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A Collaborative Filtering Recommendation Algorithm Based on Probabilistic Matrix Factorization

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Abstract:-The information of users and items is difficult to obtain due to expensive expert labeling costs and privacy issues. So the sequential behavior relationship of consumption is introduced to solve the problem of collaborative filtering recommendation. The sequential behavior relationship is extended to the item consumption network, and the calculation method of the asymmetric item-item similarity is defined. The item-item matrix is constructed via probabilistic matrix factorization to explore the potential neighborhood information of items. The neighbor information is integrated into the user-item rating matrix, and the user-item rating matrix is reorganized by matrix factorization to predict the user ratings for items. A collaborative filtering recommendation framework model based on user and item neighbors is proposed. Experiments on real data sets show that the collaborative filtering method based on two-level probability matrix factorization integrating sequential behavior can improve the accuracy of the user's rating prediction and the performance of recommendation.

Keywords: *probabilistic matrix factorization, collaborative filtering, sequential behavior, asymmetric item similarity, consumption network.*

Ⅰ. INTRODUCTION

The rapid expansion of WWW network has brought great convenience to people, but also caused serious efficiency problems. Users have to spend a lot of time to filter their desired information from massive network data. Recommendation systems play a major role in information overload and are widely used by many online services, including e-commerce, online news, etc[1]. In the recommendation system, it is very important to predict the score of new items. Collaborative filtering is the most popular method due to its relatively good predictive performance[2]. Its main idea is to recommend information that users are interested in by using preferences of like-minded groups with same experience[3]. Therefore, the neighbor selection of active users is the key in collaborative filtering recommendation. However, traditional collaborative filtering ignores the influence of sequential behavior relationship on similar users. In fact there is potential influence between users or between items. If user A consumes an item, user B also consumes the item within a certain time interval. If this happens many times, we have reason to believe that user A has a great influence on user B, and vice versa. Similarly, if item X is consumed by a certain user for several times

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and item Y is also consumed by that user within a certain interval, it is considered that item X has a great influence on item Y; otherwise, it is not. It can be seen that the influence between items or between users is not symmetrical. This paper establishes user consumption network and item consumption network based on consumer sequential behavior, defines item similarity, uses probability matrix factorization to excavate the implicit impact between potential users and between items, determines the neighbors of users and items, incorporates the neighbor information to probability matrix factorization of the score matrix, reconstructs the score matrix, and predicts user ratings for the item. The collaborative filtering method combining two-level probability matrix factorization with sequential behavior relation can improve the accuracy of recommendation effectively.

Ⅱ. RELATED WORKS

Collaborative filtering has become one of the most popular methods to provide personalized services for users. The key is to find the neighbors of users and items so that the system can provide recommendations for users[4]. Many scholars study collaborative filtering from different perspectives. In web service recommendation based on QoS, collaborative filtering models based on location[5] and user similarity perspective[6] are proposed. Luo et al. imposed a Gaussian-Gamma prior on ratings and the latent features and proposed a hierarchical Bayesian model-based CF[7] which is more robust, and the penalty terms can be adapted automatically in inference. Trust has been increasingly applied to mine implicit relations between users[8-9], and explicit and implicit trust information had been applied to improve the quality of recommendations. Many collaborative filtering schemes based on neural networks have been proposed. In literature [10], collaborative filtering schemes based on deep neural networks were proposed for top-N recommendation of items, and this model gave a good explanation after recommendation. However, such similarity-based evidence may be too rough to improve users' trust. Fu et al.[11] proposed a novel deep learning method, which imitated effective intelligent recommendation by understanding users and items in advance. Xiao et al.[12] constructed a covariance matrix to describe the changing trend of user interest. Matrix factorization-based methods have been proved to be highly accurate and extensible in solving collaborative filtering problems[13], and can reduce high-dimension matrix to low-rank matrix. Wu et al.[14] used labeled data to calculate the nearest neighbor and added the nearest neighbor relationship to the user-item feature. These methods ignore the influence between users caused by the sequential relation of user consumption. In the real world, due to expensive expert tagging and personal privacy issues, it is difficult to obtain user and item information. Collaborative filtering recommendation based on consumption sequential behavior is a concise and effective way. Sun et al.[15]proposed a collaborative filtering model based on sequential behavior relations, but ignored the impact of absolute value of score on similarity. In this paper, considering the absolute value of the score and the consumption effect of the sequential relationship between users and between items, set up the user consumption and item consumption network, using matrix factorization to calculate the users similarity and items similarity, get the TOP-N close neighbors, and then add the neighbor information to probability matrix factorization, refactoring user-item

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evaluation matrix to achieve prediction.

