Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

Application of Lotka-Volterra Model in Enterprise Competition

Ming Cao
School of Mathematics and Statistics, Shaanxi Xueqian Normal University, Xi'an, China

Abstract:

In recent years, differential equations have been widely used in many fields, such as ecological environment, engineering, medicine, economy and finance. Considering the similarity between the relationship between enterprises and the population relationship in biology, according to the competition among enterprises, the Lotka-Volterra model in differential equations is used to introduce the promotion coefficient between two enterprises for mathematical modeling. Through dynamic analysis of the model and corresponding economic explanation, how to adjust the relationship between enterprises to achieve better development is studied.

Keywords*: Lotka-Volterra, Differential equation, Enterprise competition, Stability*

I. MODEL DESCRIPTION

The process of competition among many enterprises is similar to that among biological populations. Therefore, we can establish the enterprise competition model according to the population competition model and analyze the competition relationship among enterprises.

Assuming that there are only two enterprises, A and B, which produce the same product, in a market with limited resources and environment, there will be a competitive relationship between them in the market.

Assuming that the production levels of enterprises A and B are $x(t)$ and $y(t)$, the intrinsic growth rates of output of enterprises A and B are r_1 and r_2, and their maximum outputs under the relative environment are K_1 and K_2, we can establish the output growth model of the two enterprises according to the Logistics equation [1]:

A: $\frac{dx}{dt} = r_1 x\left(1 - \frac{x}{K_1}\right)$

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

B: $\frac{dy}{dt} = r_2 y\left(1 - \frac{y}{K_2}\right)$

Considering the competitive relationship between the two enterprises, let A_{12} be the competitive coefficient of enterprise A to enterprise B, $\frac{1}{K_1}$ be the restraining effect of resources consumed by each product in enterprise A on its own output [2], and $\frac{A_{12}}{K_2}$ be the restraining effect of resources consumed by each product in enterprise B on the output of enterprise A, the competitive equation of enterprise A can be obtained:

$$\frac{dx}{dt} = r_1 x\left(1 - \frac{x}{K_1} - \frac{A_{12} y}{K_2}\right) \tag{1}$$

Let A_{21} be the competitive coefficient of enterprise B to enterprise A, $\frac{1}{K_2}$ be the restraining effect of resources consumed by each product in enterprise B on its own output, and $\frac{A_{21}}{K_1}$ be the restraining effect of resources consumed by each product in enterprise A on the output of enterprise B, the competitive equation of enterprise B can be obtained [3]:

$$\frac{dy}{dt} = r_2 y\left(1 - \frac{y}{K_2} - \frac{A_{21} x}{K_1}\right) \tag{2}$$

Juggling equations (1) and (2), and the competitive model of the two enterprises can be obtained:

$$\begin{cases} \frac{dx}{dt} = r_1 x\left(1 - \frac{x}{K_1} - \frac{A_{12} y}{K_2}\right) \\ \frac{dy}{dt} = r_2 y\left(1 - \frac{y}{K_2} - \frac{A_{21} x}{K_1}\right) \end{cases} \tag{3}$$

As we know, when two enterprises compete with each other, they also play a certain role in promoting each other. With the market pressure brought by the competitor, they will promote their own innovation and development, and accelerate the production efficiency of enterprise products. Therefore, we introduce the promotion coefficients B_{12} and B_{12} between two

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

enterprises, then $\frac{B_{12}}{K_2}$ is the promotion of resources consumed by each product in enterprise B to the output of enterprise A, and $\frac{B_{21}}{K_1}$ is the promotion of resources consumed by each product in enterprise A to the output of enterprise B, and an improved competition model is obtained [4]:

$$\begin{cases} \frac{dx}{dt} = r_1 x\left(1-\frac{x}{K_1}-\frac{A_{12}y}{K_2}+\frac{B_{12}y}{K_2}\right) \\ \frac{dy}{dt} = r_2 y\left(1-\frac{y}{K_2}-\frac{A_{21}x}{K_1}+\frac{B_{21}x}{K_1}\right) \end{cases} \tag{4}$$

II. STABILITY ANALYSIS

When two enterprises reach a balance in the process of competition, that is, when the output of the two enterprises does not change any more, $\frac{dx}{dt}=0, \frac{dy}{dt}=0$ [5]. According to the adaptive characteristics of Lotka-Volterra system, the equilibrium steady-state solution can be obtained as $P_1(0,0), P_2(K_1,0), P_3(0,K_2), P_4\left(\frac{K_1-(A_{12}-B_{12})K_2}{1-(A_{12}-B_{12})(A_{21}-B_{21})}, \frac{K_2-(A_{21}-B_{21})K_1}{1-(A_{12}-B_{12})(A_{21}-B_{21})}\right)$.

