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State Feedback Impulsive Control of a Prey-Predator System with Constant Release of Prey Population in Toxin Environment

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Abstract:

River crab is a freshwater breeding species with higher economic value, and the river crab breeding industry has developed rapidly in recent years. As the river crab is at the top of the food chain in the lake, the increasingly serious water pollution has a great impact on the growth and reproduction of the river crab population. A model is established to describe the growth and state feedback impulsive harvest of river crab in toxin environment, and the existence, uniqueness and stability of the first-order periodic solution of the state feedback impulsive system are studied. The numerical simulation at the end of this paper verifies the correctness of the theory, and the conclusion provides a theoretical support for improving the economic benefits of river crab culture.

***Keywords**: Toxin; Semi-continuous dynamic system, Order-1 periodic solution, Stability, river crab.*

I. INTRODUCTION

River crab breeding industry has formed a complete industrial chain in Jiangnan area of China, and has become the largest output value industry of single species in freshwater aquaculture. With the development of economy and the progress of industry in recent years, heavy metals and chemical pesticide residues are often detected in freshwater, which is unfavorable to the survival and reproduction of aquatic animals and plants. This not only affects food safety, but also threatens the survival and reproduction of species. Generally, the higher the level of organisms in the food chain, the greater the impact. Crabs have few natural enemies in fresh water, toxins will gradually accumulate in the population. In the long run, the production of river crabs will decline, and a large number of river crabs will die in serious cases. This will seriously affect the development of river crab breeding industry and the sustainable survival of river crab population in certain area. Considering the above factors, the cultivation of river crabs in polluted water has attracted more and more attention from experts and scholars.

In recent years, many mathematicians have get a lot of achievements in the semi-continuous dynamical system and aquaculture model with toxins. Chen et al. established a class of impulsive state feedback control model from the problem of pest control, and proposed the geometric theory of semi-continuous

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dynamic system, they also proved that the existence condition of order-1 periodic solution [1, 2, 3]. Jin Zhen et al. studied the three-dimensional Volterra predator-prey system with time delay under the condition of pollution, and gave the threshold of persistence and extinction of one kind of prey population and two kinds of predator population [4]. Wu improved the Holling functional response and set up a toxicity prey-predator system, and discussed the optimal balance capture with the extreme value determination and the Pontryagin's maximal principle [5]. Many scholars have studied the influence of polluted environment to predator-prey system, and obtained the sufficient conditions of population survival and extinction, and the thresholds of extinction and persistence [6-11]. Jiao et al. set up the stage-structured switched single population model with impulsive pollutant input and birth pulse, and studied the controlling conditions of extinction and permanence. dynamics model of toxin pulse input and pulse birth switching stage [12]. Jiang et al. obtained the control conditions of population extinction and persistence by using ordinary differential equations and differential analysis, which provided a reliable management strategy for the management of biological resources in polluted environment [13]. Zhang et al. profoundly studied the dynamic behavior of single and bilateral state-feedback impulsive models in aquaculture [14-19]. However, few work were carried out to study the freshwater aquaculture prey-predator problem in the long-term regional pollution environment.

The rest of this paper is arranged as follows: the model and some preliminaries are presented in the next section. In Sect.3, the qualitative characteristics of impulse free system are analyzed. In Sect.4, we analyze the existence, uniqueness and stability of the order-1 periodic solution. Finally, the results are verified by numerical simulations and some conclusions are summarized.

II. MODELING AND PRELIMINARIES

In this paper, we propose the following state feedback impulsive differential equations to simulate the process of crab culture and harvest in toxin environment, and the impulse is used to represent the feeding and adult harvest. When the number of adult crab reaches the threshold, an impulse will be applied. According to the actual situation, the model is as follows:

$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} &= x\left(b_1 - a_{12}y\right) - \alpha x^3 \\ \frac{dy}{dt} &= y\left(b_2 + a_{21}x - a_{22}y\right) - \beta y^2 \end{aligned}\right\} (x,y) \notin \{y = y^*, 0 < x < x^*\}, \\ \left.\begin{aligned} \Delta x &= -\gamma\left(x - x^*\right) \\ \Delta y &= -qy^* \end{aligned}\right\} (x,y) \in \{y = y^*, 0 < x < x^*\}. \end{cases} \quad (1)$$

In the model, x and y represent the population density of aquatic plants and river crabs at time t respectively; b_1 and b_2 indicate the intrinsic growth rate of aquatic plants and river crab; a_{12} is the predator's predation rate, a_{21} is the transformation efficiency of predation, and a_{22} indicates the intraspecific competition of river crab. According to the different effects of toxic substances on aquatic plants and river crabs, the influences were expressed by αx^3 and βy^2 respectively. During river crab culture,

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$M=\{(x,y)|0<x<x^*, y=y^*\}$ is a harvesting signal. When $(x,y)\notin M$, aquatic plants and river crab grow and reproduce according to the first two equations of system (1). When $(x,y)\in M$, river crabs will be harvest and aquatic plants will be invest as a necessary supplement according to the last two equations of system (1).