Ⅲ. USER AND ITEM CONSUMPTION NETWORK

In order to calculate the initial value of the user similarity matrix, a user consumption network[16] was constructed (see Fig. 1).

Calculate user similarity $SU_{i\to j}$:

$$SU_{i\to j} = \theta * \frac{W_{i\to j}}{f_U(i,j)} \qquad (1)$$

Here, $W_{i\to j}$ is the edge weight, and $f_U(i,j)$ is the union of user U_i and user U_j's consumption items.

Items influence each other and the effects are asymmetric, that is, the impact of item A on item B is usually different from the impact of B on A. In order to calculate the initial value of item similarity matrix, we define the concepts of consistent and reverse items.

consistent items: if item V_i is consumed by user U, and item V_j is also consumed by user U within a certain time interval, and user U's scores for item V_i and V_j are both greater than or equal to the median value of the given scoring interval, or both are less than or equal to the median value, then it is said that user U to item V_i->V_j score is consistent.

reverse items: If item V_i is consumed by user U, and item V_j is also consumed by user U within a certain time interval, and one of user U's scores for item V_i and V_j is greater than the median value of the given scoring interval, and the other is less than the median value, then it is said that user U to item V_i->V_j score is reverse.

On this basis, in order to better describe this asymmetry, we construct the item consumption network (see Fig. 2), which is represented by a directed graph G={V, E}, where V represents the collection of all items and E represents all consumption relations, such as user U rated to item V_i->V_j is consistent, then weight $WE_{i\text{->}j}$ on edge $E_{i\text{->}j}$ increases by 1 accordingly. The weight on $E_{i\text{->}j}$ represents the number of users who score item V_i -> V_j are consistent, which reflects the influence of item V_i on V_j. Iterate through the sequential consumption relationship among all items and construct the item consumption relationship diagram.

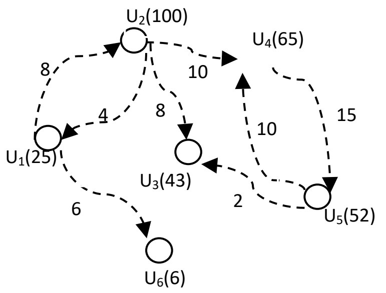

Fig. 1. User consumption network

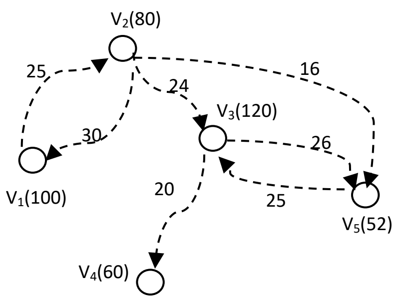

Fig. 2. Item consumption network

The weight $WE_{i\text{->}j}$ of Edge $E_{i\text{->}j}$ is defined as:

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$$WE_{i\to j} = \sum_{k=1}^{M_{i,j}} T_{i\to j} \ ,$$

$$T_{i\to j} = \begin{cases} 1 & \text{if user } U_k \text{ scores item } V_i \to V_j \text{ is consistent} \\ 0 & \text{if user } U_k \text{ scores item } V_i \to V_j \text{ is reverse} \end{cases}$$

Here $M_{i,j}$ represents the number of users who have graded the consumption of item $V_i \to V_j$ successively.

$WE_{i\text{->}j}$ represents the number of users who score item V_i->V_j are consistent among $M_{i,j}$ users, reflecting the influence of the item.