According to this stable solution, we can get the growth equilibrium line [6] of the two enterprises: $\begin{cases} x+(A_{12}-B_{12})y=K_1 \\ y+(A_{21}-B_{21})x=K_2 \end{cases}$.

(1) When $A_{12}-B_{12}>\frac{K_1}{K_2}, A_{21}-B_{21}<\frac{K_2}{K_1}$, namely $\frac{K_1}{A_{12}-B_{12}}<K_2, \frac{K_2}{A_{21}-B_{21}}>K_1$, the competitive relationship between the two enterprises is shown in Figure 1.

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

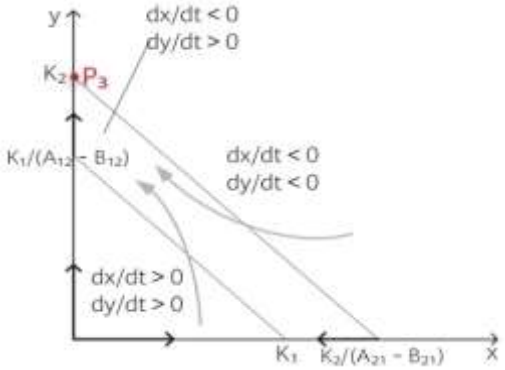

Fig 1: The relationship between A and B in Case One

Therefore, the equilibrium line of enterprise A is always located below that of enterprise B. In $\frac{dx}{dt} > 0, \frac{dy}{dt} > 0$ region, the production scale increases with time, while in $\frac{dx}{dt} < 0, \frac{dy}{dt} < 0$ region, the production scale decreases with time. Finally it is stable at $P_3(0, K_2)$ point. At this time, enterprise A will be extinct in the competition, and enterprise B will survive all the time.

(2) When $A_{12} - B_{12} < \frac{K_1}{K_2}, A_{21} - B_{21} > \frac{K_2}{K_1}$, namely $\frac{K_1}{A_{12} - B_{12}} > K_2, \frac{K_2}{A_{21} - B_{21}} < K_1$, the competitive relationship between the two enterprises is shown in Figure 2.

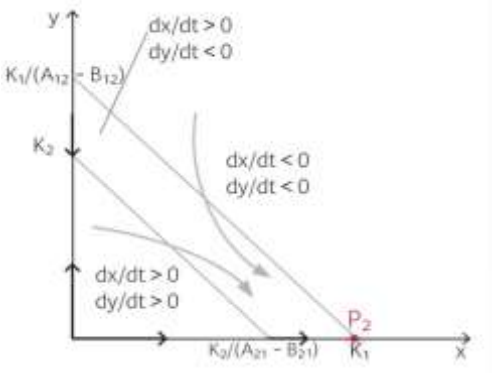

Fig 2: The relationship between A and B in Case Two

It can be seen that the equilibrium line of enterprise A is always above that of enterprise B. In $\frac{dx}{dt} > 0, \frac{dy}{dt} > 0$ region, the scale of production decreases with time, while in $\frac{dx}{dt} < 0, \frac{dy}{dt} < 0$ region, the scale of production increases with time. Finally it is stable at point $P_2(K_1, 0)$. At this

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

time, enterprise A will survive in the competition and enterprise B will be extinct.

(3) When $A_{12}-B_{12}>\frac{K_1}{K_2}, A_{21}-B_{21}>\frac{K_2}{K_1}$, namely $\frac{K_1}{A_{12}-B_{12}}<K_2, \frac{K_2}{A_{21}-B_{21}}<K_1$, the competitive relationship between the two enterprises is shown in Figure 3.

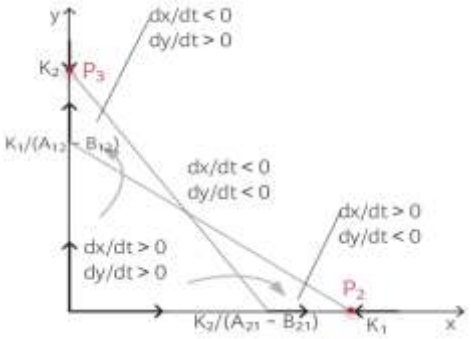

Fig 3: The relationship between A and B in Case Three

Therefore, at this time, the competition between the two enterprises is stable at point $P_2(K_1,0)$ and point $P_3(0,K_2)$, and the two enterprises will coexist in the competition.

(4) When $A_{12}-B_{12}<\frac{K_1}{K_2}, A_{21}-B_{21}<\frac{K_2}{K_1}$, namely $\frac{K_1}{A_{12}-B_{12}}>K_2, \frac{K_2}{A_{21}-B_{21}}>K_1$, the competitive relationship between the two enterprises is shown in Figure 4.

