Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

Lookback Option Pricing Formulas with Floating Interest Rate in Uncertain Environment

Jie LI, Zhaopeng LIU*, Zufeng ZHANG, Shilong FEI
College of Mathematics and Statistics, Suzhou University, Suzhou 234000, China.
*Corresponding Author.

Abstract:

Uncertainty theory is widely used in the financial field. As a path dependent option, Look back option is favored in the financial market because of its own characteristics and is often used by investors to avoid risks. Based on this, this paper proposes to use the Ornstein Uhlenbeck stock model to study the look back option price, and takes the uncertain mean reversion model as the assumption of interest rate treatment. In the traditional lognormal distribution, the one-way change of stock price with time. When using the Ornstein Uhlenbeck stock model, the stock price follows the index O-U process, this situation is avoided, and the characteristic of interest rate, the average value of interest rate fluctuation, and the interest rate follows the mean reversion model can be well reflected. Therefore, the actual situation of the financial market can be more appropriately reflected by the new stock model. Further, the pricing formula of look back option is further deduced and discussed. At the same time, it is explained through relevant special cases.

Keywords: *Floating interest rate*, *Exponential O-U process*, *Lookback options.*

I. INTRODUCTION

In recent years, there has been a significant increase in options business products and types around the world. Financial derivatives have become an important part of the financial market. For a long time, the large fluctuations of the financial market make market participants fully realize that there are great risks in the financial market. Retrospective option is favored by financial market because of its own characteristics and is often used by investors to avoid risks. A retrospective option is an option that selects the most favorable price as the agreed price during the expiration period, thus providing the investor with the lowest price to buy the target stock.

Under the framework of probability theory, traditional financial decision theory assumes that stock return rate obeies normal distribution and uses stochastic differential equation to describe the change of original asset price. As a matter of fact, stock return rate does not completely obey normal distribution. In real financial decision-making, people make decisions based on some nonlinear transformation of probability. Path dependence is a feature of backdating options whose benefits are determined by the strike price and the extreme value of the asset price. Compared with other options, call options are more attractive to investors by buying low and selling high to reduce investor regret. Black Scholes model

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

studies the pricing of callback options [1,2]. However, stochastic differential equations can not accurately describe the stock price process. Liu proposed a new theory in 2007[3], and Liu derived the European option pricing formula [4]. Under the framework of uncertainty theory, this paper introduces the uncertain stock price model, considers the influence of floating interest rate on stock price, and studies the backdating option pricing which is more consistent with the actual financial asset price change. Therefore, the research on this kind of problem is not only innovative in theory, but also has a clear application background and application prospect

II. PRELIMINARIES

In this paper, the Ornstein Uhlenbeck stock model is used to study backdating option prices and the uncertain mean regression model is used as the hypothesis of interest rate treatment. The pricing formula of retroactive option is further deduced and discussed. All equations, theorems, definitions, lemmas, propositions, inferences, examples, remarks, etc. are numbered continuously and without repetition in each part of this paper. For example, definition 2.1, lemma 2. Theory. Three

Definition 2.1 [5] Let L be a σ algebra on a set Γ. M : $L \rightarrow$ [0, 1] is called an uncertain measure If the following axioms are satisfied:

Axiom1: $M\{\Gamma\}=1$ for t For the complete set Γ.

Axiom2: $M\{\Lambda\}+M\{\Lambda^C\}=1$ for any event Λ .

Axiom3: Pair countable sequence $\Lambda_1, \Lambda_2, \cdots,$

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}$$

Axiom4:(Product Axiom) Let the triple (Γ_k, L_k, M_k) $k=1,2,\cdots$ be an uncertain space.

if $$M\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} \le \bigwedge_{k=1}^{\infty} M\{\Lambda_k\}$$

Uncertain distribution of an uncertain variable ξ as

$$\Phi(x) = M\{\xi \le x\} \quad (1)$$

Definition 2.2 [6] ξ is an variable. Then the expectation of ξ is defined as

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

$$E[\xi]=\int_0^{+\infty} M\{\xi\geq x\}dx-\int_{-\infty}^{0} M\{\xi\leq x\}dx \tag{2}$$