In the following part of this section, we will introduce some lemmas needed to prove the existence and stability of order-1 periodic solutions.

Lemma 1:If $\{[a_n,b_n]\}$ is a closed interval nest, $\lim_{n\to\infty}(b_n-a_n)=0$, then there exists a unique real number $\xi\in[a_n,b_n]$, $n=1, 2, 3, \ldots$, in addition $\lim_{n\to\infty}a_n=\lim_{n\to\infty}b_n=\xi$.

Lemma 2:[20] Consider the following ordinary differential equation

$$\frac{dx}{dt}=x(t)\left(d_1-d_2x(t)\right), \tag{2}$$

we have the following conclusion:

$$\text{(1) if } d_1>0\text{, then } \lim_{x\to\infty}x(t)=\frac{d_1}{d_2},$$
$$\text{(2) if } d_1<0\text{ ,then } \lim_{x\to\infty}x(t)=0.$$

Lemma 3: [21] If there are two points A and B on the phase set N. A coordinate system is set up on N under the action of pulse parameter α_1. Let A^* be the succeeding point of A and B^* be the succeeding point of B. The coordinates of point A and point B are a and b respectively. If $F_s^{\alpha_1}(A)>0, F_s^{\alpha_1}(B)<0$, there must exist a point C between A and B satisfying $F_s^{\alpha_1}(C)=0$, The trajectories passing C is an order-1 periodic solution (shown in Fig.1).

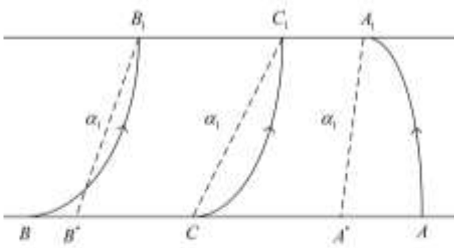

Figure 1: order-1 periodic solutions of SCDS.

Lemma 4: [22] If the Floquet multiplier μ satisfies $|\mu|<1$, where

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$$\mu = \prod_{j=1}^{n} \Delta_j \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t)) \right) \mathrm{d}t \right], \tag{2}$$

$$\Delta_j = \frac{\left(\frac{\partial B}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial B}{\partial x}\frac{\partial C}{\partial y} + \frac{\partial C}{\partial x} \right) P_+ + \left(\frac{\partial A}{\partial x}\frac{\partial C}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} \right) Q_+}{\frac{\partial C}{\partial x} P + \frac{\partial C}{\partial y} Q}, \tag{3}$$

with $P, Q, (\partial A)/(\partial x), (\partial A)/(\partial y), (\partial B/(\partial x), (\partial B)/(\partial y), (\partial C/(\partial x)$ and $(\partial C/(\partial y)$ are evaluated at the point $\left(\xi(\tau_k)\eta(\tau_k)\right)$ and $P_+ = P\left(\xi\left(\tau_j^+\right), \eta\left(\tau_j^+\right)\right), Q_+ = Q\left(\xi\left(\tau_j^+\right), \eta\left(\tau_j^+\right)\right)$, where $\tau_j (j \in N)$ are the moment of the j-th pulse action, then order-1 periodic solution of model (1) is asymptotically stable.

III. QUALITATIVE ANALYSIS OF SYSTEM (1) WITHOUT IMPULSIVE EFFECT

Getting rid of the impulsive influence of system (1), we obtain the corresponding free system:

$$\begin{cases} \frac{dx}{dt} = x\left(b_1 - a_{12}y\right) - \alpha x^3, \\ \frac{dy}{dt} = y\left(b_2 + a_{21}x - a_{22}y\right) - \beta y^2. \end{cases} \tag{4}$$

3.1 Positivity and Boundedness of the Solution

From the first equation of system (2), we have

$$x(t) = x(0)\exp[\int_0^t \left(b_1 - a_{12}y(s) - \alpha x^2(s)\right) ds], \tag{5}$$

similarly, from the second equation of system (2), we have

$$y(t) = y(0)\exp[\int_0^t \left(b_2 + a_{21}x(s) - a_{22}y(s) - \beta y(s)\right) ds]. \tag{6}$$

Then $x(t) > 0$ and $y(t) > 0$ with initial conditions $x(0) > 0$ and $y(0) > 0$.

Therefore, all solutions started from an interior point of the first quadrant will develop in it. In addition, the solution trajectory starting from $(x_0, 0)$ will remain near E_2 at all future times. Similar results are satisfied to trajectories starting from points on the positive y-axis. Therefore, $R^2 = \{(x, y) : x, y \geq 0\}$ is an

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invariant set. In order to prove the boundness of the solution, two different cases will be discussed according to the initial value $x(0)$. To prove this conclusion, the following result is necessary. To equation

$$\frac{dx}{dt} = x\left(b_1 - a_{12}y - \alpha x^2\right) \le x\left(b_1 - \alpha x^2\right) , \tag{7}$$

we have $x(t) \le x(0)\exp[\int_0^t f(s)ds]$ where $f(s) = \left(\sqrt{b_1} - \sqrt{\alpha}x(s)\right)\left(\sqrt{b_1} + \sqrt{\alpha}x(s)\right)$.

