

Solving Integrated Scheduling Problem with Precedence Constraint and Machine-State-Related Setup Time based on Improved DE

Xiaowei Zhang, Zhiqiang Xie, Yingchun Xia*

School of Computer Science and Technology, Harbin University of Science and Technology, Harbin, Heilongjiang, China

*Corresponding Author.

Abstract:

Integrated Scheduling Problem (ISP) addresses a production mode for personalized and trial products with large differences in product structure and process parameters. ISP flexibly arranges operations according to BOM sequence. As long as the conditions are met, the operation can be processed or assembled without waiting for a unified production beat. Due to the influence of temperature, precision machines need to be warmed up after a period of inactivity in order to ensure machining accuracy. The warm-up time of a machine can be considered as a kind of preparation time related to the state of the machine. To solve the ISP considering precedence constraint (PC) between operations and machine-state-related setup time (MSRST), a mathematical model is developed to minimize the total tardiness. To deal with the PC, a differential update operator (DUO), which can ensure that the operations subject to the PC, is proposed. To satisfy the MSRST, a decoding method based on the machine idle signal strategy (MIS) is proposed. At last, the improved differential evolution (DE) algorithm based on DUO and MIS is designed to solve the problem. The operation research optimizer OR-Tools is used as the control algorithm. Experimental results show that the proposed algorithm is effective and feasible and that it can get similar results to the control algorithm in a very short time.

Keywords: *Integrated scheduling, Precedence constraint, Setup time, Deferential evolution.*

I. INTRODUCTION

The problem of production scheduling has a long history and is closely related to people's life. With the advent of the modern industrial revolution, the manufacturing problem has evolved from a simple scheduling problem in the handicraft era to a complex batch product scheduling problem in the factory. In fact, the research on production scheduling problems can be traced back to Johnson's work on the FSP (Flow-shop Scheduling Problem) for two machines. In this work, Johnson gave a specific mathematical model, proposed the famous Johnson's scheduling rule and proved the applicability and global optimality

of Johnson's rule. However, in practice, production scheduling problems are often very complex, with a wide range of constraints, resulting in algorithms for one type of scheduling problem requiring extensive modifications before they can be applied to another type of scheduling problem. Therefore, it is of practical and theoretical interest to design scheduling algorithms for a variety of practical situations.

JSP (Job-shop Scheduling Problem) and FSP scheduling are the most common production scheduling problems and they are suitable for products with similar structures and parameters. There are many variants and derivatives of this type of problem, such as the flexible job shop scheduling problem in the literature [1], no-wait job shop scheduling in the literature [2], dynamic job shop scheduling in literature [3] and hybrid flow shop scheduling in the literature [4]. However, the scheduling algorithms mentioned above can only be applied to batch products with similar structures and parameters. The mathematical model in these algorithms assumes that each workpiece is a serial machining task and that there are no constraints between the workpieces, so the mathematical model of these algorithms no longer reflects the overall BOM structure of the product. This production model is suitable for the production of small batches or single parts with a similar structure.

With the development of the economy, consumers pay more and more attention to experience. Products without differentiated and personalized characteristics are difficult to attract consumers. At this time, some scholars have studied the ISP (Integrated Scheduling Problem) considering both processing and assembly [5-7]. ISP fully considers the characteristics of personalized products and organizes the operations according to the precedence constraint of the product BOM. It can flexibly arrange the processing and assembly operations without waiting for the predefined process sequence. Personalized products are characterized by a wide variety, widely varying structures, inconsistent process requirements and almost always single-piece product quantities. These factors make ISP production scheduling more suitable than classic JSP for the production of such products with widely varying structures and parameters.