Define impact weights between items $WT_{i\to j} = \frac{WE_{i\to j}}{d_U(i,j)}$, $d_U(i,j)$ represents the union of users that consume items V_i and V_j. Item similarity is defined as:

$$ST_{i\to j} = \theta * WT_{i\to j} = \theta * \frac{WE_{i\to j}}{d_U(i,j)} \qquad (2)$$

In Fig. 2, $V_1(100)$ represents 100 users consuming item V_1, and the weight of the edge V_1 -> V_2 is 25, indicating the number of users to grade consistent item V_1->V_2 is 25. Assuming θ is 1, then $WE_{1\text{->}2}$=25. Assuming that the union of users consuming V_1 and V_2 is 125, then $WT_{1\text{->}2}$=25/125=0.2, then the item similarity based on sequential behavior relationship $ST_{1\text{->}2}=\theta*WT_{1\text{->}2}$=0.2,while $ST_{2\text{->}1}=\theta*WT_{2\text{->}1}$=30/125=0.24. According to the definition,$ST_{1\text{->}2} \neq ST_{2\text{->}1}$, that is, the similarity between items is asymmetric.

Ⅳ. USER/ITEM NEIGHBOR SELECTION

1. User/Item Similarity Matrix Factorization

The user-user and item-item similarity matrices are constructed according to the previous calculation method of user and item similarity. The user-user similarity matrix is factorized to low rank matrices P and Q, and the item-item similarity matrix is factorized to low rank matrices X and Y.

Define loss function L1(S,P,Q) and calculate P and Q according to the method literature[16]. And then the user-user similarity matrix is reconstructed, and the user's *k*-nearest neighbor is obtained by sorting the user's similarity value.

In order to factorize the item-item similarity matrix $T_{M\times M}$, the optimization function $L_2(T,X,Y)$ is defined:

$$L_2(T,X,Y) = \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M} I_{ij}^{T}(T_{ij} - g(X_i^T Y_j))^2 + \frac{\beta_1}{2}\|X\|^2 + \frac{\beta_2}{2}\|Y\|^2$$

Where $g(x) = 1/(1+e^{-x})$, if the item V_i->V_j has the corresponding similarity value, then I_{ij}^{T} is 1, otherwise 0.

The gradient iteration formula is as follows

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$$\begin{cases} X_i' = X_i - \alpha_2 \cdot \frac{\partial L_2}{\partial X_i} \\ Y_j' = Y_j - \alpha_2 \cdot \frac{\partial L_2}{\partial Y_j} \end{cases}$$

The gradient is calculated as follows:

$$\begin{cases} \frac{\partial L_2}{\partial X_i} = \sum_{j=1}^{M} I_{ij}^T Y_j g'(X_i^T Y_j)\left(g(X_i^T Y_j) - T_{ij}\right) + \beta_1 X_i \\ \frac{\partial L_2}{\partial Y_j} = \sum_{i=1}^{M} I_{ij}^T X_i g'(X_i^T Y_j)\left(g(X_i^T Y_j) - T_{ij}\right) + \beta_2 Y_j \end{cases}$$

We can get the matrices X and Y.

2. User/Item K-Nearest Neighbor Selection Algorithm

The user-user/item-item similarity matrix is constructed by matrix factorization method, and then the user/item similarity is ranked, and the user/item k-nearest neighbor is obtained.

Algorithm 1: User/item k nearest neighbor selection algorithm

INPUT: Movielens-100K dataset, learning rate α_1/α_2, regularization parameters λ_1, λ_2 /β_1, β_2

OUTPUT: user/item k nearest neighbor

⑴ construct user/item consumption network according to data set;
⑵ the initial user/item similarity data set DS/IS was calculated according to formula ⑴/⑵ (θ=1).
⑶ DS/IS is divided into two parts: training set TRDS/TRIS(80%) and test set TEDS/TEIS(20%);
⑷ random initialization matrices P, Q/X, Y
⑸ do
⑹ for $(U_i,U_j,S_{ij})/(V_i,V_j,T_{ij})$ in TRDS/TRIS
⑺ calculate gradient $\frac{\partial L_1}{\partial P_i}$、$\frac{\partial L_1}{\partial Q_j}$/$\frac{\partial L_2}{\partial X_i}$、$\frac{\partial L_2}{\partial Y_j}$
⑻ calculate a new round of P_i, Q_j /X_i , Y_j
⑼ end for
⑽ calculate MAE values based on test sets TEDS/TEIS
⑾ while (MAE>ε)
⑿ reconstructed the user-user/item-item similarity matrix according to the feature vectors P_i、Q_j /X_i、Yj
⒀ sort the user/item similarity and select the K nearest neighbor of the user/item

