl})\dfrac{W(t)}{2}\dfrac{W(t)}{\tau}Kq(t-\tau) - P_{dl}(W(t)-1)\dfrac{W(t)}{\tau}Kq(t-\tau) + h[W(t)-W(t-\tau)], \\ g(W) = -C + N(1-P_{ul})\dfrac{W(t)}{\tau}. \end{cases} \tag{16}$$

Then, set $y_1(t)=W(t)-W_*$, $y_2(t)=q(t)-q_*$. The Taylor series expansion of Eq. (16) near the operating point can be obtained as follows:

$$\begin{cases} \dot{y}_1(t) = c_1 y_1(t) + c_2 y_1(t-\tau) + c_3 y_2(t-\tau) + c_4 y_1^2(t) + c_5 y_1(t) y_2(t-\tau) + c_6 y_1^2(t) y_2(t-\tau), \\ \dot{y}_2(t) = c_7 y_1(t). \end{cases} \tag{17}$$

Where,

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$$c_1 = \left.\frac{\partial f}{\partial W}\right|_{(W_0,q_0)} = h - \frac{1}{\tau}(1+P_{dl})W_0 p_0 + \frac{p_0 P_{dl}}{\tau}, \quad c_2 = \left.\frac{\partial f}{\partial W_\tau}\right|_{(W_0,q_0)} = -h,$$

$$c_3 = \left.\frac{\partial f}{\partial q_\tau}\right|_{(W_0,q_0)} = K\left(-\frac{(1+P_{dl})}{2\tau}W_0^2 + \frac{P_{dl}}{\tau}W_0\right),$$

$$c_4 = \frac{1}{2}\left.\frac{\partial^2 f}{\partial W^2}\right|_{(W_0,q_0)} = -\frac{(1+P_{dl})}{\tau\left[(1+P_{dl})W_0^2 - 2P_{dl}W_0\right]},$$

$$c_5 = \left.\frac{\partial^2 f}{\partial W \partial q_\tau}\right|_{(W_0,q_0)} = -\frac{K}{\tau}\left[(1+P_{dl})W(t) - P_{dl}\right],$$

$$c_6 = \frac{1}{3!}\left.\left(\frac{\partial^3 f}{\partial^2 W \partial q_\tau} + \frac{\partial^3 f}{\partial W \partial q_\tau \partial W}\right)\right|_{(W_0,q_0)} = -\frac{1}{3}\frac{K(1+P_{dl})}{\tau},$$

$$c_7 = \left.\frac{dg}{dW}\right|_{(W_0,q_0)} = \frac{N}{\tau}(1-P_{ul}).$$

To facilitate the study of the dynamic behavior of τ in the neighborhood of $\tau_0^{'}$, set $\tau = \tau_0^{'} + \mu$, $\mu \in R$, then when $\mu = 0$, Bifurcation occurs in the controlled system (Eq. 10). Setting $u_t(\theta) = u(t+\theta)$, $\theta \in [-\tau,\ 0]$, $u(t) = [y_1(t)\ \ y_2(t)]^T$, define space $C = C([-\tau,0], R^2)$, $\varphi(\theta) \in C[-\tau,0]$ is the initial condition. $F: R^2 \times C \to R^2$, Eq. (14) can be transformed into a functional differential equation:

$$\dot{u}(t) = L_\mu(u_t) + F(\mu, u_t). \tag{18}$$

According to [14], the property of Bifurcating periodic solution $u(t,\mu(\varepsilon))$ in Hopf bifurcating system (ε is a small parameter greater than zero) is related to $\mu(\varepsilon)$ (the position relationship between bifurcation points), bifurcation period $T(\varepsilon)$ and non-zero Floquet index $\beta(\varepsilon)$, and the above variables can be expressed as:

$$\begin{aligned} &\mu(\varepsilon) = \mu_2\varepsilon^2 + \mu_4\varepsilon^4 + \cdots, \\ &T(\varepsilon) = \frac{2\pi}{\omega_{\tilde{p}}}(1 + T_2\varepsilon^2 + T_4\varepsilon^4 + \cdots), \\ &\beta(\varepsilon) = \beta_2\varepsilon^2 + \beta_4\varepsilon^4 + \cdots. \end{aligned} \tag{19}$$

The following theorems can be obtained for the controlled system (Eq.(10)).