In practice, some machines may require a start-up process such as warming up before it is put into use, for example, a soldering furnace in a wave soldering process or a press in a tire curing process. However, keeping the machine in operation at all times during discrete production activities can significantly increase energy consumption and incur unnecessary costs. To cope with this production scheduling demand, the literature [8] investigates a job shop scheduling problem with warm-up times. Literature [8] modelled warm-up time as setup time related to machine state and proposed a genetic algorithm to solve the model, but the model targeted the FSP scheduling problem and did not consider the BOM precedence constraint. The literature [9] investigated how to reduce the setup time of grinding machines during stick production. The setup time in this study is a fixed value and does not take into account the influence of the machine state on the setup time. The literature [5-7] considers the precedence constraint but not the setup time. The literature [10] considers both precedence constraint and setup time, but its setup time is a fixed value and does not take into account the fact that the effect of the initial state of the machine on the setup time is not taken into account.

In this work, an integrated scheduling model considering both machine-state-related setup time and precedence constraint is developed. To deal with the PC constraint, a differential update operator, which can ensure that the operations subject to the PC after DUO is acted, is proposed. In order to satisfy the machine-state-related setup time (MSRST), a decoding method based on the machine idle signal strategy (MIS) is proposed. At last, a differential evolution algorithm based on DUO and MIS is designed to so the problem.

II. PROBLEM DESCRIPTION

This work investigates an integrated scheduling problem that considers both machine-state-related setup time and precedence constraint. For ease of description, the following assumptions need to be made about the problem.

1. The set $\{P_1, P_2, \dots\}$ represents all products. A product consists of the operations shown in Fig. 1. The data in the boxes in the figure indicate operation/machine/time, e.g., operation v_1 can be processed flexibly on machine d_1 or d_5 for 9h and 5h respectively. The structure and processing parameters vary significantly from product to product. By considering both machining and assembly, it is not necessary to wait for a predefined process sequence, as long as the conditions are met the next operation can be performed, i.e., the operations are subject to the precedence constraint. Any 2 operations subject to the precedence constraint need to satisfy the constraint shown in equation (1).

$$\forall v_i \prec v_j : ct_i \leq st_j \tag{1}$$

where, $v_i \prec v_j$ indicates a precedence constraint, which means that the completion time ct_i of operation v_i cannot be later than the start time st_j of operation v_j .

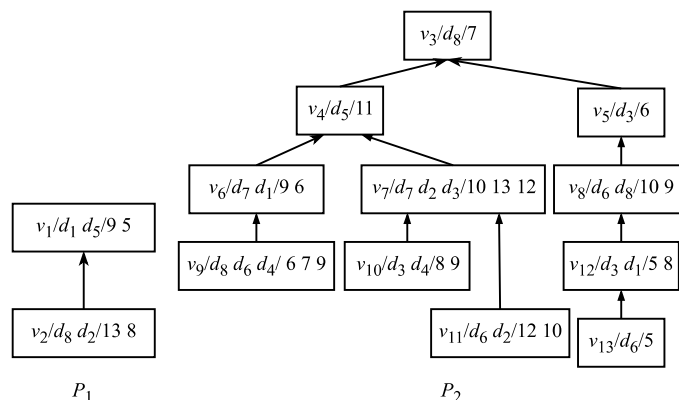


Fig 1: An instance of the precedence constraint

2. The set $\{d_1, d_2, \dots\}$ represents a collection of processing machines. A machine can only handle one operation at a time. Unless the operation is completed, the machine cannot undertake the production tasks

of other processes. From the perspective of Gantt chart, two adjacent operations on the same machine are not allowed to overlap in time, as shown in equation (2).

$$\forall v_i, v_{i+1} \in d_k : ct_i \leq st_{i+1} \quad (2)$$

3. An operation can only select one machine among the flexible machines. The symbol $s_{ik} = 1$ indicates that operation v_i is assigned to process on machine d_k , otherwise $s_{ik} = 0$. Therefore, for any operation, its machine decision variable s_{ik} needs to satisfy the constraint shown in equation (3).

$$\forall v_i : \sum_k s_{ik} = 1 \quad (3)$$

4. The set $\{A_1, A_2, \dots\}$ denotes the delivery time of the product $\{P_1, P_2, \dots\}$. Then, the tardiness of product P_k is defined as shown in equation (4).

$$due(P_k) = \max\{\max\{ct_i | v_i \in P_k\} - A_k, 0\} \quad (4)$$

where, the completion time of the last operation of the product is the completion time of that product, and the completion time of the last operation is the maximum of the completion times of all operations of that product.