Case I: $0 < x(0) < \sqrt{\frac{b_1}{\alpha}}$.

If $0 < x(0) < \sqrt{\frac{b_1}{\alpha}}$, there is $x(t) \le \sqrt{\frac{b_1}{\alpha}}$ holding for all positive t. If otherwise, there are two positive real numbers t_1 and t_2 ($t_2 > t_1$), where $x(t_1) = \sqrt{\frac{b_1}{\alpha}}$ and $x(t) > \sqrt{\frac{b_1}{\alpha}}$ hold for $\forall t \in (t_1, t_2)$. Then $\forall t \in (t_1, t_2)$, we can obtain

$$x(t) \le x(0)\exp[\int_0^{t_1} f(s)ds]\exp[\int_{t_1}^{t_2} f(s)ds], \tag{8}$$

where $x(0)\exp[\int_0^{t_1} f(s)ds] = x(t_1)$ and $f(s) < 0$ for $t \in (t_1, t_2)$. So $x(t) < x(t_1)$, which contradicts our hypothesis, and our original conclusion is correct.

Case II: $x(0) < \sqrt{\frac{b_1}{\alpha}}$.

Since $x(0) < \sqrt{\frac{b_1}{\alpha}}$, we have

$$x(t) \le x(0)\exp[\int_0^t f(s)ds] < x(0) , \tag{9}$$

where $x(t) \ge \sqrt{\frac{b_1}{\alpha}}$. If $x(t) \le \sqrt{\frac{b_1}{\alpha}}$,the conclusion clearly holds.

Therefore, we know that $x(t)$ is bounded in the above two cases. Assume M as the upper bound of $x(t)$ be, where $M = \max\left\{x(t), \sqrt{\frac{b_1}{\alpha}}\right\}$, for all t >0. Now, through the second equation of system (2), We can get

$$\frac{dy}{dt} = y\left(b_2 + a_{21}x - a_{22}y - \beta y\right) \le y\left(b_2 + a_{21}M - (a_{22} + \beta)y\right). \tag{10}$$

According to the theory of comparison, it is obvious that $y \le \frac{b_2 + a_{21}M}{a_{22} + \beta}$. From the above description, $x(t)$ is bounded. Then the model system (4) is dissipative.

3.2 Equilibria of Free System

It is obvious that system (4) has four equilibria $O(0,0)$, $E_1(0, \frac{b_2}{a_{22}+\beta})$, $E_2(\sqrt{\frac{b_1}{\alpha}}, 0)$ and $E_3(x^*, y^*)$ where

$$x^* = \frac{1}{2\alpha}\left(-\frac{a_{12}a_{21}}{a_{22}+\beta} \pm \sqrt{\left(\frac{a_{12}a_{21}}{a_{22}+\beta}\right)^2 - 4\alpha\frac{a_{12}b_2}{a_{22}+\beta} + 4\alpha b_1}\right), \tag{11}$$

and $y^* = \frac{a_{21}x^* + b_2}{a_{22}+\beta}$. O, E_1, E_2 are marginal equilibria, while E_3 is the only positive equilibrium in case of $\frac{a_{12}b_2}{a_{22}+\beta} - b_1 < 0$. In the following discussions, we assume the condition is satisfied consistently.

The Jacobian matrix of O is

$$J_{(0,0)} = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \tag{12}$$

where $D = b_1 b_2 > 0, T = b_1 + b_2 > 0,$ and $\Delta = T^2 - 4D = (b_1 - b_2)^2 \ge 0$, so $O(0,0)$ is an unstable node.

$$J_{E_1} = \begin{pmatrix} b_1 - \frac{a_{12}b_2}{a_{22}+\beta} & 0 \\ \frac{a_{21}b_2}{a_{22}+\beta} & -b_2 \end{pmatrix}, \tag{13}$$

where $D = -b_2\left(b_1 - \frac{a_{12}b_2}{a_{22}+\beta}\right) < 0$, so E_1 is a saddle.

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$$J_{E_2} = \begin{pmatrix} -2b_1 & -a_{12}\sqrt{\frac{b_1}{\alpha}} \\ 0 & b_2 + a_{21}\sqrt{\frac{b_1}{\alpha}} \end{pmatrix}, \tag{14}$$

where $D = -2b_1\left(b_2 + a_{21}\sqrt{\frac{b_1}{\alpha}}\right) < 0$, so E_2 is a saddle.

$$J_{E_3} = \begin{pmatrix} -2\alpha\left(x^*\right)^2 & -a_{12}x^* \\ a_{21}y^* & -\left(a_{22}+\beta\right)y^* \end{pmatrix}, \tag{15}$$

where $D = 2\alpha\left(x^*\right)^2\left(a_{22}+\beta\right)y^* + a_{12}a_{21}x^*y^* > 0$ and $T = -2\alpha\left(x^*\right)^2 - \left(a_{22}+\beta\right)y^* < 0$. So E_3 is a locally stable node or focus.