Ⅴ. COLLABORATIVE FILTERING ALGORITHM AND RECOMMENDATION FRAMEWORK

1. User-item Score Matrix Factorization

The larger user-user similarity or item-item similarity, the more similar the feature vectors are. In other words, the feature vectors of users and items are determined by their K nearest neighbors. Therefore, after

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obtaining the k-nearest neighbors of users and items, the feature vectors of users and items are expressed as follows:

$$\begin{cases} \widetilde{U}_u = \sum_{v \in N_u} S_{v \to u} U_v \\ \widetilde{V}_i = \sum_{j \in N_i} T_{j \to i} V_j \end{cases}$$

In the formula N_u is the nearest neighbor set of user U and N_i is the nearest neighbor set of item i. In order to solve the user/item feature matrix and decompose the user-item score matrix, a loss function L(R,S,T,U,V) influenced by the nearest neighbor users is defined.

$$L(R,S,T,U,V) = \frac{1}{2}\sum_{u=1}^{N}\sum_{i=1}^{M} I_{u,i}^{R}\left(R_{u,i} - g(U_u^T V_i)\right)^2 + \frac{\lambda_u}{2}\|U\|^2 + \frac{\lambda_v}{2}\|V\|^2 + \frac{\lambda_S}{2}\sum_{u=1}^{N}\left((U_u - \sum_{v \in N_u} S_{v \to u} U_v\right)^T (U_u - \sum_{v \in N_u} S_{v \to u} U_v)) + \frac{\lambda_T}{2}\sum_{i=1}^{M}\left((V_i - \sum_{j \in N_i} T_{j \to i} V_j\right)^T (V_i - \sum_{j \in N_i} T_{j \to i} V_j))$$

The gradient can be calculated by:

$$\begin{cases} \frac{\partial L}{\partial U_u} = \sum_{i=1}^{M} I_{u,i}^{R} V_i g'(U_u^T V_i)\left(g(U_u^T V_i) - R_{u,i}\right) + \lambda_U U_u + \lambda_S (U_u - \sum_{v \in N_u} S_{v \to u} U_v) - \lambda_S \sum_{\{v | u \in N_v\}} S_{u \to v}(U_v - \sum_{w \in N_v} S_{w \to v} U_w) \\ \frac{\partial L}{\partial V_i} = \sum_{u=1}^{N} I_{u,i}^{R} U_u g'(U_u^T V_i)(g(U_u^T V_i) - R_{u,i}) + \lambda_V V_i + \lambda_T (V_i - \sum_{j \in N_i} T_{j \to i} V_j) - \lambda_T \sum_{\{j | i \in N_j\}} T_{i \to j}(V_j - \sum_{k \in N_j} T_{k \to j} V_k) \end{cases} \quad (3)$$

The gradient iteration formula is as follows:

$$\begin{cases} U'_u = U_u - \alpha \frac{\partial L}{\partial U_u} \\ V'_i = V_i - \alpha \frac{\partial L}{\partial V_i} \end{cases} \quad (4)$$

2. Collaborative Filtering Recommendation Algorithm

According to the above analysis, a recommendation algorithm is proposed.