5. If an operation is to be processed with high precision on one machine, the machine needs a certain amount of setup time, and only after this can the process begin. The length of the setup time is the machine-state-related setup time, i.e., the machine can only start working when it is in full state and no operation can be processed until it is in full state.

To describe this machine-state-related setup time, a high-precision machine tool in a workshop is used here as an example. In such workshops, the machining of high-precision products is often encountered, where not every part of the product needs to be machined with high precision, but where the precision of those highly demanding parts needs to be guaranteed. For example, some precision machines require consideration of the thermal balance affecting machining accuracy. Especially affected by the climate and temperature, if the idle waiting time of the precision machine tool exceeds a certain time after processing, the machine needs to be preheated for a period of time when it starts working next time to reduce the impact of thermal expansion and cold contraction on the processing accuracy. The warm-up time of the machine tool is affected by its idle waiting time. For example, TABLE I and Fig. 2 show the relationship between idle waiting time and preheating time of machines in a workshop.

TABLE I. Relationship between waiting time and preheating time

waiting time/h	setup time/min
less 1	0
less 3	15
greater or equal 3	30

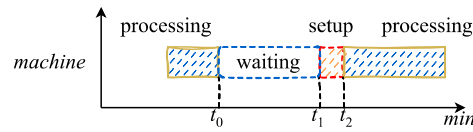


Fig 2: Schematic diagram of waiting time and setup time

In Fig. 2, an operation is completed at t_0 , and the materials for the next operation have not yet arrived. The machine is waiting for processing. At t_1 , a new operation arrives, but it needs high-precision processing, so the machine needs a certain setup time to start processing. The length of the setup time t_2-t_1 is related to the size of the waiting time t_1-t_0 . When the setup is completed, the new operation starts at t_2 . Therefore, for two adjacent operations on the same machine, they need to meet not only the constraint relationship shown in equation (2), but also the constraint relationship shown in equation (5).

$$\forall v_i, v_{i+1} \in d_k \wedge v_{i+1} \in high : ct_i + waiting + setup \leq st_{i+1} \tag{5}$$

where v_{i+1} is an operation that requires high precision processing and the waiting time and setup time are shown in Fig. 2.

Based on the above description, the objective function of the integrated scheduling problem considering both machine-state-related setup time and precedence constraint is shown in equation (6).

$$\min(f_1) = \min(\sum due(P_k)) \tag{6}$$

where f_1 denotes the total tardiness time for all products.

III. IMPROVED DIFFERENTIAL EVOLUTION ALGORITHM BASED ON DUO AND MIS

Storn and Price proposed a simple and efficient differential evolution for global optimization over continuous spaces [11]. The algorithm utilizes the differential between individuals to perturb the target individual, then a new population is obtained through a crossing and selection mechanism. However, the above differential evolution method can only be used in continuous optimization problems. The integrated scheduling problem with the precedence constraint and machine-state-related setup time is a discrete optimization problem. So, the framework of the differential evolution method must be designed for this case. The following improvements of the differential evolution are proposed to solve the integrated

scheduling problem.

2.1 The Encoding Method based on Random Schedulable Operation

By observing the precedence constraint shown in Fig. 1, it can be found that the leaf node is a schedulable operation. Randomly select an operation from the schedulable operations, put it into the encoding queue Q , and then update the schedulable operations. A complete operation string can be obtained by repeating the above process. For the precedence constraint shown in Fig. 1, the difficulty is the encoding of the operation sequence, whereas the encoding of the machine is simple and will not be repeated.