The trajectories in the nearby area of equilibrium point $E*$ is roughly as shown in Fig.2. Summarizing the above analysis, Table 1 shows the detail of the equilibria of the model (4).

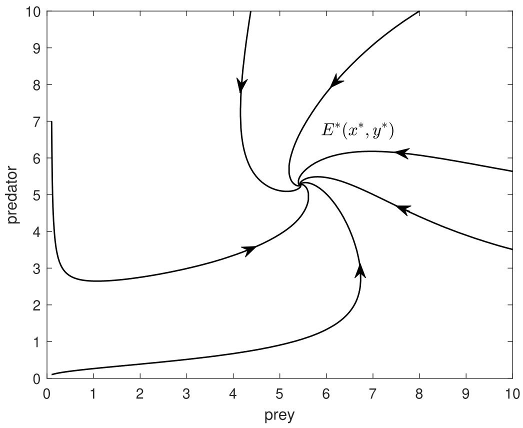

Figure 2: E_3 trajectory diagram.

Table 1. Stability of model (3.1) equilibrium

Equilibrium	Existence	Stability
O	Always exist	unstable node
E_1	Always exist	saddle
E_2	Always exist	saddle

E_3	$\frac{a_{12}b_2}{a_{22}+\beta}-b_1<0$	locally stable node or focus

3.3 Inexistence of Limit Cycle

From system (4), we can find that $y=\frac{-\alpha x^2+b_1}{a_{12}}$ and $x=0$ are vertical isoclinic lines. And the horizontal isoclinic lines are $y=\frac{a_{21}x+b_2}{a_{22}+\beta}$ and $y=0$. The bounded region will be selected following the succeeded statement (Fig.3).

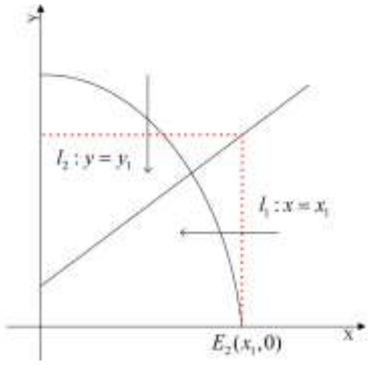

Figure 3: region G.

Let the straight line $l_1: x-x_1=0$ (where $x_1=\sqrt{\frac{b_1}{\alpha}}$), we have $\frac{dx}{dt}\Big|_{x=x_1}=-a_{12}x_1y<0$, if $y>0$. Hence the straight line l_1 is nontangent, and the trajectories of system (4) always travel from right side to the left of l_1. And let the straight line $l_2: y-y_1=0$, where $y_1=\frac{a_{21}x_1+b_2}{a_{22}+\beta}$, we have

$$\frac{dy}{dt}\Big|_{y=y_2}=y_2\left(b_2+a_{21}x-a_{22}y_2-\beta y_2\right)<0, \quad (16)$$

while $x\in(0,x_1)$. So the straight line l_2 is nontangent, the straight of system (4) from the upper of l_2 through l_2 into the lower. Therefore, the system (4) is bounded domain. It can be seen from system (4)

$$\begin{cases}\frac{dx}{dt}=x\left(b_1-a_{12}y\right)-\alpha x^3=P(x,y),\\ \frac{dy}{dt}=y\left(b_2+a_{21}x-a_{22}y\right)-\beta y^2=Q(x,y),\end{cases} \quad (17)$$

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Assume Dulac function $B(x,y)=\frac{1}{xy}$. It is obvious that $\frac{\partial(BP)}{\partial x}+\frac{\partial(BQ)}{\partial y}=-(\frac{2\alpha x}{y}+\frac{a_{22}}{x}+\frac{\beta}{x})<0$, according to Dulac Theory, we know the system (4) does not exist a limited cycle.