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Theorem 3: Hopf bifurcation characteristics of the controlled system (Eq.(10)) are determined by the parameters in Eq.(19), Where, μ_2 determines the direction of the bifurcation, if $\mu_2 > 0(<0)$, then the Hopf bifurcation is supercritical (subcritical), and when $\tau > \tau_0^{'}$, there is a periodic solution (when $\tau < \tau_0^{'}$, there is a periodic solution). β_2 determines the stability of bifurcation periodic solutions. If $\beta_2 > 0(<0)$, then the bifurcation periodic solution is unstable (the bifurcation periodic solution is stable). T_2 determines the period of the bifurcation periodic solution. If $T_2 > 0(<0)$, the period is increasing (period is decreasing). The values of the parameters μ_2, β_2, T_2 can be obtained by the following formula ($C_1(0)$ is the Lyapunov coefficient):

$$\begin{cases} C_1(0) = \dfrac{i}{2\omega_{\tilde{p}}}(g_{20}g_{11} - 2|g_{11}|^2 - \dfrac{|g_{02}|^2}{3}) + \dfrac{g_{21}}{2}, \\ \mu_2 = -\dfrac{\operatorname{Re}\{C_1(0)\}}{\operatorname{Re}\lambda^{'}(0)}, \\ T_2 = -\dfrac{\operatorname{Im}\{C_1(0)\} + \mu_2 \operatorname{Im}\lambda^{'}(0)}{\omega_{\tilde{p}}}, \\ \beta_2 = 2\operatorname{Re}\{C_1(0)\}. \end{cases}$$

After the above parameters are calculated, the bifurcation properties of the controlled system (Eq.(10)) when Hopf bifurcation occurs can be determined according to Lemma 3.

Ⅴ. SIMULATION EXPERIMENT

In this section, simulation experiments are used to verify the correctness of the above analysis and the effectiveness of the proposed algorithm. The number of TCP connections $N = 80$, the ratio of packet dropping/marking probability to queue length $K = 0.001$, bottleneck link capacity $C = 1500$ packets/s, and the probability of upstream and downstream marking $P_{dl} = P_{ul} = 0.1$ are selected. The simulation experiment will be divided into two parts: uncontrolled system and controlled system.

(1) Investigate the uncontrolled system

By calculating Eq. (4) and Eq. (7), it can be obtained that $\tau_0 = f(\tau)$ that is a subtraction function of τ, and the intersection point $\tau_c = 0.1864$ between $\tau_0 = f(\tau)$ and $\tau_0 = \tau$. As shown in Fig.2, when the communication delay $0 \le \tau < \tau_c$, then the communication delay τ of the uncontrolled

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system is less than the critical value τ_0 of Hopf bifurcation. On the contrary, when $\tau > \tau_c$, the delay τ is greater than the critical value τ_0 of Hopf bifurcation.

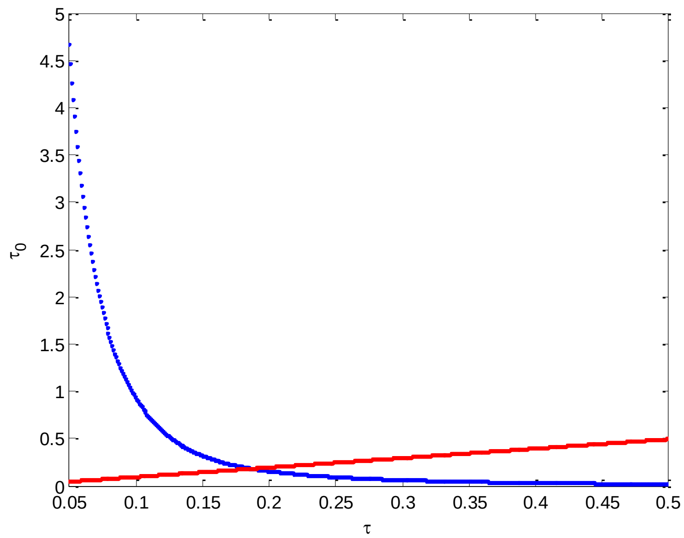

Fig.2 The relation between τ and τ_0

① Setting $\tau = 0.16$, we can acquire $W_0 = 3.3333$, $p_0 = 0.1731$, $q_0 = 173.1$, $a_1 = -3.8582$, $a_2 = -0.0361$, $a_3 = 450$, $\omega_p = 3.2296$. The bifurcation critical value of communication delay in uncontrolled system $\tau_0 = 0.2706$, so it can thus be seen, $\tau < \tau_0$. As shown in Fig. 3, the uncontrolled system is asymptotically stable at the operating point.