2.2 Differential Update Operator

In this work, a differential update operator (DUO) is proposed to simulate the mutation of the differential evolution, as shown in equation (7)-(9).

$$B = F(X_{r_2}(g) - X_{r_3}(g)) \quad (7)$$

$$V_i(g+1) = B(X_{r_1}(g) + X_i(g)) \quad (8)$$

$$X_i(g+1) = \text{fit}(X_i(g)) < \text{fit}(V_i(g+1)) ? V_i(g+1) : X_i(g) \quad (9)$$

where, individual r_1 is selected by roulette wheel selection from g -th generation population, and individual r_2 and r_3 is selected by complete random from g -th generation population, $i \neq r_1 \neq r_2 \neq r_3$. B is Hemming distance vector, which means whether the operations at the same location are the same, with 1 meaning different and 0 meaning the same. The scaling factor F randomly keeps the count of 1 in the vector B below a half. Equation 8 indicates the process of controlling the transmission of information from individual r_1 to individual i according to the 0-1 vector B . The $\text{fit}(X)$ indicates the fitness of the individual X .

To implement the main setup of DUO shown in equation (8), the following method is proposed.

Method 1 equation (8) implement.

Input: $X_{r_1}(g)$, $X_i(g)$, B .

Output: $V(g+1)$.

Step 1 Set 0-value array $Visited_{r_1}$ and $Visited_i$ with the same count as vector B .

Step 2 for index w in B do:

if $B[w] = 0$ do:

index the first position in $Visited_i$ where value is equal to 0, assume it is p

$V(g + 1)[w] = X_i(g)[p]$;

$Visited_i[p] = 1$;

index the position in $X_{r1}(g)$ where value is equal to $X_i(g)[p]$, assume it is q ;

$Visited_{r1}[q] = 1$;

else do

index the first position in $Visited_{r1}$ where value is equal to 0, assume it is q ;

$V(g + 1)[w] = X_{r1}(g)[q]$;

$Visited_{r1}[q] = 1$;

index the position in $X_i(g)$ where value is equal to $X_{r1}(g)[q]$, assume it is p ;

$Visited_i[p] = 1$;

end if

end for

Step 3 output $V(g + 1)$.

2.3 Decoding Method Based on the Machine Idle Signal Strategy

In order to satisfy the machine-state-related setup time and precedence constraint at same time, a decoding method based on the machine idle signal strategy (MIS) is proposed, the main steps are shown as follows.

Method 2 MIS decoding method.

Input: n operations, m machines, operation string V and machine string M .

Output: solution S .

Step 1 Set queue Q_Idle and queue $Q_Dispatched$ for each machine. For instance, $Q_Idle[d_k]$ indicates the set of operations assigned to machine d_k . Place all operations in Q_Idle according to V and M .

Step 2 Traversing each machine at idle signal time t_k . Taking machine d_k as an example, we first check whether the d_k has released a new schedulable process at moment t_k , then determine whether the head of $Q_Idle[d_k]$ is schedulable, and then determine whether high-precision is required.

If the head operation of $Q_Idle[d_k]$ does not require high-precision processing, the start time of this operation is determined to be t_k , and the operation is dequeued from $Q_Idle[d_k]$ and joins $Q_Dispatched[d_k]$. Mark the next idle signal time of d_k as the completion time of the operation.

If the head operation of $Q_Idle[d_k]$ does require high-precision processing: the setup time is determined from TABLE I and Equation (5), and then insert a virtual setup operation, which start time is t_k , into $Q_Dispatched[d_k]$. At last, insert the head operation of $Q_Idle[d_k]$ into $Q_Dispatched[d_k]$. Mark the start time of the head operation of $Q_Idle[d_k]$ as the completion time of the virtual setup operation. Update the next idle signal time of d_k .