3.4 Global Asymptotically Stability

Let's assume that the Liapunov function is

$$V=\left[\left(x-x^*\right)-x^*\log\frac{x}{x^*}\right]+h\left[y-y^*-y^*\log\frac{y}{y^*}\right]. \tag{18}$$

We know $V>0, V\left(x^*,y^*\right)=0$, so wo can know

$$\begin{aligned}\frac{dV}{dt}&=\frac{x-x^*}{x}\frac{dx}{dt}+h\frac{y-y^*}{y}\frac{dy}{dt},\\&=(x-x^*)f(x)+h(y-y^*)g(x).\end{aligned} \tag{19}$$

Where

$$\begin{aligned}f(x)&=b_1-a_{12}y-\alpha x^2,\\g(x)&=b_2+a_{21}x-a_{22}y-\beta y.\end{aligned} \tag{20}$$

Consider the equilibrium equation

$$\begin{cases}b_1-a_{12}y^*-\alpha\left(x^*\right)^2=0,\\b_2+a_{21}x^*-a_{22}y^*-\beta y^*=0,\end{cases} \tag{21}$$

we can find that

$$\begin{aligned}\frac{dV}{dt}&=\frac{x-x^*}{x}\frac{dx}{dt}+h\frac{y-y^*}{y}\frac{dy}{dt},\\&=-\left[H(x,y)+G(x,y)+Q(x,y)\right],\end{aligned} \tag{22}$$

Where

$$\begin{aligned}H(x,y)&=\alpha\left(x-x^*\right)^2\left(x+x^*\right),\\G(x,y)&=h\left(a_{22}+\beta\right)\left(y-y^*\right)^2,\\Q(x,y)&=\left(a_{12}-a_{21}h\right)\left(x-x^*\right)\left(y-y^*\right).\end{aligned} \tag{23}$$

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While $\frac{dV}{dt} < 0$ in case of $h = \frac{a_{12}}{a_{21}}$, the system (4) is globally asymptotically stable.

IV. ORDER-1 PERIODIC SOLUTION

In this section, we will prove the existence, uniqueness and stability of the order-1 limit cycle of system (1).

Theorem 1 If $\frac{a_{12}b_2}{a_{22}+\beta} - b_1 < 0$ holds, for $\gamma > 1$, system (1) has an order-1 limit cycle.

Proof: In system, to any $\gamma > 1$, imagine set $N: x^* \le x, y = (1-q)y^*$, and impulsive set $M: y = y^*$, $0 \le x < x^*$. For the trajectory from point A_1, M_{A_1} is the intersection of the trajectory from point A_1 and the pulse set A_2 is the pulse point of M_{A_1}. We can find a point M_{B_1} in the pulse set, and the abscissa of M_{B_1} is ε. According to the properties of the trajectory, we can find the starting point M_{B_1} of B_1, where the x_{B_1} □ γx^*.Similarly, we can get the impulsive point B_2 of M_{B_1} (see Fig.4). So the successor function of A_1 is

$$F(A_1) = x_{A_2} - x_{A_1} > 0. \tag{24}$$

The successor function of B_1 is

$$F(B_1) = x_{B_2} - x_{B_1} < 0. \tag{25}$$

It's obvious that the following function is continuous. Following the successor function in preparatory knowledge, there is at least one point S between A and B such that $F(S)$ is equal to 0. That's the complete proof.

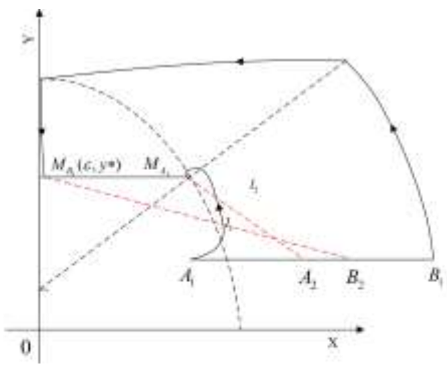

Figure 4: the successor function is monotonically decreasing.

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Theorem 2 If $\frac{a_{12}b_2}{a_{22}+\beta}-b_1<0$ holds, for $\gamma>1$, the order-1 limit cycle of system (1) is unique.

Proof: Find two arbitrary points in the phase set A_1 and B_1, and let $x_{A_1}<x_{B_1}$. There must be two trajectories from A_1 and B_1, denoted by l_1 and l_2 respectively (see Fig.5).

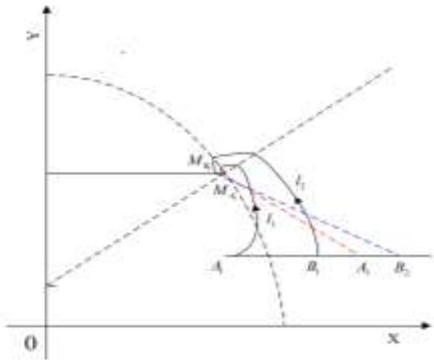

Figure 5: trajectory of controlled system (1).

These two trajectories can reach the pulse set M. We use M_{A_1} and M_{B_1} to represent the intersection point respectively. It is obvious that M_{B_1} must locate on the left side of M_{A_1}. Then the impulse function ϕ maps them to A_2 and B_2 respectively, where $x_{A_2}=\gamma x^*+x_{M_{A_1}}(1-\gamma)$ and $x_{B_2}=\gamma x^*+x_{M_{B_1}}(1-\gamma)$ .it is easy to obtain $x_{B_2}>x_{A_2}$. In order to make the whole pulse process more intuitive, we make appropriate simplification (see Fig.6).