Theorem 2.3 [5] If ξ has normal uncertainty distribution Φ. Then ξ of expectation is

$$E[\xi]=\int_0^{+\infty}(1-\Phi(x))dx-\int_{-\infty}^{0}\Phi(x)dx \tag{3}$$

Theorem 2.4 [7] If ξ is an uncertain variable with regular uncertainty distribution Φ,

$$E[\xi]=\int_0^{1}\Phi^{-1}(\alpha)d\alpha \tag{4}$$

Definition 2.5 [6] C_t is to be a canonical Liu process if

(i) $C_0=0$, almost all sample paths are Lipschitz continuous;

(ii) C_t With stable independent increment;

(iii) For each increment $C_{s+t}-C_t$ Its normal uncertainty distribution is

$$\Phi(x)=\left(1+\exp\left(\frac{-\pi x}{\sqrt{3}t}\right)\right)^{-1}. \tag{5}$$

Definition 2.6 [6] Let C_t be a Liu process and f g to be two real functions.

Then

$$dX_t=f(t,X_t)dt+g(t,X_t)dC_t \tag{6}$$

is an uncertain differential equation.

Definition 2.7 [5] Let α be a number with $(0<\alpha<1)$. An uncertain differential equation

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t \tag{7}$$

is said to have an α−paths X_t^α it solves the corresponding ordinary differential equation

$$dX_t^\alpha = f(t, X_t^\alpha)dt + |g(t, X_t^\alpha)|\Phi^{-1}(\alpha)dt \tag{8}$$

$\Phi^{-1}(\alpha)$ is the inverse distribution of the standard normal uncertainty distribution, Namely

$$\Phi^{-1}(\alpha) = \frac{\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}$$

Theorem 2.8 [6] Let X_t and X_t^α be the solution and α−path of the uncertain differential equation

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t \tag{9}$$

respectively. Then, the solution X_t has an inverse uncertainty distribution $\Psi^{-1}(\alpha) = X_t^\alpha$.

And for s >0 and strictly increasing function $J(x)$, $\int_0^s J(X_t)dt$ has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = \int_0^s J(X_t^\alpha)dt. \tag{10}$$

And if $J(x)$ is a strictly decreasing function, $\int_0^s J(X_t)dt$ has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = \int_0^s J(X_t^{1-\alpha})dt. \tag{11}$$

Theorem 2.9[6] Let X_t and X_t^α be the solution and α−path of the uncertain differential equation

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t$$

respectively. Then, for $s > 0$ and strictly increasing function $J(x)$, the supremum

$$\sup_{0 \le t \le s} J(X_t)$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

has an inverse uncertainty distribution

$$\Psi_s^{-1}(\alpha) = \sup_{0\le t\le s} J\left(X_t^{\alpha}\right). \tag{12}$$

and

$$\inf_{0\le t\le s} J(X_t)$$

has a distribution

$$\Psi_s^{-1}(\alpha) = \inf_{0\le t\le s} J\left(X_t^{\alpha}\right). \tag{13}$$

Theorem 2.10 [5] Let X_t and X_t^{α} be the solution and α −path of the uncertain differential equation

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t$$

$\sup_{0\le t\le s} J(X_t)$ respectively. Then, for s >0 and strictly decreasing function $J(x)$, the supremum

$$\sup_{0\le t\le s} J(X_t)$$

has an inverse uncertainty distribution

$$\Psi_s^{-1}(\alpha) = \sup_{0\le t\le s} J\left(X_t^{1-\alpha}\right). \tag{14}$$

and the infimum

$$\inf_{0\le t\le s} J(X_t)$$

has an inverse uncertainty distribution

$$\Psi_s^{-1}(\alpha) = \inf_{0\le t\le s} J\left(X_t^{1-\alpha}\right). \tag{15}$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

Theorem 2.11 [7] Assume that $X_{1t}, X_{2t}, \cdots X_{nt}$ are some independent uncertain processes. If the function $f(x_1, x_2, \cdots x_n)$ is strictly increasing relating to $x_1, x_2, \cdots x_m$ and strictly decreasing relating to $x_{m+1}, x_{m+2}, \cdots x_n$ then the uncertain process $X_t = f(X_{1t}, X_{2t}, \cdots X_{nt})$ has an α−path

$$X_t^\alpha = f\left(X_{1t}^\alpha, \cdots, X_{mt}^\alpha, X_{m+1,t}^\alpha, \cdots X_{nt}^\alpha\right).$$