Step 3 The next idle signal time t_{k+1} according to the idle time of each machine, and the busy status of each machine is updated. The above process is repeated until the Q_Idle of all machines is empty, and the scheduling scheme is output at this time.

For ease of understanding, this is illustrated with the data in Fig. 1 and TABLE I. Assume that there are operation string $F = \{v_9, v_2, v_1, v_{10}, v_{13}, v_6, v_{11}, v_{12}, v_7, v_4, v_8, v_5, v_3\}$ and machine string $M = \{d_6, d_8, d_1, d_4, d_6, d_1, d_6, d_3, d_2, d_5, d_6, d_3, d_8\}$, where operation $\{v_1, v_2, v_5, v_6, v_7, v_8, v_9\}$ have high accuracy requirements. Assume that product P_1 in Fig. 1 has a due date of 25h and P_2 has a due date of 55h. As the decoding process is shown in Fig. 3.

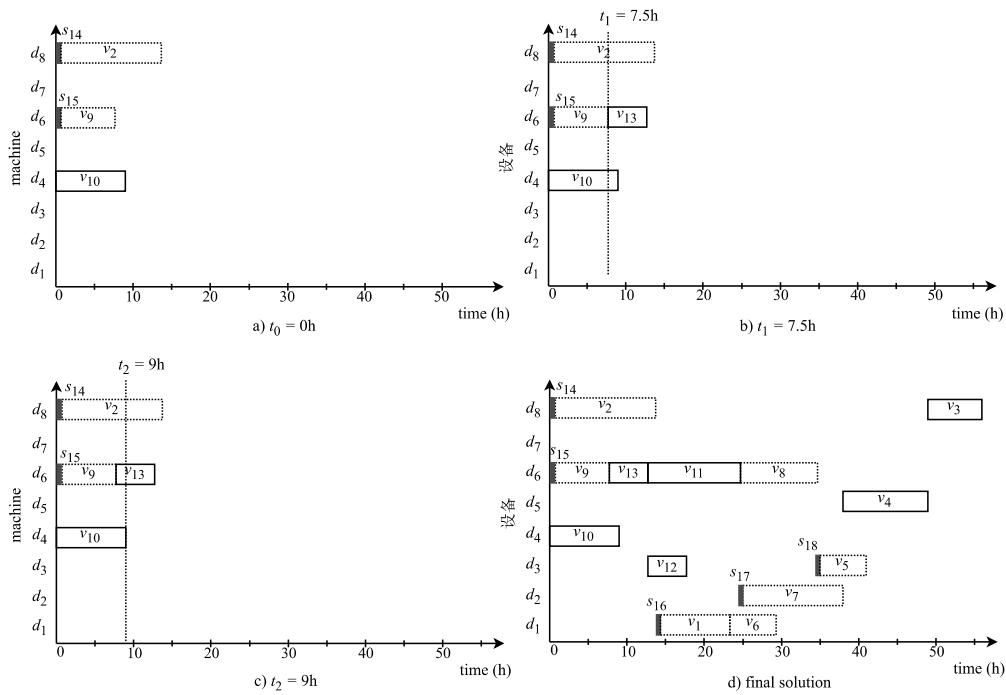


Fig 3: An instance of the decoding method

Then, at the moment $t_0 = 0h$, all machines are traversed.

d_1 is idle, but $Q_Idle[d_1] \rightarrow first = v_1$ is non-schedulable.

d_5 is idle, but $Q_Idle[d_5] \rightarrow first = v_4$ is non-schedulable.

d_8 is idle, $Q_Idle[d_8] \rightarrow first = v_2$ is schedulable and has high accuracy requirements. As $t_0 = 0h$, it needs the maximum warm-up time. Based on the data in TABLE I, we set the length of the virtual setup operation to 0.5h, which is s_{14} , i.e., the start time is 0h and the finish time is 0.5h, and put s_{14} into $Q_Dispatched[d_8]$. Set the start time of v_2 to 0.5h and the finish time to 13.5h, and v_2 is dequeued from $Q_Idle[d_8]$ and inserted into $Q_Dispatched[d_8]$. d_8 's next idle signal time is 13.5h.