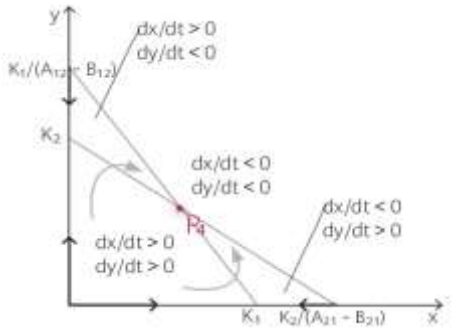

Fig 4: The relationship between A and B in Case Four

Therefore, at this time, the competition between the two enterprises is stable at point

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

$P_4\left(\frac{K_1-(A_{12}-B_{12})K_2}{1-(A_{12}-B_{12})(A_{21}-B_{21})},\frac{K_2-(A_{21}-B_{21})K_1}{1-(A_{12}-B_{12})(A_{21}-B_{21})}\right)$, and the two enterprises will coexist in the competition. But the competitiveness of the two enterprises is different.

III. PARAMETER ANALYSIS

When $\frac{K_1}{A_{12}-B_{12}}<K_2,\frac{K_2}{A_{21}-B_{21}}>K_1$ or $\frac{K_1}{A_{12}-B_{12}}>K_2,\frac{K_2}{A_{21}-B_{21}}<K_1$, that is to say, when the cost of enterprise A is higher than that of enterprise B or enterprise B is higher than that of enterprise A, either enterprise A loses to enterprise B's resource utilization rate and is extinct in the competition, or enterprise B loses to enterprise A's resource utilization rate and is extinct in the competition. In short, one of the two enterprises will be extinct in the process of competition. At this time, the disadvantaged party should rationally adjust its management mode, actively innovate products, improve resource utilization rate and strive to survive.

When $\frac{K_1}{A_{12}-B_{12}}<K_2,\frac{K_2}{A_{21}-B_{21}}<K_1$ or $\frac{K_1}{A_{12}-B_{12}}>K_2,\frac{K_2}{A_{21}-B_{21}}>K_1$, that is, the resource utilization rates of enterprise A and enterprise B being low or high, the two enterprises are in a state of coexistence in the competition. Although they can coexist in the competition, they should constantly learn and make progress, restrain each other's growth through various competition methods, gain more market share for themselves, increase their competitive advantage, and compete reasonably in the competition, so as to avoid excessive competition between the two enterprises, which will lead to a decline in the productivity of both parties.

IV. INSTANCE ANALYSIS

To verify the feasibility of the model and related theories established above, we introduce specific examples for analysis. We take the book audio-visual market of e-commerce model as an example. E-commerce dominated by online retail is called B2C. In the online retail market of Chinese publications B2C, Dangdang and Amazon China are two competing e-commerce platform companies. This paper selects these two companies to analyze the market share of B2C online retail of some Chinese publications from 2009 to 2014 [7]. See Table I for specific data.

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

TABLE I. B2C online retail market share of Chinese publications

YEAR	DANGDANG	AMAZON CHINA
200901	52.01%	35.27%
200902	50.02%	34.81%
200903	51.69%	34.29%
200904	53.00%	33.00%
201001	54.81%	36.42%
201002	52.12%	37.85%
201003	51.00%	36.86%
201004	49.27%	35.33%
201101	45.55%	33.87%
201102	41.00%	32.28%
201103	37.32%	30.73%
201104	32.88%	28.30%
201201	29.39%	27.50%
201202	29.89%	26.42%
201203	31.33%	27.32%
201204	33.70%	25.93%
201301	34.36%	24.43%
201302	37.39%	23.40%
201303	39.09%	23.70%
201304	42.92%	24.11%
201401	43.20%	23.41%
201402	40.39%	25.90%
201403	43.21%	25.51%
201404	43.51%	23.79%

In the B2C online retail market of Chinese publications with limited resources and environment, Dangdang and Amazon China almost monopolize the market, and the product types and target customers of the two companies are roughly the same, so there is a competitive relationship between them in the market. We can establish a Lotka-Volterra competition model to describe their relationship, namely

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

$$\begin{cases} \dfrac{dx}{dt} = r_1 x\left(1 - \dfrac{x}{K_1} - \dfrac{A_{12}y}{K_2} + \dfrac{B_{12}y}{K_2}\right) \\ \dfrac{dy}{dt} = r_2 y\left(1 - \dfrac{y}{K_2} - \dfrac{A_{21}x}{K_1} + \dfrac{B_{21}x}{K_1}\right) \end{cases} \tag{5}$$