Algorithm 2: Improved collaborative filtering algorithm

Inputs: Movielens-100K dataset, user/item K nearest neighbors, learning rate α, regularization parameters λu, λv, λs, λt

Outputs: user and item feature matrix U and V

(1) divide the user-item score data set into training set TR(80%) and test set TE(20%)

(2) the feature matrices U and V of users and items are initialized, satisfying the (0,1) gaussian distribution

(3) do

(4) for (U_i,V_j,R_{ij}) in TR

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(5) calculate gradient $\frac{\partial L}{\partial U_i}$、$\frac{\partial L}{\partial V_j}$ according to formula (3)

(6) calculate U_i and V_j a new round according to formula (4)

(7) end for

(8) calculate MAE based on the test set TE

(9)while (MAE>ε)

(10)output the final user and item feature vectors U_i,V_j

After obtaining the user/item feature matrix U and V, the user-item score matrix is reconstructed, and the unknown score data is predicted.

3. Collaborative Filtering Recommendation Framework

Based on the above analysis, a collaborative filtering recommendation framework is presented (see Fig. 3). In this recommendation framework, the k-nearest neighbor and feature vectors of user/item are obtained by two-level matrix factorization.

4. Time Complexity Analysis

In the collaborative filtering model based on sequential behavior, the computation costs are mainly in two aspects: ① constructing user/item consumption network to mine the potential relationship between users and between items, and selecting user/item's neighbor. ②According to the obtained influence relation, the gradient calculation is carried out to realize the recommendation.

As for ①, assume an item is consumed by $\hat{r}$ users on average, and the time complexity of creating the consumption adjacency list of M items is $O(M\hat{r}log\hat{r})$. Assume each user consumes $\hat{t}$ items on average, and the time complexity of creating the consumption adjacency list of N users is $O(N\hat{t}log\hat{t})$. Assume a user affects at most g users and an item can affect at most s items, the time complexity is $O(M\hat{r}g + N\hat{t}s)$ to compute user-user and item-item similarity based on the consumption adjacency lists. Therefore, the total time complexity in ① is $O(M\hat{r}log\hat{r} + N\hat{t}log\hat{t} + M\hat{r}g + N\hat{t}s)$. Assume the vector dimension of matrix factorization is k and the number of neighbors is l, and the time complexity of computing the gradient is $O(M\hat{r}k + Ml^2k + N\hat{t}k + Nl^2k)$[15,17].

Creating a consumption network requires only one traverse the rating data and no iteration. In general, $\hat{r}$、$\hat{t}$、g、s、l and k is small, so the total time complexity of this method is not large and can be applied to large data processing.

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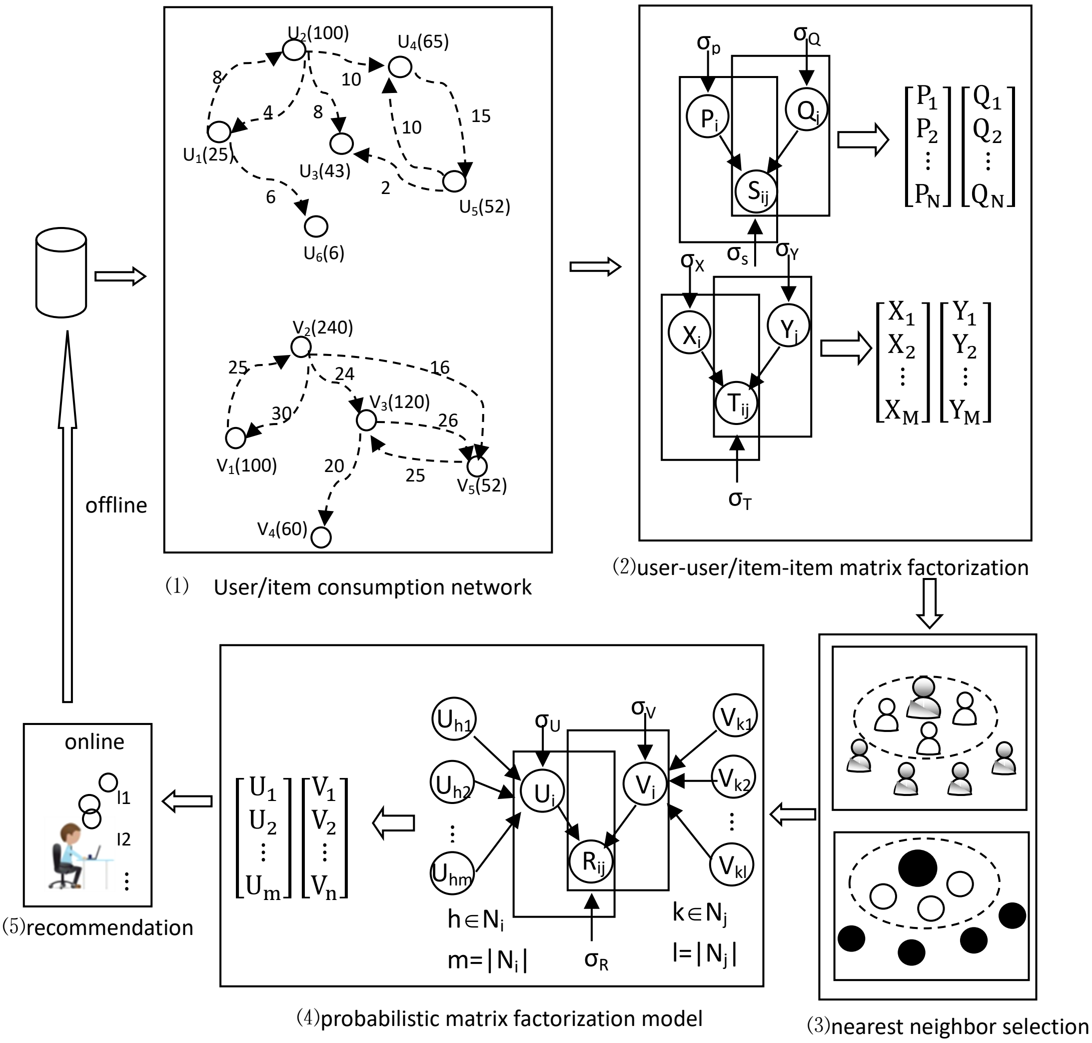