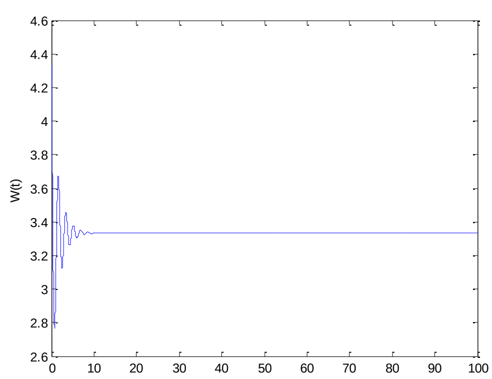

Fig.3(a) The waveform of w(t) at $\tau = 0.16$

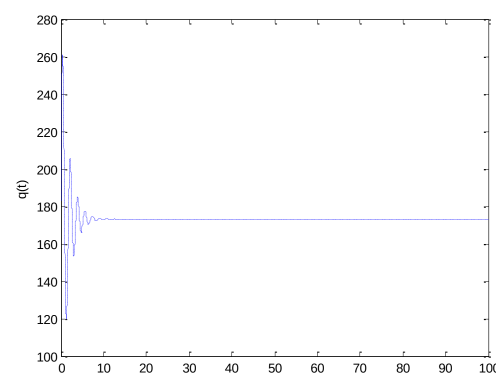

Fig.3(b) The waveform of q(t) at $\tau = 0.16$

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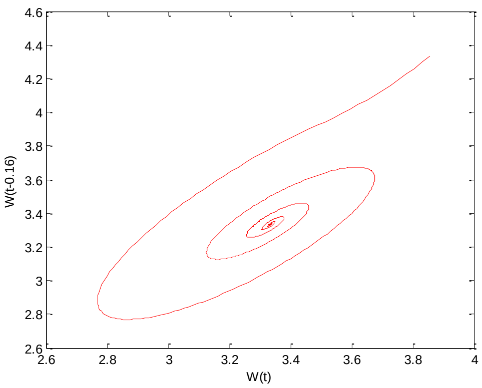

Fig.3(c) The phase diagram of W(t) at $\tau = 0.16$

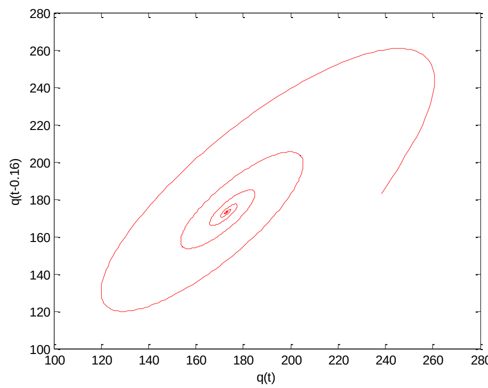

Fig.3(d) The phase diagram of q(t) at $\tau = 0.16$

② Setting $\tau = 0.19$, we can acquire $W_0 = 3.9583$, $p_0 = 0.1216$, $q_0 = 121.6$, $a_1 = -2.7233$, $a_2 = -0.0433$, $a_3 = 378.9474$, $\omega_p = 3.6199$. The bifurcation critical value of communication delay in uncontrolled system $\tau_0 = 0.1782$, so it can thus be seen, $\tau < \tau_0$. As shown in Fig. 4, when $\tau = 0.19$, the uncontrolled systems are unstable at the operating point and Hopf bifurcation occurs.