III. UNCERTAIN EXPONENTIAL ORNSTEIN-UHLENBECK MODEL WITH FLOATING INTEREST RATE

Share prices have fluctuated around their long-term average. Considering the long-term fluctuations of stock prices and changes of interest rates, Sun et al. [6] constructed an uncertain stock model with mean regression to calculate stock prices and interest rates, which is characterized by floating interest rates.

$$\begin{cases} dr_t = \left(m_1 - a_1 r_t\right) dt + \sigma_1 dC_{1t} \\ dX_t = \left(m_2 - a_2 X_t\right) dt + \sigma_2 dC_{2t} \end{cases} \tag{16}$$

However, the afore-mentioned model is linear mean reversion. In this part, we will make some improvements to the above stock model. So as to simulate stock prices more accurately, we shall give the nonlinear model and add a new uncertain exponential Ornstein-Uhlenbeck model with floating interest rate,

$$\begin{cases} dr_t = \left(b - a_1 r_t\right) dt + \sigma_1 dC_{1t} \\ dX_t = \mu\left(1 - c \ln X_t\right) X_t dt + \sigma_2 X_t dC_{2t} \end{cases} \tag{17}$$

where b means the average interest rate, σ_1 means the interest rate diffusion, σ_2 means the stock price diffusion, $a, b, c, \mu, \sigma_1, \sigma_2$ are some positive real numbers, and C_{1t} and C_{2t} are independent canonical Liu processes.

IV.LOOKBACK OPTION PRICING FORMULAS

4.1 Look Back Calloption

The payoff of the look back call option:

$$\left(\sup_{0 \le t \le T} X_t - K\right)^+.$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

Based on the time value of money, the return of option holders is

$$-f_c+\exp\left(-\int_0^T r_s ds\right)\left(\sup_{0\le t\le T} X_t-K\right)^+.$$

The option issuer's return is

$$f_c-\exp\left(-\int_0^T r_s ds\right)\left(\sup_{0\le t\le T} X_t-K\right)^+.$$

So

$$E\left[-f_c+\exp\left(-\int_0^T r_s ds\right)\left(\sup_{0\le t\le T} X_t-K\right)^+\right]=E\left[f_c-\exp\left(-\int_0^T r_s ds\right)\left(\sup_{0\le t\le T} X_t-K\right)^+\right]. \quad (18)$$

Definition 4.1 Considering the lookback option has execution price and maturity time. So what's the price is

$$f_c=E\left[\exp\left(-\int_0^T r_s ds\right)\left(\sup_{0\le t\le T} X_t-K\right)^+\right]. \quad (19)$$

Theorem4.2 Given the look back option of uncertain stock model (2), its execution price is K and its maturity time is T. So what's the price is

$$f_c=\int_0^1 \exp\left(-\int_0^T r_s^{1-\alpha} ds\right)\left(\sup_{0\le t\le T} X_t^\alpha-K\right)^+ d\alpha. \quad (20)$$

Where

$$r_s^{1-\alpha}=r_0\exp(-as)+\left(\frac{b}{a}+\frac{\sqrt{3}\sigma_1}{\pi a}\ln\frac{1-\alpha}{\alpha}\right)\left(1-\exp(-as)\right).$$

$$X_t^\alpha=\exp\left(\exp\left(-\mu ct\ln X_0+\left(1-\exp(-\mu ct)\right)\right)\left(\frac{1}{c}+\frac{\sqrt{3}\sigma_2}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right).$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

Proof

$$dr_t = (b - ar_t)dt + \sigma_1 dC_{1t}$$

has an α -path

$$r_t^\alpha = r_0 \exp(-at) + \left(\frac{b}{a} + \frac{\sqrt{3}\sigma_1}{\pi a}\ln\frac{\alpha}{1-\alpha}\right)(1 - \exp(-at))$$

By calculation

$$dr_t^\alpha = (b - ar_t^\alpha)dt + \sigma_1 \frac{\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}dt$$

Similarly

$$dX_t = \mu(1 - c\ln X_t)X_t dt + \sigma_2 X_t dC_{2t}$$

has also an α−path

$$X_t^\alpha = \exp\left(\exp(-\mu ct \ln X_0 + (1 - \exp(-\mu ct)))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right).$$

By calculation

$$dX_t^\alpha = \mu(1 - c\ln X_t^\alpha)X_t^\alpha dt + \sigma_2 X_t^\alpha \frac{\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}dt$$

It can be obtained from theorem 2.10 that the α−path of $\int_0^T r_s ds$ is $\int_0^T r_s^\alpha ds$.