d_2 is idle, but $Q_Idle[d_2] \rightarrow first = v_7$ is non-schedulable.

d_3 is idle, but $Q_Idle[d_3] \rightarrow first = v_{12}$ is non-schedulable.

d_7 has an empty Q_Idle .

d_6 is idle and $Q_Idle[d_6] \rightarrow first = v_9$ is a schedulable operation with a high accuracy requirement. Insert a virtual setup operation s_{15} , with a start time of 0h and a completion time of 0.5h, and place s_{15} into $Q_Dispatched[d_6]$. Set the start time of v_9 to 0.5h and the finish time to 7.5h, and place v_9 into

Q_Dispatched[d_6]. d_6 's next idle signal time is 7.5h.

d_4 is idle and Q_Idle[d_4]->first = v_{10} is a regular operation, directly set v_{10} with start time 0h and finish time 9h. d_4 's next idle signal time is 9h.

At this moment all machines at t_0 are processed, as shown in Fig. 3a. Calculate the next earliest idle signal time, obviously $t_1 = 7.5$ h.

At the moment $t_1 = 7.5$ h. Due to v_9 having been finished, operation v_6 becomes schedulable. Similarly, the scheduling result at moment t_1 can be seen in Fig 3b. Fig. 3c shows the scheduling result when $t_2 = 9$ h. Fig. 3d shows the final solution.

IV. SIMULATION EXPERIMENT

In order to verify the effectiveness of the proposed differential evolution algorithm based on DUO and MIS in this work, an instance X1 was collated from a real workshop. Where X1 contains 5 products, as the DOT format data are shown in Appendix. In the instance X1, shown in Appendix, product P_1 is v_1 to v_{11} with a due time of 60h; product P_2 is v_{12} to v_{27} with a due time of 70h; product P_3 is v_{28} to v_{49} with a due time of 85h; product P_4 is v_{50} to v_{55} with a due time of 30h and product P_5 is v_{56} to v_{75} with a due time of 80h. The operations $\{v_1, v_2, v_4, v_6, v_{14}, v_{15}, v_{17}, v_{18}, v_{29}, v_{30}, v_{44}, v_{47}, v_{49}, v_{50}, v_{61}, v_{64}, v_{68}, v_{70}\}$ have high precision demands.

The following method was used as a control method for the experiment: the objective functions and constraints shown in equations (1) to (6) were transformed into a mathematical model that could be processed by the operations research optimizer OR-Tools. And after obtaining a mathematical model that OR-Tools can handle, the instance X1 is solved using OR-Tools for 12h and then the result is output and recorded as OR-12. The OR-12 result is then compared with the proposed method. The population size of the proposed differential evolution is 100, max generation is 150.

Fig. 4 is the Gantt chart of instance X1 according to the proposed method in this paper, where the total tardiness is 31.25h and the total setup time is 0.75h. The virtual setup operation is s_{76} and s_{77} , where the tardiness of P_1 is 2h, 11h for product P_2 , 5h for product P_3 , 12h for product P_4 , and 1.25h for product P_5 .

Fig. 5 shows the Gantt chart obtained by the OR-12 method, where the total tardiness is 26h and the total setup time is 0h. The tardiness of product P_1 is 1h, P_2 is 9h, P_3 is 6h, P_4 is 4h and P_5 is 6h.