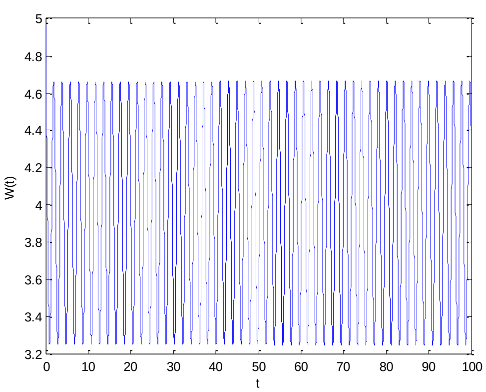

Fig.4(a) The waveform of w(t) at $\tau = 0.19$

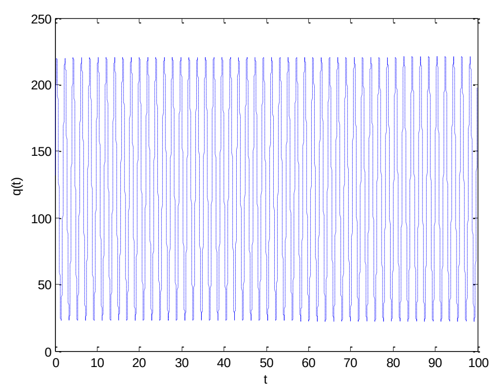

Fig.4(b) The waveform of q(t) at $\tau = 0.19$

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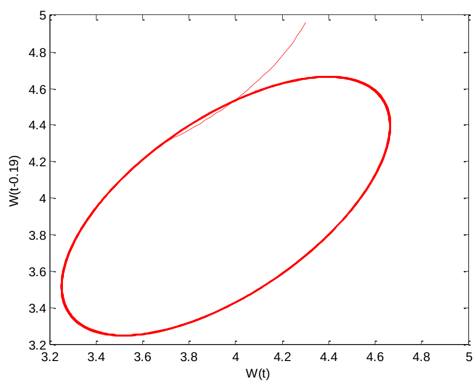

Fig.4(c) The phase diagram of W(t) at $\tau = 0.19$

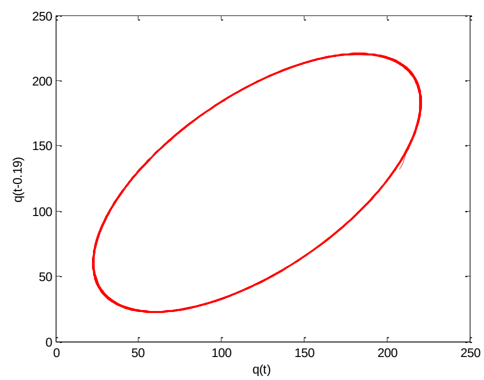

Fig.4(d) The phase diagram of q(t) at $\tau = 0.19$

(1) Investigate the controlled system

Above, Hopf bifurcation occurred when $\tau = 0.19$ in the uncontrolled system. Now, a delay feedback controller is added to the original uncontrolled system to study the dynamic characteristics change of the controlled system at this point. In the controlled system, the relevant parameters are set as in Simulation 1, and the delay feedback gain $h = -10$ is obtained.

① Invesigate $\tau_0' = f(\tau)$

By calculating Eq. (12), Eq. (14) and Eq. (15), $\tau_0' = f(\tau)$ can be obtained which is a subtraction function of τ. The intersection point $\tau_c = 0.347$ between $\tau_0' = f(\tau)$ and $\tau_0' = \tau$, as shown in Fig. 5. When $0 \le \tau < \tau_c$, the delay τ is less than the critical value τ_0' of Hopf bifurcation. On the contrary, when $\tau > \tau_c$, the delay τ is greater than the critical value τ_0' of Hopf bifurcation.

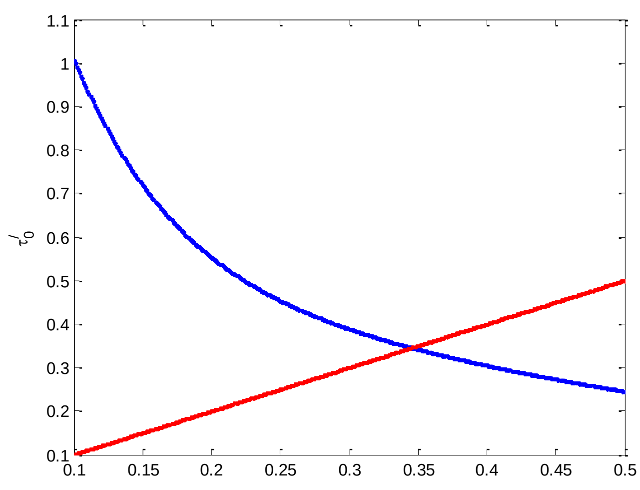

Fig.5 The relation between τ and τ_0'