The discount rat

$$\exp\left(-\int_0^T r_s ds\right)$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

has an α -path

$$\exp\left(-\int_0^T r_s^{1-\alpha} ds\right).$$

The supremum

$$\sup_{0\le t\le T} J\left(X_t\right) = \sup_{0\le t\le T} X_t$$

has α-path

$$\sup_{0\le t\le T} J\left(X_t^\alpha\right) = \sup_{0\le t\le T} X_t^\alpha = \sup_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu ct}\ln\frac{\alpha}{1-\alpha}\right)\right)\right).$$

Since is

$$\left(\sup_{0\le t\le T} X_t - K\right)^+$$

$$\left(\sup_{0\le t\le T} X_t^\alpha - k\right)^+ = \left(\sup_{0\le t\le T}\left(exp\left(exp(-\mu ct)\, ln\, X_0 + (1-exp(-\mu ct))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu c\pi}\, ln\frac{\alpha}{1-\alpha}\right)\right)\right) - k\right)^+.$$

an increasing function with respect to $\sup_{0\le t\le T} X_t$, it has an α -path

Hence, the present value for the option

$$\exp\left(-\int_0^T r_s ds\right)\left(\sup_{0\le t\le T} X_t - K\right)^+$$

has an α -path

$$\exp\left(-\int_0^T r_S^{1-\alpha} ds\right)\left(\sup_{0\le t\le T} X_T^\alpha - K\right)^+$$

from Theorem 2.11. the price is

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

$$f_c = \int_0^1 \exp\left(-\int_0^T r_S^{1-\alpha} ds\right)\left(\sup_{0\le t\le T} X_t^\alpha - K\right)^+ d\alpha$$

from Theorem 2.4 and 2.8.

Theorem4.3 Let f_c be the lookback call option price of the uncertain stock model (2).

Then

(1) f_c is an increasing function of X_0;

(2) f_c is a decreasing function of K.

(3)

Proof From Theorem 4.2,

$$f_c = \int_0^1 \exp\left(-\int_0^T r_S^{1-\alpha} ds\right)\left(\sup_{0\le t\le T} X_t^\alpha - K\right)^+ d\alpha$$

Where

$$r_t^{1-\alpha} = \sup_{0\le t\le T} X_t^\alpha = r_0 \exp(-at) + \left(\frac{b}{a} + \frac{\sqrt{3}\sigma_1}{\pi a}\ln\frac{1-\alpha}{\alpha}\right)(1-\exp(-at))$$

$$X_t^\alpha = \exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu ct}\ln\frac{\alpha}{1-\alpha}\right)\right).$$

(1) Since $\exp(-\mu ct) > 0$, the function $\exp(-\mu ct)\ln X_0 > 0$ is increasing respecting X_0.

(2) Since the function

$$\sup_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right) - k$$

is decreasing respecting K and the lookback call option price f_c is decreasing respecting the execution price K. This represents that the higher the execution price, the lower the lookback call option price.

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

Example 1 Given the parameters of the stock price are $c=1, \mu=3, \sigma_2=\pi, X_0=4$, and the parameters of the interest rate are $a=0.06$, $b=0.05$, $\sigma_1=0.04$, $r_0=0.03$, the execution price $K=4$ and the expiration time $T=1$. Then a lookback call option price is f_c= 2.3859.