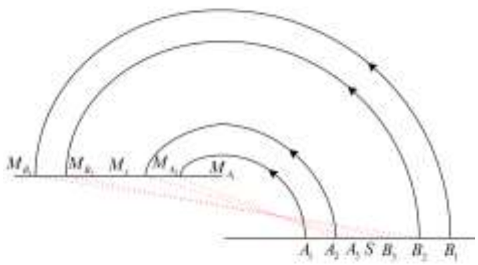

Figure 6: impulse diagram of A_n and B_n.

The points A_2 and B_2 are their subsequent points of A_1 and B_1 respectively. The points A_2 and B_2 have A_3 and B_3 as their subsequent points respectively. And so on, we can get sequence and A_n and B_n. which sequence A_n has upper bound and sequence has B_n lower bound. So we can get a nest of intervals

$$\{[A_n,B_n]\},n=1,2,3,\cdots. \qquad (26)$$

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According to the closed interval nest theorem in preparatory knowledge, we can know that there exists a point S belonging to $[A_n, B_n]$, such that the successor function $F(S)=0$ .i.e. the subsequent point of S' is point S . Therefore the order-1 limit cycle of system is unique. That's the complete proof.

Theorem 3 If $y^*(a_{22}+\beta)(q+\gamma-q\gamma) > b_2\gamma + a_{21}\xi_0\gamma$ and $\frac{a_{12}b_2}{a_{22}+\beta} - b_1 < 0$ holds, for $\gamma > 1$, system (1) has an orbitally asymptotically stable order-1 periodic solution.

Proof: Let the order-1 periodic solution of the system (1) be $SM_S S^+ = ((\xi(t), \eta(t)))$ and its period be T. Denote

$$\begin{aligned} S &= (\xi(0), \eta(0)) = (\xi_0, (1-q)y^*), \\ M_S &= (\xi(T), \eta(T)) = (\xi_1, y^*), \\ S^+ &= (\xi(T^+), \eta(T^+)) = (\xi_0, (1-q)y^*), \end{aligned} \tag{27}$$

where $\xi_0 = (1-\gamma)\xi_1 + \gamma x^*$. Calculate the Floquet multiplier through the analysis of Poincare criterion μ, we have

$$\begin{aligned} &P(x,y) = x\left(b_1 - a_{12}y - \alpha x^2\right), \\ &Q(x,y) = y\left(b_2 + a_{21}x - a_{22}y - \beta y\right), \\ &A(x,y) = \gamma(x^* - x), \\ &B(x,y) = -qy, \\ &C(x,y) = y - y^*. \end{aligned} \tag{28}$$

By calculation, we have

$$\begin{aligned} &\frac{\partial P}{\partial x} = (b_1 - a_{12}y - 3\alpha x^2), \\ &\frac{\partial Q}{\partial y} = b_2 + a_{21}x - 2(a_{22}+\beta)y, \\ &\frac{\partial A}{\partial x} = -\gamma, \frac{\partial A}{\partial y} = 0, \\ &\frac{\partial B}{\partial x} = \frac{\partial B}{\partial x} = 0, \\ &\frac{\partial C}{\partial x} = 0, \frac{\partial C}{\partial y} = 1. \end{aligned} \tag{29}$$

We also have $\Delta_1 = \frac{(1-\gamma)Q_+}{Q}$. From

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$$\int_0^T \frac{\partial P}{\partial x}(\xi(t),\eta(t))dt = \ln\left(\frac{\xi_1}{\xi_0}\right) - \int_0^T \left(2\alpha x^2\right)dt, \tag{30}$$

And

$$\int_0^T \frac{\partial Q}{\partial y}(\xi(t),\eta(t))dt = \ln\left(\frac{1}{1-q}\right) - \int_0^T \left(a_{22}+\beta\right)ydt, \tag{31}$$

then we have

$$\begin{aligned}\mu &= \Delta_1 \cdot \exp\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t))\right)dt \\ &\le \frac{(1-\gamma)Q_+}{Q}\frac{1}{1-q}\frac{\xi_1}{(1-\gamma)\xi_1+\gamma x^*} \\ &\le \frac{(1-\gamma)Q_+}{(1-q)Q}\end{aligned} \tag{32}$$

Substitute $Q(\xi(T^+),\eta(T^+))$ and $Q(\xi(T),\eta(T))$ into the above equation, then if condition

$$y^*(a_{22}+\beta)(q+\gamma-q\gamma) > b_2\gamma + a_{21}(\xi_1+\xi_0(\gamma-1)), \tag{33}$$

we have $|\mu|<1$. Therefore, the order-1 periodic solution of system (1) is orbitally asymptotically stable, if

$$y^*(a_{22}+\beta)(q+\gamma-q\gamma) > b_2\gamma + a_{21}\xi_0\gamma. \tag{34}$$

That's the complete proof.

V. NUMERICAL SIMULATION

In this section, we propose and discuss several special examples to test the theoretical results we have previously proved.