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② Setting $h = -10$, $\tau = 0.19$, we can acquire that $W_* = 3.9583$, $p_* = 0.1216$, $q_* = 121.6$, $a_1 = -2.7233$, $a_2 = -0.0433$, $a_3 = 378.9474$, $\omega_{\tilde{p}} = 2.0191$, $\tau_0^{'} = 0.5795$. Obviously, $\tau < \tau_0'$. Thus, after the introduction of the controller, the bifurcation critical value of communication delay increases without changing the original operating point of the system. As is shown in Fig.6, Fig. 6(a), (b) are the waveform diagrams of $W(t)$, $q(t)$ respectively, and Fig. 6(c), (d) are the phase diagrams of $W(t)$, $q(t)$ respectively. By comparing Fig. 5 and Fig. 6, it can be seen that the bifurcation is delayed and the system turns from unstable to stable after introducing the delay feedback control which expands the stable region of the system.

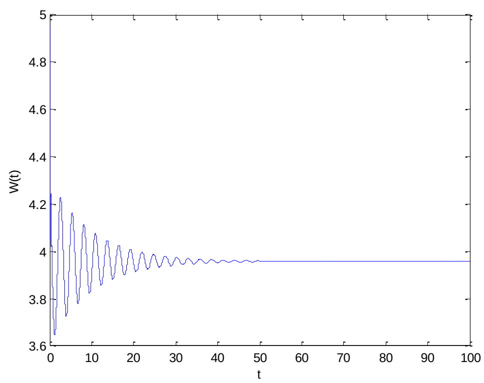

Fig.6(a) The waveform of w(t) at $h = -10$, $\tau = 0.19$

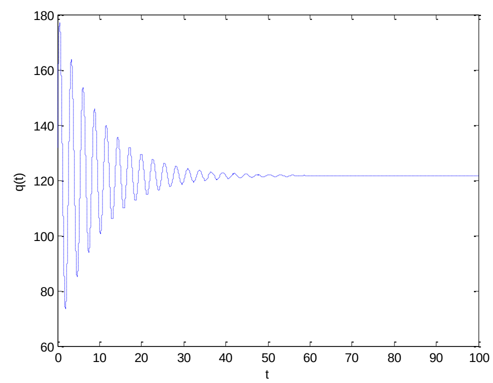

Fig.6(b) The waveform of q(t) at $h = -10$, $\tau = 0.19$

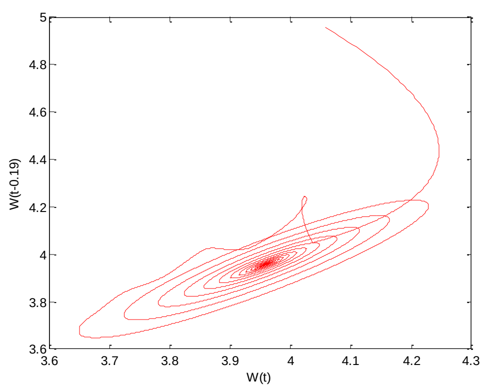

Fig.6(c) The phase diagram of W(t) at $h = -10$, $\tau = 0.19$

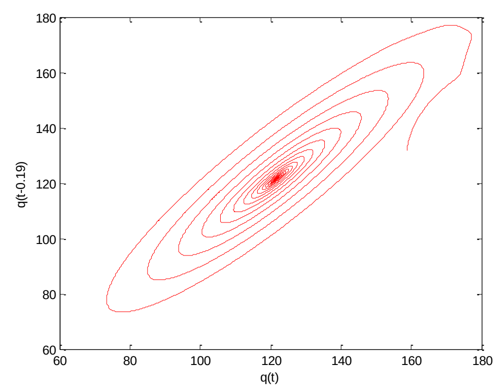

Fig.6(d) The phase diagram of q(t) at $h = -10$, $\tau = 0.19$

③ Setting $h = -10$, $\tau = 0.35$, we can acquire that $W_* = 7.2917$, $p_* = 0.0623$, $q_* = 62.3$, $a_1 = -0.7937$, $a_2 = -0.0815$, $a_3 = 205.7143$, $\omega_{\tilde{p}} = 3.2293$, $\tau_0^{'} = 0.5795$. Obviously, $\tau > \tau_0'$. Hopf bifurcation occurs in the controlled system near the operating point. Further analysing the bifurcation properties, $\mu_2 = 0.04325$, $\beta_2 = -0.07952$, $T_2 = 11.05962$ can be obtained. According to Theorem 3, it

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can be deduced that Hopf bifurcation is supercritical and bifurcation periodic solution is stable, and the period of bifurcation periodic solution is increasing. The above analysis is verified by the waveform diagrams of $W(t)$, $q(t)$ and the phase diagram of $W(t)$, $q(t)$ shown in Fig. 7.