4.2Lookback put option

If the look back put option has a fixed execution price K and an expiration time T, the stock price at time t is X_t. Its income is given by the following formula

$$\left(K-\inf_{0\le t\le T} X_t\right)^+.$$

Let f_p means the price of lookback calloption, the option holder's return is

$$-f_p+\exp\left(-\int_0^T r_s ds\right)\left(K-\inf_{0\le t\le T} X_t\right)^+.$$

And the option issuer's return is

$$f_p-\exp\left(-\int_0^T r_s ds\right)\left(K-\inf_{0\le t\le T} X_t\right)^+.$$

Based on the fair price characteristics of options, the issuer and the holder should have the same expected return, so

$$E\left[-f_p+\exp\left(-\int_0^T r_s ds\right)\left(K-\inf_{0\le t\le T} X_t\right)^+\right]=E\left[f_p-\exp\left(-\int_0^T r_s ds\right)\left(K-\inf_{0\le t\le T} X_t\right)^+\right]. \tag{21}$$

Definition 4.4 Given the lookback option has an execution price K and an expiration time T. Then the price of the lookback put option is

$$f_p=E\left[\exp\left(-\int_0^T r_s ds\right)\left(K-\inf_{0\le t\le T} X_t\right)^+\right]. \tag{22}$$

Theorem4.5 Given a lookback option of the uncertain stock model (2) has a execution price K and an expiration time T . Then the price of the lookback put option is

$$f_p=\int_0^1\exp\left(-\int_0^T r_s^{1-\alpha} ds\right)\left(K-\inf_{0\le t\le T} X_t^{1-\alpha}\right)^+ d\alpha. \tag{23}$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

Where

$$r_t^{1-\alpha} = r_0 \exp(-at) + \left(\frac{b}{a} + \frac{\sqrt{3}\sigma_1}{\pi a} \ln \frac{1-\alpha}{\alpha} \right) (1 - \exp(-at))$$

$$X_t^{1-\alpha} = \exp\left(\exp(-\mu ct) \ln X_0 + (1 - \exp(-\mu ct)) \left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu ct} \ln \frac{1-\alpha}{\alpha} \right) \right).$$

Proof From the Yao-Chen formula, the uncertain differential equation

$$dr_t = (b - ar_t) dt + \sigma_1 dC_{1t}$$

has also an α−path

$$r_t^{\alpha} = r_0 \exp(-at) + \left(\frac{b}{a} + \frac{\sqrt{3}\sigma_1}{\pi a} \ln \frac{\alpha}{1-\alpha} \right) (1 - \exp(-at))$$

By solving the equation

$$dr_t^{\alpha} = (b - ar_t^{\alpha}) dt + \sigma_1 \frac{\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha} dt$$

Similarly, the equation

$$dX_t = \mu (1 - c \ln X_t) X_t dt + \sigma_2 X_t dC_{2t}$$

has also an α−path

$$X_t^{\alpha} = \exp\left(\exp(-\mu ct) \ln X_0 + (1 - \exp(-\mu ct)) \left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu ct} \ln \frac{\alpha}{1-\alpha} \right) \right)$$

by solving the differential equation

$$dX_t^{\alpha} = \mu (1 - c \ln X_t^{\alpha}) X_t^{\alpha} dt + \sigma_2 X_t^{\alpha} \frac{\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha} dt$$

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

From theorem 2.8 we can get the α−path of $\int_0^T r_s ds$ is $\int_0^T r_s^\alpha ds$.

Since $y=\exp(-x)$ is strictly decreasing with respect to x, the discount rate

$$\exp\left(-\int_0^T r_s ds\right)$$

has an α−path

$$\exp\left(-\int_0^T r_s^{1-\alpha} ds\right).$$

Since $J(x)=x$ is strictly increasing with respect to x, the infimum

$$\inf_{0\le t\le T} J(X_t)=\inf_{0\le t\le T} X_t$$

has an α−path

$$\inf_{0\le t\le T} J(X_t^\alpha)=\inf_{0\le t\le T} X_t^\alpha=\inf_{0\le t\le T}\left(\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma_2}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right).$$

Since is

$$\left(K-\inf_{0\le t\le T} X_t\right)^+$$

an decreasing function respecting $\inf_{0\le t\le T} X_t$, it has an α-path

$$\left(K-\inf_{0\le t\le T} X_t^{1-\alpha}\right)^+=\left(K-\inf_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0+(1-\exp(-\mu ct))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma_2}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right)\right)^+.$$

The present value of the option

$$\exp\left(-\int_0^T r_s ds\right)\left(K-\inf_{0\le t\le T} X_t\right)^+$$

has an α−path

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 633-647
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

$$\exp\left(-\int_0^T r_s^{1-\alpha} ds\right)\left(K - \inf_{0\le t\le T} X_t^{1-\alpha}\right)^+$$

from Theorem 2.11. We have the price of the lookback put option

$$f_p = \int_0^1 \exp\left(-\int_0^T r_s^{1-\alpha} ds\right)\left(K - \inf_{0\le t\le T} X_t^{1-\alpha}\right)^+ d\alpha.$$

from Theorem 2.4 and 2.8.