Fig. 3 Collaborative filtering recommendation framework

Ⅵ. EXPERIMENT ANALYSIS

1. The Data Set

The experiment here uses the public Movielens-100K data set, with 943 users, 1682 items and 100,000 rating data. Each record contains the user number, item number, score, and timestamp. The user rated the item between 1 and 5.

2. Evaluation Indicators

To evaluate the prediction quality of our proposed method, MAE and RMSE are used as evaluation indexes. MAE and RMSE can be calculated as follows:

$$MAE = \frac{1}{T}\sum_{i,j}\left|R_{i,j} - \bar{R}_{i,j}\right|$$

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$$RMSE = \sqrt{\frac{1}{T}\sum_{i,j}(R_{i,j} - \bar{R}_{i,j})^2}$$

3. Analysis of Experimental Results

3.1 Impact of User/Item Neighbor k

The MAE and RMSE values vary with the number of neighbors. Fig.4 shows the changes of MAE and RMSE when the number of neighbors between users and between items changes from 1 to 10. We can see MAE and RMSE is the smallest when k is 5.

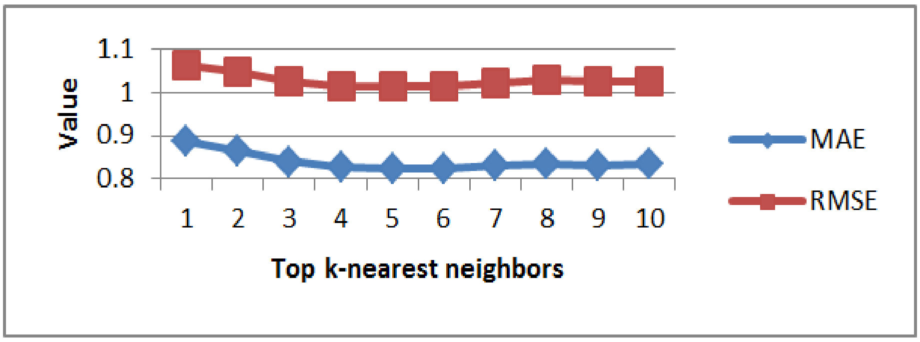

Fig. 4. Impact of k nearest neighbors on MAE and RMSE

3.2 Comparison of Several Methods

We use the method based on Sequential Behavior Relationship (SBR) proposed in this paper to calculate the initial similarity matrix with the methods of ACOS, Jaccard, PIP[18] and NHSM[4]. Then the similarity between user-user/item-item is reconstructed by matrix factorization, and the K nearest neighbor of user/item is obtained. The user/item k-nearest neighbor is integrated into the matrix factorization of user-item score matrix, and the user-item score matrix is reconstructed to realize the prediction.