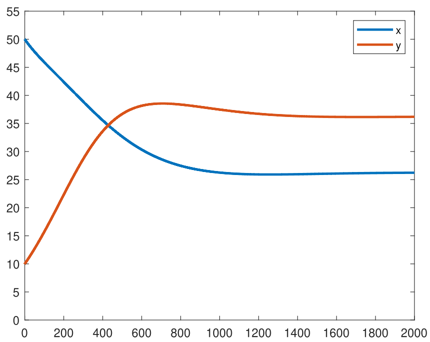

Figure 7: x and y with respect to t sequence graphs without impulses.

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Let $b_1 = 0.5, b_2 = 0.2, a_{21} = 0.06, a_{12} = 0.05, a_{22} = 0.05, \alpha = 0.5, \beta = 0.2$,the phase set and impulse set are $N = \{(x, y) | x^* \le x, y = (1-q)y^*\}$ and $M = \{(x, y) | y = y^*, 0 \le x < x^*\}$ respectively, the condition of pulse coefficient $\gamma > 1$ is satisfied. We take two groups of data of q and γ for comparison, the initial value point is always in the phase set, and the value of *x*-axis remains unchanged. In the absence of pulse, the initial point a is selected to simulate with MATLAB. Fig.7 is a time series diagram without impulse. Although it is stable, people can not get enough supplies of river crabs. Under the condition of impulsive control, the existence and stability of first-order periodic solutions are proved by numerical simulation. (see Fig.8, 9, 10, 11)

Through the simulation results of four groups of data, we can find the following conclusions. First, we can find that the order-1 periodic solution does not depend on the value of p from Fig. 8 and Fig. 9 (or Fig. 10 and Fig. 11) Second, the stable position of the order-1 periodic solution is related to γ, but it is nolinear. (see Fig. 8 and Fig. 10) Through the analysis of these data, we can get some information in the breeding process. In the process of crab breeding, the higher the value of q the more we harvest. However, according to the biological significance, we can know that the maximum value of q can only be taken as 0.5. If the value of *q* exceeds 0.5, the normal growth and reproduction of river crab will be affected [23, 24, 25]. At the same time, the smaller the value γ, the better the case will be. On the one hand, it can reduce the working time of breeding personnel, on the other hand, it can reduce the cost of bait. Only in this way can we get the maximum economic benefits.

VI. CONCLUSION

The system (1) describes the whole process of crab culture and harvest in toxin environment, which describes the harvest of river crab with impulsive mode. As we all know, the individual weight has a great impact on the price of the crab, thus we take this important factor into account when building our model. We start harvesting when the crab population reaches the threshold we set the second time instead of harvesting when it reaches the threshold first time. This can not only ensure the quantity of crab when harvest, but also ensure the weight of a single crab. From the previous detailed proof, it can be seen that the state feedback impulsive control model (1) has asymptotically stable order-1 periodic solution which means that we can achieve long-term harvest by setting a proper threshold and manual intervention, and this operation can supply sustainable economic benefits.

So we can conclude that if the crabs are in toxic waters for a long time, the population will be affected. We set a threshold for the number of river crabs. When the number of river crabs exceeds this threshold, we will start to harvest mature river crabs through the state feedback pulse model for adjustment. By studying the uniqueness of the first-order periodic solution, the feasibility of increasing the yield of river crab and obtaining higher economic benefits are explored. This is of great practical significance to maintain the profits of aquaculture households and the stability of market economy.

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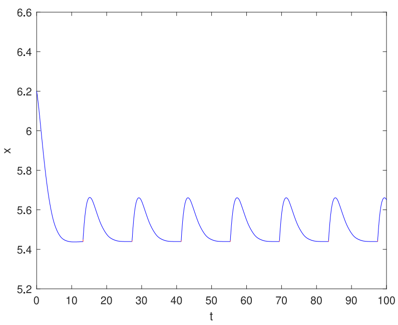
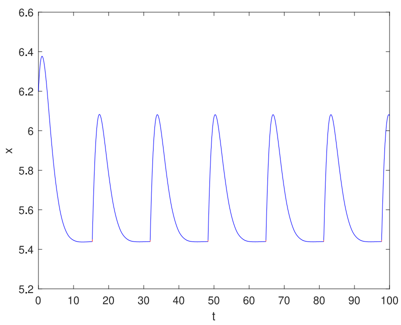
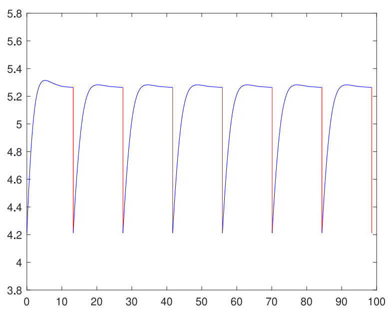


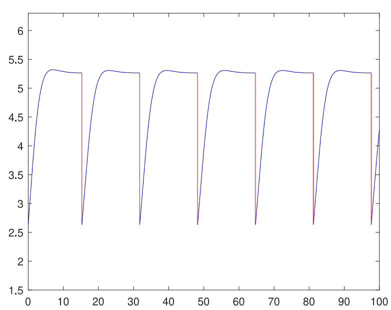
(8a) Time series diagram of x and t.