Theorem4.6 Let f_p be the lookback put option price of the uncertain stock model (2). Then

(1) f_p is a decreasing function of X_0;

(2) f_p is an increasing function of K.

Proof From Theorem 4.5,

$$f_p = \int_0^1 \exp\left(-\int_0^T r_s^{1-\alpha} ds\right)\left(K - \inf_{0\le t\le T} X_t^{1-\alpha}\right)^+ d\alpha.$$

Where

$$r_t^{1-\alpha} = r_0 \exp(-at) + \left(\frac{b}{a} + \frac{\sqrt{3}\sigma_1}{\pi a}\ln\frac{1-\alpha}{\alpha}\right)\left(1-\exp(-at)\right)$$

$$X_t^{1-\alpha} = \exp\left(\exp(-\mu ct)\ln X_0 + \left(1-\exp(-\mu ct)\right)\left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu c\pi}\ln\frac{1-\alpha}{\alpha}\right)\right).$$

(1) Since $\exp(-\mu ct) > 0$, the function $-\exp(-\mu ct)\ln X_0 > 0$ is decreasing respecting X_0 and the lookback put option price f_p is decreasing respecting the initial stock price X_0. So, the higher the initial stock price, the lower the lookback put option price.

(2) Since the function

$$K - \inf_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0 + \left(1-\exp(-\mu ct)\right)\left(\frac{1}{c} + \frac{\sqrt{3}\sigma_2}{\mu c\pi}\ln\frac{1-\alpha}{\alpha}\right)\right)\right)$$

is increasing respecting K and the lookback put option price f_p is increasing with respect to the execution price K. The higher the execution price, the higher the lookback put option price.

Example 2 Given the parameters of the stock price are $c=1$, $\mu=3$, $\sigma_2=\pi$, $X_0=4$, and the parameters of the interest rate are $a=0.06, b=0.05$, $\sigma_1 0.04, r_0=0.03$, the execution price $K=4$ and the expiration time $T=1$. Then a lookback put option price is f_c= 1.5659.

V.CONCLUSIONS

In this paper, the Ornstein Uhlenbeck stock model is used to study backdating option prices and the uncertain mean regression model is used as the hypothesis of interest rate treatment. The pricing formula of retroactive option is further deduced and discussed. At the same time, the relevant special cases are illustrated

ACKNOWLEDGEMENTS

This work has been supported by the humanities and Social Sciences project (No.: sk2020a025, sk2021a0709, sk2021a0695), the university youth talent project (gxyq2017092, gxyqzd2016340, 2021fzjj14).

REFERENCES

[1] M. B. GOLDMAN, H. B. SOSIN, M. A. GATTO. Path dependent optins: buy at the low, sell at the high. J Finance, **34**:1111-1127(1979)
[2] R. C. HEYNEN, H. M. KAT. Lookback options with discrte partial and monitoring of the underlying price. Appl Maths Finance, **2**: 263–274(1995)
[3] M DAI, H.Y. WONG, Y.K. KWOK. Quanto lookbalk ptions. Math Finance, **14**: 445– 467(2004)
[4] BLIU. Uncertainty Theory. Springer, Berlin, (2007)
[5] KYAO, XCHEN. Anumerical method for solving uncertain differental equations. J Intell Fuzzy Syst. ,25(3):825-832(2013)
[6] B LIU. Uncertainty distibution and indepedence of uncetain processes. Fuzzy Optim Decis Mak.,13(3): 259-271(2014)
[7] KYAO.Uncertaincontorprocessanditsapplicationistockmodelwithfloatinginterest rate. Fuzzy Optim Decis Mak., 14(4):399-424(2015)