Collaborative filtering based on different methods was labeled as ACOS-CF, Jaccard-CF, NHSM-CF, PIP-CF and SBR-CF respectively, and then accuracy prediction was carried out using these five methods. MAE and RMSE values are shown in Fig. 5 and Fig. 6.

It can be seen from FIG. 5 and FIG. 6 that the SBR-CF method has better effect than other methods, which indicates that the collaborative filtering recommendation proposed by us based on the two-level probability matrix factorization to mine the nearest neighbor relationship by calculating the similarity of sequential consumption behavior is effective.

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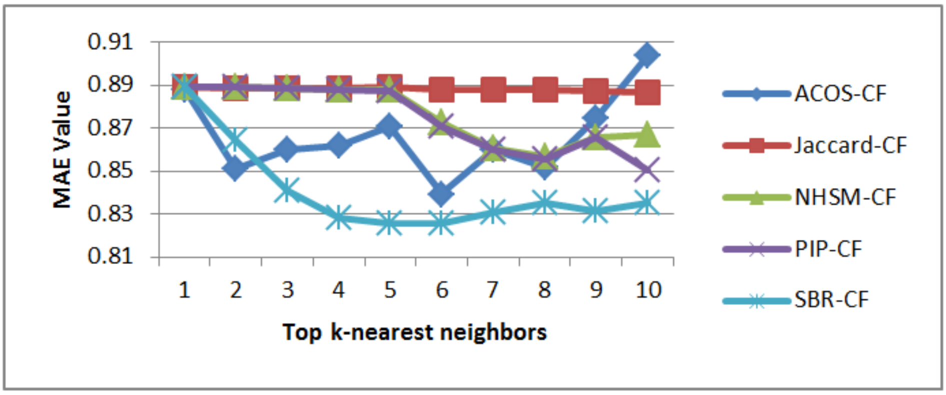

Fig.5. Comparision of MAE based on different methods

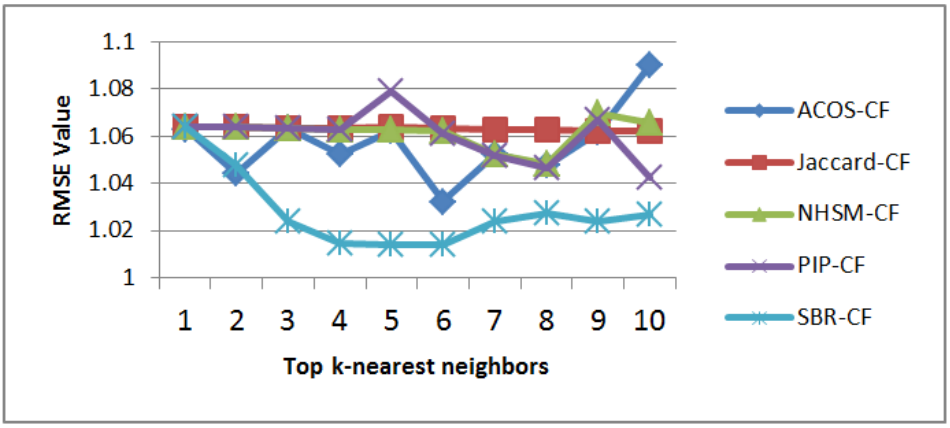

Fig.6. Comparision of RMSE based on different methods

Ⅶ. CONCLUSION

When it is difficult to obtain personal privacy information, it is a simple and effective method to integrate the consumption sequential behavior of users and items into the collaborative filtering based on two-level probability matrix factorization to predict users' rating of items. With the development of social networks, the next step is to further study personalized recommendation based on the information resources of social networks.

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