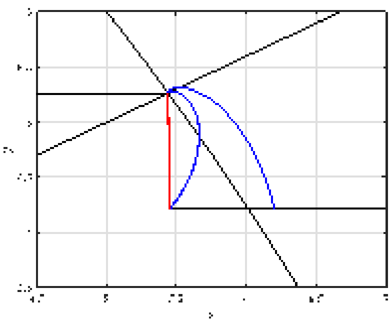
(9a) Time series diagram of x and t.



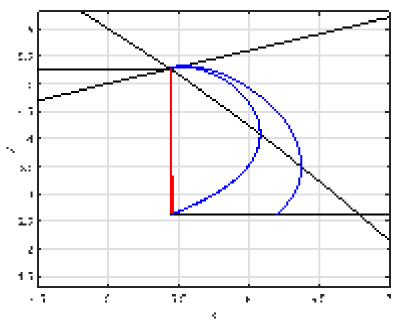
(8b) Time series diagram of y and t.



(9b) Time series diagram of y and t.



(8c) Phase diagram of the system (1).



(9c) Phase diagram of the system (1).

Figure 8: the phase diagram and time series diagram of the system (1), the initial value point is $(6.2, 0.8y^*)$, $p = 0.2$ and $\gamma = 10$.

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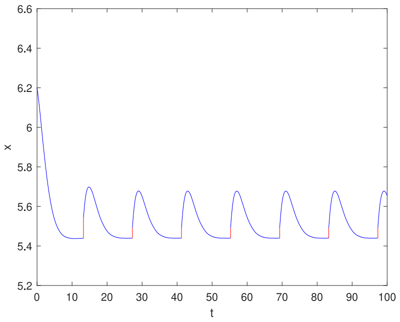
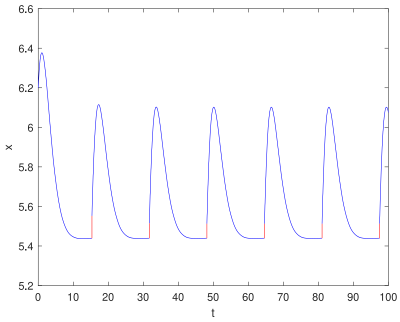
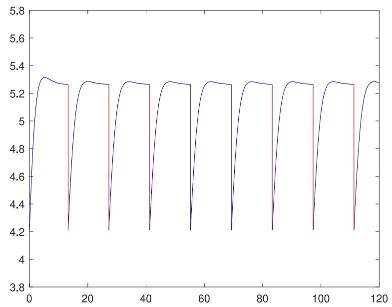
Figure 9: the phase diagram and time series diagram of the system (1), the initial value point is $(6.2, 0.5y^*)$, $p = 0.5$ and $\gamma = 10$.



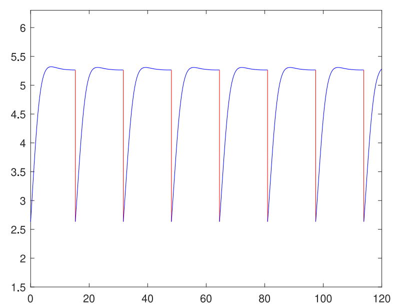
(10a) Time series diagram of x and t.



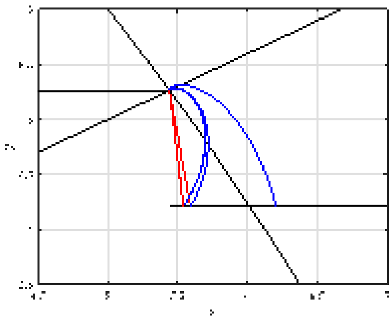
(11a) Time series diagram of x and t.



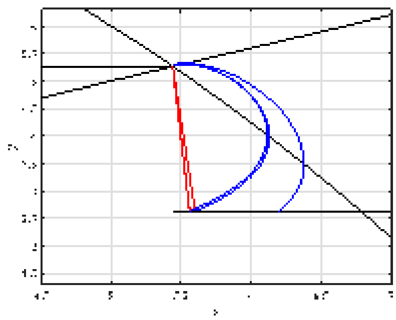
(10a) Time series diagram of y and t.



(11a) Time series diagram of y and t.



(10c) Phase diagram of the system (1).



(11c) Phase diagram of the system (1).

Figure 10: The phase diagram and time series diagram of the system (1), the initial value point is $(6.2, 0.8y^*)$, $p = 0.2$ and $\gamma = 100$.

Figure 11: The phase diagram and time series diagram of the system (1), the initial value point is $(6.2, 0.5y^*)$, $p = 0.5$ and $\gamma = 100$.

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