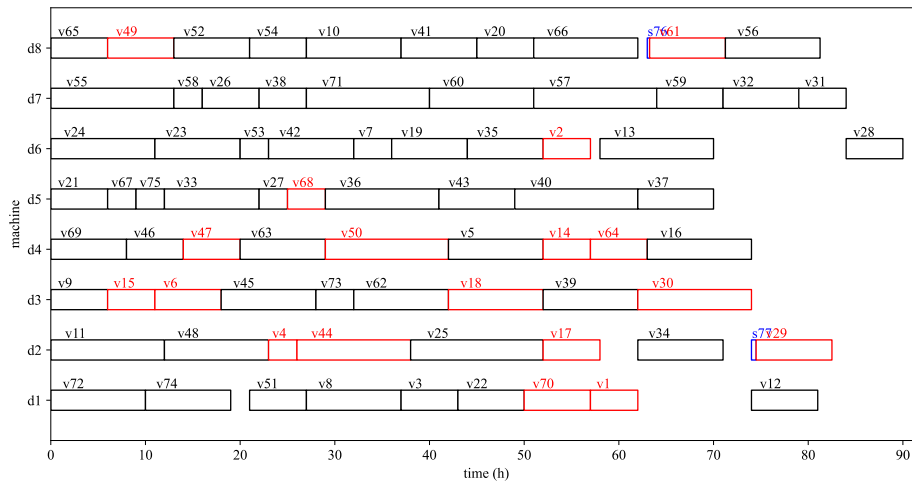


Fig 4: Gantt chart of the instance X1 by proposed method

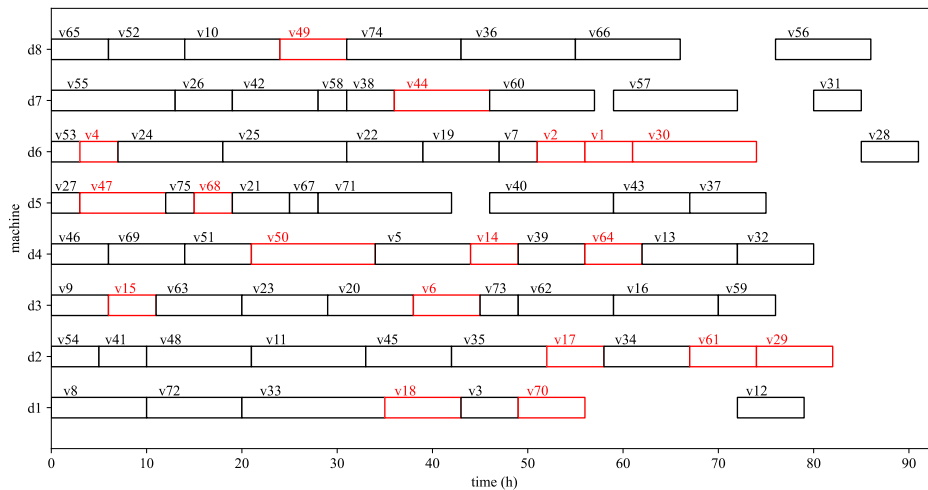


Fig 5: Gantt chart of the instance X1 by OR-12 method

In the result of the algorithm proposed in this work, the tardiness for P_3 and P_5 are shorter than OR-12. However, for the most influential product P_4 , the proposed algorithm achieves tardiness of 12 h, which is higher than the 4 h obtained by the OR-12 method. Because of its very high accuracy as an operational optimizer, it is appropriate to use OR-12 as a reference for comparison. X1 was run independently 10 times by the proposed method and the results shown in Fig. 4 were obtained 8 times, indicating that the algorithm proposed in this paper is highly stable and can achieve similar scheduling quality to the OR-12 method in only a few tens of seconds.

V. CONCLUSION

In this work, we investigate the integrated scheduling problem considering both precedence constraint and machine-state-related setup time and proposed a mathematical model for the problem. This work takes into account integrated scheduling problem while other scheduling studies focused on Job shop or Flow

shop in which there is no precedence constraint and machine-state-related setup time. The proposed DUO provides a population updating strategy without parameters. The proposed MIS can decode the operation string to a solution that satisfies precedence constraint and machine-state-related setup time. Compared with the OR-12 method, the proposed differential evolution algorithm based on DUO and MIS in this paper can give similar results to it in a very short time.

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