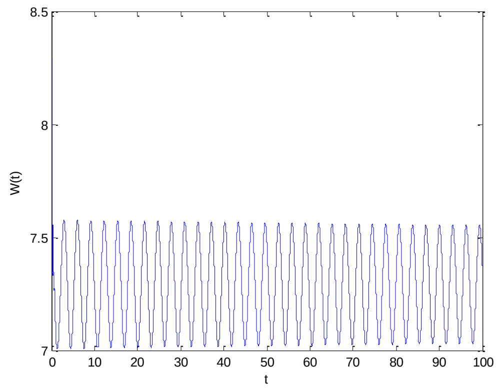

Fig.7(a) The waveform of w(t) at $h=-10$, $\tau=0.35$

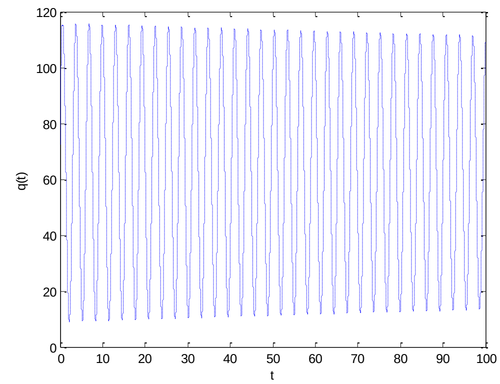

Fig.7(b) The waveform of q(t) at $h=-10$, $\tau=0.35$

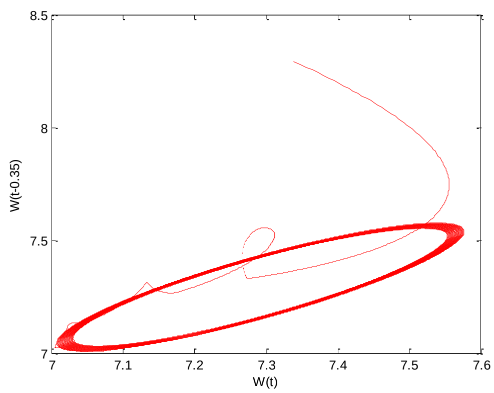

Fig.7(c) The phase diagram of W(t) at $h=-10$, $\tau=0.35$

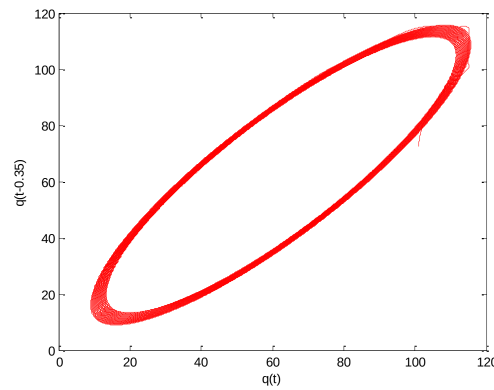

Fig.7(d) The phase diagram of q(t) at $h=-10$, $\tau=0.35$

Ⅵ. CONCLUSION

In this paper, the dynamic analysis and control problem of congestion control system based on wireless access network are studied. The communication delay τ is selected as the bifurcation parameter. By discussing the distribution of characteristic roots of uncontrolled system, it is proved that Hopf bifurcation will occur when τ increases the crossing bifurcation critical value τ_0. Then, in order to delay the occurrence of Hopf bifurcation and expand the stability range of the system without changing the original operating point of the system, delay feedback control strategy is proposed to construct the controlled system, and the critical value τ_0' of Hopf bifurcation is obtained in the controlled system. The formula for judging the direction of Hopf bifurcation and stability of bifurcation periodic solution of the controlled system is derived. Finally, the correctness of the analysis and the effectiveness of the proposed control strategy are verified by simulation.

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