Wherein, Dangdang and Amazon China have sales levels of $x(t)$ and $y(t)$, intrinsic growth rates of sales of r_1 and r_2, maximum sales of K_1 and K_2 in relative environment, competition coefficients of A_{12} and A_{12}, and promotion coefficients of B_{12} and B_{21}. We may rewrite them into general form:

$$\begin{cases} \dfrac{dx}{dt} = r_1 x\left(1 - \dfrac{x}{K_1} - \dfrac{A_{12}y}{K_2} + \dfrac{B_{12}y}{K_2}\right) \\ \dfrac{dy}{dt} = r_2 y\left(1 - \dfrac{y}{K_2} - \dfrac{A_{21}x}{K_1} + \dfrac{B_{21}x}{K_1}\right) \end{cases} \tag{6}$$

It can be learned that $A_{12} - B_{12} = \dfrac{c_1 K_2}{b_1 K_1}$, $A_{21} - B_{21} = \dfrac{c_2 K_1}{b_2 K_2}$.

Aussming that $K_1 = K_2 = 0.01$, the collected data are empirically analyzed and calculated by Eviews software, and the data [8] is obtained after iteration by Marquardt judgment rule, as shown in Table II:

TABLE II. The Parameter of Dangdang and Amazon China

	PARAMETER		
	a	b	c
DANGDANG	0.053405	-0.003805	0.012918
AMAZON CHINA	-0.010151	0.000269	0.000276

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

As can be seen from the table

$$A_{12}-B_{12}=\frac{0.012918}{-0.003805}\times\frac{0.01}{0.01}<\frac{K_1}{K_2}=1,\ A_{21}-B_{21}=\frac{0.000276}{0.000269}\times\frac{0.01}{0.01}>\frac{K_2}{K_1}=1.$$

According to the above analysis, we know that Dangdang will always survive in the competition. From the collected data, we can also see that Dangdang has an advantage over Amazon China in market share, and its development ability is stronger than Amazon China.

Similarly, we select Tmall platform and JD.COM platform to analyze the share of B2C online retail market from 2011 to 2018, and get data Table III:

TABLE III. The Parameter of Tmall and TD.COM

	PARAMETER		
	a	b	c
TMALL	1.382733	0.82983	-0.71938
JD.COM	1.087363	1.39259	-0.65491

Similarly, $K_1=K_2=0.01$. It can be obtained from the table

$$A_{12}-B_{12}=\frac{-0.71938}{0.82983}\times\frac{0.01}{0.01}<\frac{K_1}{K_2}=1,\ A_{21}-B_{21}=\frac{-0.65491}{1.39259}\times\frac{0.01}{0.01}<\frac{K_2}{K_1}=1.$$ This is the fourth case in the above analysis. Tmall and JD.COM will coexist in the process of competition. Seen from the current market, we also find that Tmall and JD.COM are neck and neck in the competition.

If the two enterprises are competing with each other, the weaker party should adjust its management mode according to the situation, actively innovate products, improve the utilization rate of resources, and constantly improve its competitive advantage to avoid being eliminated by the market. In the process of competition, if both parties are equal, they should constantly learn and make progress, gain more market share for themselves through various methods, and pay attention to avoiding the loss of interests caused by excessive competition between them.

Forest Chemicals Revew
www.forestchemicalsreview.com
ISSN: 1520-0191
July-August 2021 Page No. 1568-1577
Article History: Received: 12 May 2021 Revised: 25 June 2021 Accepted: 22 July 2021 Publication: 31 August 2021

ACKNOWLEDGEMENTS

This research was supported by Shaanxi Science and Technology Plan Project (2020JQ-914).

REFERENCES

[1] Li J, Jin ZM (2006) Industrial cluster development based on ecological theory. Scientist, 2006:431-437
[2] He JS, Dai WM (2005) Ecological model and ecological balance analysis of industrial clusters. Journal of Beijing Normal University (Social Science Edition) (01): 126-132
[3] Zhu YZ, Zong G (2014) An empirical study on the relationship between competition and cooperation for enterprise development-from the perspective of biological population theory. Journal of Anhui Academy of Governance 5(06): 80-88
[4] Ye QW, Qiang X, Song GX (2014) Research and explanation of LV competition model between e-commerce platforms. Commercial Research (06): 37-41
[5] Cui LY (2020) Research on the existing B2C platform competition based on L-V model. Guangxi Quality Supervision Guide (03): 89-90
[6] Liu RH, Han XS (2008) Competitive ecosystem of construction enterprises based on predation model. Journal of Harbin Institute of Technology 40(12): 2016-2019
[7] Shang XF, Chen HM (2009) A summary of the research on competition strategy and regulation of enterprises with bilateral market characteristics. Industrial Economics Research 4: 89-93
[8] Zhao YH (2012) Research on the model of dependence cooperation and reciprocity based on Lotka-Volterra equation. Technological Development of Enterprise 31(23): 53-54